NEAR-SURFACE VELOCITY RECONSTRUCTION
USING SURFACE WAVE INVERSION

by

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A thesis submitted to the faculty of
the University of Utah
in partial fulfillment of the requirements for the degree of

Master of Science
in
Geophysics

Department of Geology and Geophysics
The University of Utah
June 1990
The University of Utah Graduate School

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ABSTRACT

This thesis examines the feasibility of inverting surface waves in common shot point (CSP) seismic data for S- and P-wave velocities. Several approaches to surface wave inversion are examined: 1) separate Love and Rayleigh wave inversions for S- and P-wave velocities; 2) Love wave inversion for S-wave velocities that is then followed by Rayleigh wave inversion for S- and P-wave velocities; and 3) Love wave inversion for S-wave velocities that are then used in Rayleigh wave inversion for P-wave velocities. Inversion of synthetic data suggests that a combination of Love and Rayleigh wave inversion will provide the best results, especially if the Love waves are first used to reconstruct the S-wave velocities and the Rayleigh waves are then used to recover the P-wave velocities. Results also suggest that S-wave velocities inverted from Love waves may be more reliable than those from Rayleigh wave inversion, and that combining Love and Rayleigh wave inversion will provide the most accurate S-wave velocity reconstruction from field data. Density determination appears impractical using surface wave inversion.

To verify the practicality of surface wave inversion, a nine-component surface wave experiment was performed in northeast Texas (courtesy of Arco Research). Tau-p and Fourier transforms are applied to the YY (cross-line component source recorded on cross-line component geophones) and ZZ (z-component source recorded on z-component geophones) CSP gathers to recover Love and Rayleigh wave dispersion curves respectively. These curves show clearly the fundamental and higher harmonic modes from about 2 Hz to 14 Hz. Inversion of the fundamental mode data suggests shear velocities of 180 m/s near the surface with an abrupt change to 400 m/s within the first 10 meters followed by a gradual increase to 650 m/s at a depth of 65 meters. This is in good agreement with the shear velocities measured from vertical seismic profile (VSP) data. Results suggest that either Love or Rayleigh wave inversion can be used to provide shear wave statics and near-surface layer velocities, but that a combination of the two can improve the reliability of the velocity reconstruction.
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I wish to acknowledge the assistance and advice of many people who contributed in making this thesis possible.

Dr. Schuster, Dr. Arabasz, and Dr. Bruhn, as members of my thesis committee, were helpful in critiquing and editing of this paper. Each provided input to this thesis, and their suggestions were exceedingly helpful.

Dr. Schuster, as my thesis advisor, was especially helpful with critiquing the writing style of this thesis.

I am indebted to Nigel Watrus and Jim DiSiena of Arco Research for their assistance in this project, and to Arco Oil and Gas Company for allowing participation in its nine-component surface wave experiment.

This research was funded primarily by the University of Utah Tomography Development Project, and by a special educational grant from Arco. Sponsors of the Tomography Development Project include: Amoco, Arco, British Petroleum, Chevron, Conoco, Exxon, Marathon, Mobil, Oyo, Phillips, Texaco, and the Gas Research Institute.
CHAPTER 1

INTRODUCTION

In many common depth point (CDP) seismic data sets, poor delineation of near-surface velocities contributes to a variety of processing problems including loss of data coherency, cycle skips, and false structures. Too often, residual statics, refraction statics, and/or hand statics fail to remedy these problems. This difficulty is especially severe in CDP shear wave surveys, which lack uphole velocity control and where the shear wave static corrections can be many times larger than the compressional static corrections. The inability to correct for these static errors can lead to erroneous structural and stratigraphic interpretations. It is therefore desirable to develop a method that could provide independent and accurate constraints on the velocity structure above the first good seismic reflector. It would also be desirable if this method could be applied to existing CDP seismic data.

Such a method may be velocity inversion from surface wave data. This is a compelling choice because surface wave amplitudes are usually an order of magnitude larger than those of other events in a CDP shot record. Moreover, they are primarily influenced by the near-surface shear velocities, which are the primary culprits in shear wave static problems.

There are several advantages inherent in velocity inversion of surface wave data. First, since surface waves penetrate to a depth of several wavelengths, velocities can be reconstructed at depths much greater than those evaluated with uphole times. Second, because the surface waves are recorded at many geophone stations, the subsurface can be much more densely sampled than uphole measurements would allow. Third, surface waves may reveal detail in layered rocks such as lava flows, clinker beds, or near-surface limestone formations that are notoriously difficult for conventional statics methods. Finally, the results are completely independent of other statics methods such as uphole or refraction, thus providing an additional source of information about the critical shallow velocities. The main disadvantage is the assumption of homogeneous lateral velocities over the length of the shot.
record. Hence, it is a pseudo one-dimensional inversion method.

**Previous Work**

For years, seismologists have inverted earthquake-generated Love and Rayleigh waves to investigate S-wave velocity distributions on a crustal scale (Schwab and Knopoff, 1972; Braille and Keller, 1975). Dorman and Ewing (1962) were the first to invert dispersion curves extracted from surface waves for S-wave velocities. Since then, most research has been directed to the inversion of either Love or Rayleigh wave dispersion curves interpreted from either phase or group velocities. The relatively few studies using both Love and Rayleigh waves are surprising in light of the complementary information available from each. More recently, several researchers have begun using surface wave inversion in the investigation of engineering scale problems for depths of less than 20 meters (Nazarian and Stokoe, 1986; Barrows and Gahr, 1987; Song et al., 1989). However, the usefulness of surface waves on a scale of interest to the oil industry has been investigated and described in only a few published papers (Mokhtar et al., 1988; Watrups, 1989). In particular, the inversion of combined Love and Rayleigh wave information is virtually unexplored. These data may have an important bearing on near-surface anisotropy, S- and P-wave statics, and earthquake hazard site evaluation.

One recent example of surface wave inversion of engineering seismic data is the work of Song et al. (1989). Five layers within 20 meters of the surface were first identified using refraction data. Three inversion procedures were then carried out, of which the Love wave inversion for S-wave velocities was the most accurate. The S-wave velocities inverted from two independent Rayleigh wave data sets, differing mostly in geophone dominant frequency and station spacing, showed considerable discrepancies with each other as well as with the velocities measured from refraction data. It was demonstrated that a Rayleigh wave study designed for higher frequencies (40 Hz geophones and 0.61 m (2 ft) station spacing) tended to resolve only shallow velocities while a lower frequency design (10 Hz geophones and 9.14 m (30 ft) spacing) recovered only large scale formation details. One section on method sensitivity concluded that practical depths of resolution for a two-layer model with a 20% velocity variation would be 43% of the maximum Rayleigh wavelength and 25% of the maximum
Love wavelength, or that the depth of resolution for Rayleigh waves is nearly twice that of Love waves.

Another recent relevant engineering study is that of Barrows and Ghar (1987), who inverted the group velocities of the Rayleigh wave fundamental mode to recover shallow S-wave velocities and layer thicknesses for model depths to 7 meters. The inversion algorithm also provides statistical information about the solution reliability. Group velocity data were said to be less susceptible to random noise than phase velocity data, but the group velocity contour plots from which dispersion curves were measured appear to be subject to gross errors in interpretation. The use of 100 Hz geophones also precludes application to models hundreds of meters in depth due to the 6-12 db/octave attenuation below the dominant frequency. In contrast, field work for this thesis incorporates exploration geophones with a dominant frequency of 4.5 Hz, which should facilitate velocity inversion to depths of several hundred meters. A potential difficulty in dispersion curve interpretation arises in Barrow and Ghar’s use of two adjacent seismic traces to calculate group velocities. The use of such closely spaced traces results in desired localized velocity determination, but with only two traces, random and coherent (reflected, refracted, and channeled) energy can lead to incorrect group velocity interpretations. The majority of published inversion research uses this two-station approach. This thesis uses a technique that includes many more traces to reduce noise at the expense of reduced horizontal velocity resolution.

The most recently published work using large scale CDP style data with cable lengths of several kilometers is that of Mokhtar et al. (1988). Their work in Saudi Arabia attempted to invert deep (=1.0 km) S-wave velocities and Q$_s$ (shear wave quality factor) from Rayleigh wave data. In their inversion process, both phase and group velocities are used in calculating the Jacobian partial derivative matrix. In Barrows and Gahr (1987), it was found that group velocities are more difficult to reliably extract from typical field data than are phase velocities; therefore, only phase velocities are used in this thesis. The Mokhtar algorithm was also constrained to minimize the differences in the inter-layer model changes. No such constraints are used here.
One problem with the research of Mokhtar et al. is that the inverted velocities were not verified by comparison to actual velocities. Instead, they extracted Rayleigh wave dispersion curves from seismic shot records, inverted for S-wave velocities, computed a synthetic shot record based on these velocities, and compared it to the original. They then claimed that the similarity between the synthetic and recorded shot records proved the validity of the velocity reconstruction. Also disturbing is the assumption that S- and P-wave velocities were linearly related with a constant Poisson's ratio of 0.25 from the surface to a depth of 1.0 km over a line 1000 km long.

This thesis tests the feasibility of applying surface wave inversion methods to CDP seismic data on a scale (depth = 0 - .3 km) beneficial to oil exploration and earthquake site amplification modeling. Toward this end, synthetic surface wave seismic data is used to invert for S- and P-wave velocities in the upper few hundred meters of the earth. The techniques are then applied to a real data set to evaluate the practical aspects of the process. Results show that this method may supply accurate S-wave statics information when conventional statics methods fail. Such a technique may also provide an economical means to determine velocity information for predicting site amplification within earthquake-prone areas.

This thesis is organized into six chapters. Chapter 2 presents the theory of surface wave inversion along with details of practical implementation. Chapter 3 tests the effectiveness of the inversion method with synthetic Love and Rayleigh wave dispersion curves, followed by Chapter 4, which describes the method for extracting surface wave dispersion curves from shot records. Chapter 5 examines inversion for shallow velocities from field data, and Chapter 6 presents the conclusion.
CHAPTER 2

THEORY OF SURFACE WAVE INVERSION

This chapter presents the methodology of inverting Love and Rayleigh wave data for near-surface velocities. The optimization technique is a Gauss-Newton gradient method, and the earth model is assumed to be isotropic, elastic, and layered. The seismic data are in the form of common shot point (CSP) records. The two records of particular interest are the YY gather (Love waves generated by a cross-line component source and recorded by cross-line component geophones) and the ZZ gather (Rayleigh waves generated by a z-component source and recorded on a z-component geophone). From these seismograms, the dispersion curves are extracted by a combined tau-p and Fourier transform (McMechan and Yedlin, 1981). The initial velocity model is then adjusted until the synthetic dispersion curves match the observed dispersion curves.

The S- or P-wave velocities in the \( i \)th layer are denoted by \( V_i \). The CSP seismic records in \((x, t)\) space are slant stacked and Fourier transformed in the tau variable to yield dispersion curves, or phase velocity with respect to frequency \( C(\omega, V) \). There is a nonlinear relationship (Aki and Richards, 1980) between the actual velocity model \( V(z) \) and the dispersion curves \( C(\omega, V) \). The velocity model \( V(z) \) can be reconstructed from the data \( C(\omega, V) \) by using a Gauss-Newton method, consisting of four steps;

Step 1: The S- and P-wave phase velocities \( C(\omega, V) \), where \( \omega \) is the frequency, are expanded in a Taylor Series about an initial \( N/2 \) layer velocity model \( V^0 = (V_1^0, V_2^0, \ldots, V_N^0) \):

\[
C(\omega, V) = C(\omega, V^0) + \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} (V_i^0 - V_i) + \text{higher order terms in } (V_i^0 - V_i).
\]
Rearranging terms we get

\[ \Delta C(\omega, V) = \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} \Delta V_i + \cdots \]  

(1)

where \( \Delta C(\omega, V) = C(\omega, V) - C(\omega, V^0) \) and \( \Delta V_i = V_i^0 - V_i \). The equation is linearized by assuming that the initial guess is close enough to the actual model so that the higher order terms in \( \Delta V_i \) can be neglected. Therefore:

\[ \Delta C(\omega, V) = \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} \Delta V_i \]  

(2)

Step 2: In equation (2) there are \( N \) unknown values of velocity \((V_1, V_2, V_3, \ldots, V_N)^T = V\) which are related to the \( M \) data samples measured from the dispersion curves by

\[ \Delta C(\omega_1, V) = \sum_{i=1}^{N} \frac{\partial C(\omega_1, V)}{\partial V_i} \Delta V_i , \]

\[ \Delta C(\omega_2, V) = \sum_{i=1}^{N} \frac{\partial C(\omega_2, V)}{\partial V_i} \Delta V_i , \]

\[ \Delta C(\omega_3, V) = \sum_{i=1}^{N} \frac{\partial C(\omega_3, V)}{\partial V_i} \Delta V_i , \]

\[ \vdots \]

\[ \Delta C(\omega_M, V) = \sum_{i=1}^{N} \frac{\partial C(\omega_M, V)}{\partial V_i} \Delta V_i , \]

or
\[ \Delta C = G \Delta V \]  

(3)

where \( \Delta C \) is the \( M \times 1 \) column vector of the data, \( \Delta V \) is the \( N \times 1 \) column vector of the model parameters, and \( G \) is the \( M \times N \) sensitivity matrix where:

\[ (G_k)_k = \frac{\partial C(\omega_k, V)}{\partial v_k} \]

A finite difference approximation is used to determine \( G_k \), i.e., the \( k^{th} \) element of \( V \) is perturbed, the dispersion curve is recalculated, the difference between this new curve and the original dispersion curve is computed, and the result is divided by the velocity perturbation to give

\[ \frac{\partial C(\omega_k, V)}{\partial v_k} = \frac{\Delta C(\omega_k, V)}{\Delta V_k} \] for all \( \omega \). The dispersion curves are calculated using an algorithm presented in Schwab and Knopoff (1972).

Step 3: Find \( \Delta V = (G^T G)^{-1} G^T \Delta C \). In practice, a singular value decomposition method is used to compute \( (G^T G)^{-1} \), with the singular vectors excluded if they are associated with eigenvalues below a given threshold (generally 1 - 10% of the largest eigenvalue).

Step 4: Update the velocity model \( V \) by \( \Delta V \) and repeat steps 2 - 4. It is advisable to normalize the columns of \( G \) prior to the inversion to minimize computational round off errors. The stopping criteria for this iterative procedure is when the dispersion curve values forward modeled from the inverted velocities are within the picking error range for each data point interpreted for the dispersion curves extracted from the seismic shot records.
CHAPTER 3

INVERSION OF SYNTHETIC DATA

This chapter examines the use of three different methodologies for inverting layer velocities from synthetic surface wave data. Separate inversions of Love and Rayleigh wave data for S- and P-wave velocities will first be examined, followed by combined inversions of Love and Rayleigh data. A summary of the results from this chapter is given in Table 1. The velocity model used for the synthetic tests has 6 layers with a roughly linear velocity gradient (Figure 1). This velocity model is representative of velocities found in the Williston Basin of North Dakota. Forward modeling of this velocity distribution results in the 80 point Love and Rayleigh wave dispersion curves in Figure 2, which are used for the synthetic tests. To avoid introducing any bias, the starting S-wave velocities are 0.35 km/sec for all layers, and the P-wave velocities are initialized at 1.5 km/sec.

<table>
<thead>
<tr>
<th>Inversion Method</th>
<th>Velocities Reconstructed</th>
<th>Velocities Given</th>
<th>Acceleration Applied</th>
<th>Inversion Results</th>
</tr>
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<td>Love</td>
<td>S-wave</td>
<td>-</td>
<td>No</td>
<td>Fair</td>
</tr>
<tr>
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<td>-</td>
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<tr>
<td>Rayleigh</td>
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<td>P-wave</td>
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<tr>
<td>Rayleigh</td>
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<td>-</td>
<td>No</td>
<td>Very Poor</td>
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<tr>
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<td>-</td>
<td>Yes</td>
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</tbody>
</table>
FIGURE 1: Synthetic test velocity model. S-wave velocities on the left, P-wave velocities on the right.
FIGURE 2: Fundamental mode dispersion curve data points generated using Figure 1 model.
Separate Inversion Algorithms

Love Wave Inversion

Figure 3 depicts the RMS data error vs. iteration number for shear velocity inversion from a 20 point sample of the Love wave dispersion curve in Figure 2. The solid and dashed lines represent the RMS error in the model velocities, while the dotted and dot-dashed lines represent the RMS data error. The 'Standard' curves represent the results of a standard Gauss-Newton algorithm, while the 'Fast' curves represent results from an accelerated Gauss-Newton algorithm in which the \((G^T G)^{-1} G^T A C\) step length is multiplied by a value proportional to the number of iterations up to a limit of 20. The 'Standard' inversion converges relatively quickly for the first 200 iterations, and thereafter, much more slowly. In the accelerated algorithm, all S-wave velocities converged to within 0.5% of the true model values in under 200 iterations.

Rayleigh Wave Inversion

Figure 4 represents the RMS error vs. iteration number for inverting S-wave velocities from a 20 point sample of the Rayleigh wave dispersion curve; in this case the P-wave velocities are maintained at their correct values. It can be seen that the convergence rate is much more rapid and complete than that in Figure 3. The Rayleigh wave inversion is also robust for a wider range of starting model velocities. As with the Love wave data, an accelerated Gauss-Newton method significantly increases the convergence rate. All layer velocities converged to within 0.1% of their correct values within 100 iterations using the accelerated algorithm.

The RMS error vs. iteration number for simultaneously inverting S- and P-wave velocities from Rayleigh wave data is given in Figure 5. After approximately 250 iterations, the inversion process has converged to incorrect S- and P-wave velocity values. Small errors in the S-wave velocities have offset large errors in the P-wave velocities resulting in a dispersion curve similar to the original. To correctly recover P-wave velocities from Rayleigh wave data, it is clear that the significant influence of errors in S-wave velocities must first be remedied.

Assuming a correct S-wave model, Figure 6 depicts the RMS error vs. iteration number for inverting P-wave velocities from Rayleigh wave data. An additional step length multiplication factor
FIGURE 3: RMS error for S-wave velocity inversion from Love wave data using standard and accelerated algorithms. The solid and dashed lines represent RMS errors from the model layer velocities for standard and accelerated algorithms respectively, while the dotted and dot-dashed lines represent the RMS errors from the 20 Love wave data points sampled from Figure 2.
FIGURE 4: RMS error for S-wave inversion from Rayleigh wave data given correct P-wave velocities using standard and accelerated algorithms. The solid and dashed lines represent RMS errors from the model layer velocities for standard and accelerated algorithms respectively, while the dotted and dot-dashed lines represents the RMS errors from the 20 Rayleigh wave data points sampled from Figure 2.
FIGURE 5: RMS error for simultaneous S- and P-wave velocity inversion from Rayleigh wave data using standard and accelerated algorithms.
FIGURE 6: RMS error for P-wave velocity inversion from Rayleigh wave data given correct S-wave velocities using standard and accelerated algorithms.
of 100 was incorporated for P-wave velocities to significantly increase the convergence rate over the standard algorithm. All inverted velocities were within 3% of the true layer velocities within 100 iterations. It is clear that a fast convergence rate could not have been achieved without the accelerated algorithm. However, Figure 5 and Appendix B suggest that the P-wave velocity inversion is highly sensitive to errors in S-wave velocities, meaning that inversion for P-wave velocities from real data will be difficult. Use of multiple sets of high quality data and the inclusion of higher order mode information would be useful to help recover these P-wave velocities. The next section will examine the feasibility of inverting for velocities from combinations of Love and Rayleigh wave data.

**Combined Love and Rayleigh Wave Inversion Algorithms**

**Simultaneous S- and P-wave Inversion**

Inverting a combination of Love and Rayleigh wave data appears to hold some promise for recovering both S- and P-wave velocities. Since Love waves are independent of P-wave velocities, the first step is to invert for shear wave velocities using Love waves. The first test inverts Love wave data for the 6 S-wave layer velocities followed by Rayleigh wave inversion for all 12 S- and P-wave velocity parameters. Figure 7 shows the results from a combined process in which Love waves are first inverted for S-wave velocities (until the RMS data error changes by no more than $1.0 \times 10^{-6}$ km/sec) followed by Rayleigh wave inversion for S- and P-wave velocities simultaneously. This process is performed using 20, 40, and 80 dispersion curve data points over a common frequency range. This combination of processes results in recovery of reasonable P-wave velocities (within 10% of the true values for all layers) from surface wave inversion. It can also be seen that while the 20 data point inversion is characterized by a slower convergence rate than when 40 or 80 data points are used, it does not warrant the large increase in computing time required to invert the more highly sampled data.

**Separate S- and P- Wave Inversion**

The final test (Figure 8) starts with S-wave inversion of Love wave data for the first 80 iterations until an update threshold of $1.0 \times 10^{-6}$ km/sec in RMS S-wave data error is achieved. The
FIGURE 7: Combined process: Love wave inversion for S-wave velocities followed by Rayleigh wave inversion for S- and P-wave velocities simultaneously.
FIGURE 8: Combined process: Love wave inversion for S-wave velocities followed by Rayleigh wave inversion for P-wave velocities.
Rayleigh wave algorithm then inverts for the P-wave velocities, achieving an RMS P-wave data error update threshold of of $1.0 \times 10^{-6}$ km/sec after another 250 iterations. By iteration 350, all inverted S- and P-wave velocities are within 0.1% and 10% of the true layer velocities respectively. The inversion process then continues to 2000 iterations with little improvement over the results in the previous example.

In summary, synthetic tests suggest that S-wave velocities can be optimally recovered by Love wave inversion due to the insensitivity to P-wave velocities. Love wave inversion for S-wave velocities followed by Rayleigh wave inversion for P-wave velocities appears to be a successful strategy for noiseless synthetic data. In practice, however, recovering P-wave velocities from Rayleigh wave data may be difficult due to the significant influence of S-wave velocity errors. Inversion for densities appears impractical based on their minimal contribution to dispersion curve character (Appendix B).
CHAPTER 4

DISPERSION CURVE EXTRACTION FROM SYNTHETIC SHOT RECORDS

This chapter describes the transformations that convert CSP shot records into dispersion curves. By performing a tau-p (intercept time-phase slowness) slant stack transformation of a shot record, followed by a Fourier transform from time to frequency over the tau parameter, one will recover a frequency-phase slowness record representing the phase velocity dispersion curves contained in the original data. The major benefit is that errors introduced by reflections and random noise will be minimized due to the inclusion of a large number of traces. The disadvantage is the assumption of homogeneous lateral velocities over the length of the shot record. This chapter will show the results of using the software packages based on this algorithm applied to synthetic seismic data. A more complete description of all related software, hardware, and their use is included in Appendix A.

McMechan and Yedlin (1981) described a method for extracting phase velocity dispersion curves from CSP records. The method consists of performing a slant stack on the shot record to transform it into tau-p space, followed by a Fourier transform over the tau parameter to obtain a record in frequency-p space. The frequency-p stack of a surface wave can be written as:

$$U(p, \omega) = \sum_{i=1}^{n} A(\omega, r_i) e^{j(\theta_i + \omega p r_i)}$$

where $A(\omega, r_i)$ is the amplitude of trace $i$ at an offset $r_i$ and $\phi_i$ is its phase. If the modulus of $U(p, \omega)$ is plotted as a function of $\omega$ and $p$, the largest values of $U(p, \omega)$ will correspond to the phase velocity dispersion curves in the absence of aliasing.

The first example is a 24 trace synthetic seismic section containing only Love wave fundamental mode dispersive energy (Figure 9). The synthetic section was generated by superimposing a suite
of single frequency energy packets across the traces at the delay time associated with that particular frequency. The dispersive nature of the seismic waves is clearly demonstrated. The lowest frequency energy is seen to be propagating fastest and arriving earliest at the far offsets. Progressively higher frequency energy arrives at increasing delay times until the highest frequencies traveling in the slowest shallow layer arrives last followed by noise. The strong noise level at the near offsets results from the trace by trace normalization. The synthetic seismic section is then transformed into the tau-p domain followed by a transformation into the frequency-p domain by a Fourier transform in the tau parameter. The resulting extracted dispersion curve is shown in Figure 10. Since only fundamental mode data were used in the modeling, all other curves represent spatial aliasing effects.

The second example, presented in Figures 11-16, has twice as many traces over the same offset, and was generated using a finite difference program for the SH Love wave and a reflectivity program for the P-SV Rayleigh wave shot records. The velocity model is from a preliminary vertical seismic profile (VSP) study for an Arco test site in northeastern Texas. Figures 11 and 12 are the synthetic SH and P-SV records respectively. The extracted dispersion curves (Figures 13 and 15) can be compared to the forward modeled results (Figures 14 and 16) generated with a program based on Schwab and Knopoff, 1972, which computes all theoretical modes for Love and Rayleigh waves given layer velocities and densities. Both the Love and Rayleigh wave dispersion curves extracted from the synthetic seismograms match the forward modeled curves exactly. The fundamental mode values can clearly be picked down to about 2 Hz. The first higher modes can also be matched to the forward modeled values, though their amplitudes with respect to the fundamental mode are greatly diminished.

Several problems should be addressed. The first is the problem of spatial aliasing. Both synthetic examples have maximum offsets of 1.5 km (5000 ft). The example in Figure 9 has 24 traces spaced at 64 m (210 ft) while the example in Figure 11 has 48 traces spaced at 32 m (105 ft). The aliasing manifests itself as additional curves in the lower and upper right hand corners of the plots in Figures 10, 13, and 15. It can be seen that the finer sampling in the 48-trace example forces these curves away from the fundamental mode curve, helping to expose the higher modes.
FIGURE 9: Synthetic seismic section created using fundamental mode Love wave energy only. Dispersive wave character is evident in lower frequencies arriving earlier at farthest offsets.
FIGURE 10: Dispersion curves extracted from the synthetic seismogram in Figure 9 using tau-p and Fourier transforms.
FIGURE 11: Synthetic SH wave seismic section generated with Arco test site VSP velocities and the finite difference program sh.f.
FIGURE 12: Synthetic P-SV wave seismic section generated with Arco test site VSP velocities and the reflectivity program refl.f.
FIGURE 13: Love wave dispersion curves from synthetic in Figure 11. Comparison to theoretical fundamental mode can be made using Figure 14.
FIGURE 14: Forward modeled Love wave dispersion curves generated using Arco test site VSP velocities.
FIGURE 15: Rayleigh wave dispersion curves from synthetic in Figure 12. Comparison to theoretical fundamental mode can be made using Figure 16.
FIGURE 16: Forward modeled Rayleigh wave dispersion curves generated using Arco test site VSP velocities.
When they are extractable, higher order modes would enhance the reliability of the reconstructed velocities. For example, the lower frequencies of the Rayleigh wave first higher mode are significantly influenced by P- as well as S-wave velocities. The higher order modes are, however, much weaker than the fundamental mode as evidenced in Figures 13 and 15, and writing a program that could automatically identify and invert higher order modes, given an unbiased starting velocity model and real world noise, would be very difficult.

The resolution (width) of the extracted dispersion curves, and subsequently the ease of picking frequency data, is greatly affected by the maximum source-receiver offset in the shot record. Figure 13 shows a significant widening in the fundamental mode curve below 2 Hz. A 48-trace record with a maximum offset of 2.25 km (1.4 miles) results in a thinning of the dispersion curve in Figure 17 down to 1.5 Hz, while a 48-trace 4.8 km (3.0 miles) record enhances the dispersion curve resolution for frequencies as low as 0.5 Hz (Figure 18). It can also be demonstrated that the width of the entire dispersion curve increases with decreasing receiver aperture. In addition, since reconstruction of deeper layer velocities requires lower frequency data, limitations at frequencies below 2 - 4 Hz in oil industry and engineering seismographs and geophones will limit the method’s effectiveness at depths greater than several hundred meters.
FIGURE 17: Rayleigh wave dispersion curves extracted from 48-trace 2.25 km- (1.4 mile-) wide synthetic.
FIGURE 18: Rayleigh wave dispersion curves extracted from 48-trace 4.8 km- (3.0 mile-) wide synthetic.
CHAPTER 5

INVERSION OF FIELD DATA

The seismic field data for this report were collected by Arco at their test site several miles west of Sulfur Springs, Texas. The source was an ARIS multidirectional hydraulic impact type. A vertical source response is simulated by summing two records recorded with impacts in opposing diagonal directions so that the horizontal components vectorially cancel. Similarly, a horizontal source response is simulated by computing the difference between the same two records so that the vertical displacement components vectorially cancel. The geophones in this experiment consisted of 11 groups of 6 horizontal and 6 vertical 4.5 Hz elements spaced at 9.144 m (30 ft) intervals. The geophones were planted at one end of the line while the source, starting at a near offset of 161.5 m (530 ft), was moved down each line at 100.6 m (330 ft) intervals to a maximum offset of 1469 m (4820 ft). The S- and P-wave velocities as a function of depth were determined from a previous VSP experiment shot at the geophone end of the line. The traces in Figures 19 and 20 represent the YY (Love wave) and ZZ (Rayleigh wave) component shot records. Both raw data records exhibit dominant frequencies in the range of 5-10 Hz, and show clear signs of dispersive surface waves.

The YY component shot record is transformed into the tau-p domain, and then into the frequency-p domain to yield dispersion data in frequency-phase slowness space (Figure 21). The peaks of strongest coherency are interpreted at 1 Hz increments (asterisks) to yield the fundamental mode Love wave dispersion curve data. The fundamental mode curve is clearly represented between 4 and 12 Hz, but is subject to ambiguous interpretation at higher frequencies. Higher mode curves are apparent at $P = 1.5$ sec/km above 7 Hz. The extracted Rayleigh wave dispersion curves with the fundamental mode interpretation are displayed in Figure 22. The energy of the fundamental mode is once again significantly attenuated above 12 Hz. Dispersion curve information from 10 - 15 Hz was interpreted from the XZ (radial component source recorded on a z-component geophone) gather which
FIGURE 19: Arco test site YY Love wave shot record. Station spacing is 9.14 m (30 ft).
FIGURE 20: Arco test site ZZ Rayleigh wave shot record. Station spacing is 9.14 m (30 ft).
FIGURE 21: Extracted Love wave dispersion curves with fundamental mode interpretation given by asterisks.
FIGURE 22: Extracted Rayleigh wave dispersion curves with fundamental mode interpretation given by asterisks.
contained stronger signal energy at these frequencies. Several signal enhancement techniques including raw data muting, filtering, AGC, and limiting receiver aperture have been used in an effort to improve the signal-to-noise ratio. The lack of coherency at higher frequencies may be attributable to attenuation within the substantial offset of 162 m (530 ft) between the first shot point and the nearest geophone of the array. Based on previous synthetic results, the lack of dispersion curve energy below 3 Hz is attributed primarily to finite receiver aperture (1.5 km (4820 ft)).

Three methods of inversion for S-wave velocities are presented: Love wave inversion, Rayleigh wave inversion, and alternating Love and Rayleigh wave inversion. The resulting RMS data error vs. iteration curves are presented in Figure 23. As in the synthetic tests, the fastest method is the Rayleigh wave inversion, which achieves a lower RMS data error than that of the Love wave inversion. The alternating inversion quickly reaches a point after which velocities oscillate and little additional improvement is achieved, indicating that Love and Rayleigh wave inversions are attempting to converge to different layer velocities. A simultaneous approach for inverting Love and Rayleigh wave dispersion curves was attempted, but was unstable for the same number of layers used in the separate algorithms. Using fewer layers, in an attempt to stabilize the inversion, results in a poorly modeled problem with poor convergence. The instability problem may be a result of anisotropy contributing to a discrepancy in S-wave layer velocities apparent to the Love and Rayleigh wave sections of the simultaneous inversion.

The S-wave velocity reconstructions from each method are compared in Figure 24. All three velocity reconstructions are in fair agreement. Especially significant is the agreement between the independent Love and Rayleigh wave methods and their general agreement with the VSP results. The possibility of anisotropic S-wave velocities is suggested by the discrepancy between Love and Rayleigh wave values for some layers, along with the relatively short 1 S.D. error bars. The larger Love wave values of shear velocity might be explained by vertical fracture planes which are perpendicular to the Love wave particle motion and parallel to the Rayleigh wave particle motion. Analysis of data recorded orthogonal to this data set could resolve this conflict. The discrepancy between inverted and VSP velocities can be attributed in part to the coarse layer sampling in the VSP interpretation and the
FIGURE 23: RMS data error vs. iteration number for Love, Rayleigh, and alternating Love and Rayleigh wave dispersion curve inversion.
FIGURE 24: Shear wave velocity reconstruction using Love, Rayleigh, and alternating Love and Rayleigh wave inversion methods. Error bars represent 1 standard deviation resulting from dispersion curve picking errors.
fact that the VSP profile samples only a small area at the end of the seismic line. In comparing the static corrections, or two-way travel time for each method to a depth of 0.1 km (330 ft), the Love wave static is 388 ms, the Rayleigh wave static is 412 ms, the alternating Love and Rayleigh wave static is 418 ms, and the VSP static is 438 ms. Compared to the VSP static, the Love wave static differs by less than 13% and the Rayleigh wave by less than 6%.

Although the three static estimates are in general agreement, an advantage of the alternating method is demonstrated in the next six figures. Ideally, in the absence of S-wave anisotropy, all three velocity models should yield dispersion curves which closely match those extracted from the data. It can be seen in Figure 25 that the Love wave dispersion curves forward modeled from the S-wave velocities reconstructed from the Arco Love wave data do match the real data picks (asterisks), but the Rayleigh wave curves generated from the same S-wave velocities (Figure 26), along with the VSP P-wave values, show significant discrepancies when compared to the Rayleigh wave dispersion curve interpretation. A similar result can be seen when the forward modeled Rayleigh wave dispersion curves are compared to the real data. While the Rayleigh wave forward modeled curves match the Rayleigh wave field data interpretation (Figure 27), the Love wave curves generated from the same velocity model show significant discrepancies (Figure 28). Using only Love or Rayleigh wave data may succeed in generating acceptable dispersion curves that match those from that particular data set, but it is difficult to estimate their reliability unless velocity models reconstructed from both Love and Rayleigh wave data are compared. By alternating Love and Rayleigh S-wave inversion, a more accurate overall result can be achieved. Figures 29 and 30 demonstrate the alternating method results. Both sets of curves are in error by a smaller amount, and do not exhibit the large errors characterized by separate Love or Rayleigh wave inversion.

The accuracy of the Love wave inversion can also be demonstrated by the correlation of the forward modeled and extracted higher order modes (Figure 31). The first higher mode, though not part of the inversion process, displays a remarkable fit; while the second and third higher modes are both within 0.05 km/sec of their expected values. The higher order Rayleigh modes do not match as well (Figure 32), and can not be made to match by varying P-wave velocities and densities. It is
FIGURE 25: Forward modeled Love wave dispersion curves generated using velocity model inverted from Arco Love wave data (Figure 24). Asterisks represent extracted Love wave dispersion curve interpretation.
FIGURE 26: Forward modeled Rayleigh wave dispersion curves generated using velocity model inverted from Arco Love wave data (Figure 24). Asterisks represent extracted Rayleigh wave dispersion curve interpretation.
**FIGURE 27:** Forward modeled Rayleigh wave dispersion curves generated using velocity model inverted from Arco Rayleigh wave data (Figure 24). Asterisks represent extracted Rayleigh wave dispersion curve interpretation.
FIGURE 28: Forward modeled Love wave dispersion curves generated using velocity model inverted from Arco Rayleigh wave data (Figure 24). Asterisks represent extracted Love wave dispersion curve interpretation.
FIGURE 29: Forward modeled Love wave dispersion curves generated using velocity model inverted from alternating Love and Rayleigh wave inversion (Figure 24). Asterisks represent extracted Love wave dispersion curve interpretation.
FIGURE 30: Forward modeled Rayleigh wave dispersion curves generated using velocity model inverted from alternating Love and Rayleigh wave inversion (Figure 24). Asterisks represent extracted Rayleigh wave dispersion curve interpretation.
FIGURE 31: Forward modeled Love wave dispersion curves, generated using Love wave inversion results, superimposed on the Love wave dispersion curve data extracted from the Arco test site shot record. Dots represent extracted Love wave dispersion curve interpretation.
FIGURE 32: Forward modeled Rayleigh wave dispersion curves, generated using Rayleigh wave inversion results, superimposed on the Rayleigh wave dispersion curve data extracted from the Arco test site shot record. Dots represent extracted Rayleigh wave dispersion curve interpretation.
suspected that inaccurate modeling of layer boundaries is responsible. (The XZ component Rayleigh wave dispersion curves are included because of their higher signal-to-noise ratio in the higher order modes.)

Useful P-wave velocities could not be recovered from this particular data set because of the apparent anisotropic S-wave layer velocities, the of the lack of access to multiple seismic data sets shot at the site, the sensitivity of the dispersion curves to errors in S-wave velocities, and errors introduced in the interpretation of the extracted dispersion curves. Future studies, which include additional data sets meticulously acquired with closer source-receiver offsets, may be able to approach the synthetic ideal of P-wave velocity reconstruction. However, the results presented in this chapter demonstrate the reliability with which shallow shear wave velocities can be measured using surface wave inversion of CDP data.
CHAPTER 6

CONCLUSIONS

Synthetic tests demonstrate the feasibility of reconstructing near-surface S-wave velocities from Love and Rayleigh wave data. An accelerated Gauss-Newton method is robust and accurate in inverting dispersion curves for realistic near-surface S-wave velocity models. The inversion of P-wave velocities from Rayleigh waves is much more difficult because Rayleigh waves are highly sensitive to errors in the shear velocities. Though demonstrated on synthetic data, inversion for P-wave velocities was inconclusive due in part to large source-receiver offsets, lack of multiple data sets, and possible anisotropic effects. Density inversion from Love or Rayleigh wave data appears to be impractical.

The S-wave velocities reconstructed from independent Love and Rayleigh wave data sets correlate well and compare favorably with the velocities measured from a VSP experiment. It is shown that an alternating combination of the two methods improves the reconstructed velocity model. Also, there is very good agreement between the observed and predicted static correction (two-way travel-time) to the deepest reconstructed layer with an error of less than 5% when using the alternating Love and Rayleigh wave velocity model.

The significance of this study is that it demonstrates the feasibility of using both Love and Rayleigh wave data to invert for near-surface S-wave velocities. Important applications of this method include use as a statics method independent of conventional methods which rely on body waves or refraction data. Surface wave inversion has an advantage over reflection and refraction methods in that it does not require significant velocity discontinuities to generate data, and is therefore useful for evaluating velocities in near-surface unconsolidated soils.

An important application relates to earthquake hazard studies concerned with site amplifications that are strongly controlled by near-surface S-wave velocities. The ability to economically acquire CDP data containing Love and Rayleigh surface waves in parks and other open areas throughout the
Wasatch Front valleys, followed by inversion for shear wave velocities to several hundred meters in depth, will allow for much more detailed earthquake hazard evaluation than was previously possible.
APPENDIX A

FLOWCHART OF SURFACE WAVE INVERSION PROGRAMS

This appendix describes the equipment and software at the University of Utah which has been assembled to carry out velocity analyses using surface wave inversion. The flowchart in Figure 33 outlines the process. Major segments include acquisition of seismic data with the Bison 9024 portable seismograph; first pass seismic data processing for sorting, frequency enhancement, and statics using Kansas Geological Survey (K.G.S.) software; reformatting of the K.G.S. output for processing and plotting on Sun work stations with MATLAB and laser printers; synthetic data generation; and the tau-p, frequency-p, and inversion process for velocity extraction from surface waves.

Seismic Data Acquisition

The Bison 24-channel seismograph was acquired in the summer of 1989 under a National Science Foundation grant garnered in part with the results of seismic work performed at Hill A.F.B. in the summer of 1988. Two 24-channel geophone cables with 15 meter spacings were also purchased with NSF monies, and a roll-a-long box was constructed in house to provide a light weight and portable interface for the hardware. One hundred groups of 12 element/8 Hz vertical component geophone strings, and an additional 100 elements for repairs have been donated by Mobil Oil; and 100 groups of 6 element/4.5 Hz horizontal component geophone strings were acquired from Arco. In addition, a Zenith 80386 laptop computer was purchased with the N.S.F. grant, and can be used to process the seismic data in the field. Two sources are currently in use. The first being a sledge hammer and an aluminum block, and the second, a 12 gauge shotgun which is welded into a metal plate. The seismic recordings are initiated by a trigger securely fastened to the source.
FIGURE 33: Flowchart of surface wave inversion process.
Portable Computer Data Processing

The binary or floating point seismic data from the Bison 9024 can be uploaded using the Procomm modem software on the Zenith 80386 hard drive and a 25/9 pin RS-232 cable. The data are first converted to K.G.S. format using 90002kgs. The majority of the processing steps for static corrections can be bypassed in most cases, but the sorting, filtering, deconvolution, and other frequency enhancing steps could prove useful on many data sets. The ASCII data files can then be easily uploaded to the mainframes again using Procomm with Kermit protocol. Binary data uploaded from the Bison can be uploaded directly to the mainframes, but file size limitations encountered when using the UNIX octal dump command have caused problems in ASCII conversion.

Data Reformatting and Plotting

The ASCII shot records on the mainframe can then be plotted to check for accuracy, and reformatted using the MATLAB script file kgs2mat.m. The output file a.mat contains 2 MATLAB binary arrays c and balmax which are extracted and converted to ASCII using the MATLAB program translate. C.dat then contains the trace amplitudes for the shot in trace sequential order. The first five values in balmax.dat represent the number of traces, the number of samples per trace, the sampling rate (seconds), the near offset (km), and the station spacing (km) respectively. The remaining values are trace by trace normalization factors.

The files c.dat and balmax.dat are then converted into SEG-Y format with the FORTRAN program mat2segy.f. The two input file names must be specified, and the output files include the SEG-Y binary file shot and an information file synth.out which is not used in further processing. For convenience, the Rayleigh wave shot record can then be renamed vert and the Love wave record renamed sh. These files may be plotted using the rsx software run from a suntools window on a Sun workstation. The SEG-Y file is first converted to the rsx.ah format using the FORTRAN program seg2ah.f with the indicated UNIX command, followed by the rsx initiation command in which the filename from seg2ah is specified.
The indicated rsx command will load the file and display a list of viewing parameters. Each can be changed by pointing to the parameter using the mouse, clicking button #1, and entering the new value followed by RETURN. Once the values are acceptable, a pulldown window will reveal a number of options including 'trace' which will change displays to reveal the shot record. The parameter window can be recovered with the pulldown option 'parameter'. A scrcendump is available from a second pulldown window. Scrcendump will create a file rsx.aft which can be printed by converting the file to Postscript format and piping the output to a specified laser printer. These are only a few of the capabilities of the program, and a manual for rsx is available on the Suns.

Synthetic Data Generation

The flowchart includes three methods of generating synthetic seismic sections. Dispgen.m generates surface wave sections by starting with a suite of single frequency wave packets, and superimposing them across the traces using the time delay associated with that particular frequency. Dispgen.m allows the user to generate and examine shot records containing one or several dispersion curve modes. The necessary frequency-p data pairs can be generated with the forward modeling program discurv.f which requires a velocity model and frequency bounds.

Figure 9 in Chapter 4 was generated by using the forward modeling program discurv.f to generate the frequency-phase velocity data pairs from which fundamental mode values were picked at 1/8 Hz intervals and input into the MATLAB program script dispgen.m. Dispgen.m then generated the synthetic seismic section containing only the fundamental mode surface wave which was then transformed into SEG-Y format using mat2seg.y.f. The SEG-Y data were input into taup.f to calculate the tau-p data, which were then input into tauplt.f for plotting and formatting, then into zconj.m for the Fourier transform over tau, and finally into zdec.m for dispersion curve plotting. The extracted dispersion curve is plotted Figure 10 of Chapter 4.

The second method uses the finite difference programs psvr2.f and sh.f which, while easy to work with, are limited in model size by computer memory. Figure 11 of Chapter 4 was generated using sh.f. The Rayleigh wave program psvr2.f requires input from the program mod.f which in
turn requires two input files *indat* and *inmod*. *Indat* contains the discretization parameters and layer velocities, and *inmod* contains the layer thicknesses. The output file *psvz* is converted into the ASCII files *c.dat* and *balmax.dat* with the FORTRAN program *psvr2mat.f*. Love wave shot records can be created in a similar way by specifying the geology in *sh.in* and *inmod*, running *mod.f*, followed by *sh.f* to generate the file *sh1* which is then converted by *sh2mat.f* to *c.dat* and *balmax.dat*. Because of memory and speed requirements, these programs should be run on the Stardent mini-supercomputer under the compiling option 03.

A third method generates synthetics using the reflectivity program *ref1.f* (Figure 12 of Chapter 4), but the large number of parameters and extensive code make it a difficult program to implement. The file *crfl.doc* gives a brief description of the various flags and variables input from the file *crfl.dat*. *Ref1.f* is capable of running much larger simple models, with greater frequency content, in less time than the finite difference programs; but in its current configuration, will not compile correctly on the Stardent using any option higher than 01. The output files *crfl.psv* and *crfl.sh* are converted to SEG-Y format using *ref2seg.y.f*. The program will identify the file type and output the proper *vert* or *sh* file. *Synth.out* once again contains shot and trace information, but is not used further in the process.

The fourth synthetic program *discurve.f* generates dispersion curves from a one-dimensional model specified within the program code. Four files are output: *rayl.XX* containing Rayleigh wave phase velocities, *omegar.XX* containing the frequencies associated with the Rayleigh wave values, and *love.XX* and *omegal.XX* which are the Love wave counterparts. These ASCII files are then plotted using the MATLAB scripts *dispr.m* for Rayleigh wave curves and *displ.m* for Love wave curves.

**Data Transformation and Inversion**

*Taup.f* performs the tau-p transformation on SEG-Y data, and requires interactive input. The output file *tp* and scaling file *tau.in* are used by *tauplot.f* which creates a Calcomp or Tectronix plot and reformats the tau-p data to be read by the MATLAB script file *zconj.m*. The plot can be bypassed by directing the program output into a temporary file. The tau-p programs are based on
those described in Matulevich (1989). A Fourier transform over the tau parameter is performed by
zconj.m and is then saved in b.mat, which can then be read by zdec.m and plotted. After interpreting
and hand picking dispersion curve data values, the program invrt.f can then invert for the correct
velocities. Layer thicknesses in invrt.f are fixed throughout the inversion process, so a poor match
between the actual layer thicknesses and the estimated layer thicknesses will result in poor RMS error
convergence or even divergence.
APPENDIX B

DISPERSION CURVE EXAMPLES FOR PARAMETER VARIATIONS IN TWO LAYER MODELS

This appendix examines how variations in S-wave velocity, P-wave velocity, and density affect the behavior of dispersion curves. Figure 34 represents the reciprocal of Love and Rayleigh wave phase velocities (p) as a function of frequency. These curves were generated using a simple 2 layer model and an algorithm described in Schwab and Knopoff (1972). The shallow layer has an S-wave velocity of 0.2 km/sec, a P-wave velocity of 1.0 km/sec, and a density of 2.0 gm/cc. The bottom layer has an S-wave velocity of 0.7 km/sec, a P-wave velocity of 2.0 km/sec, and a density of 2.2 gm/cc. The depth of the interface is at 0.1 km. The longest curve is that of the fundamental mode, while subsequent curves to the lower right represent the higher order modes.

As expected, when the frequency approaches 0 Hz and the wavelength increases, the Love wave is influenced by deeper and deeper formations resulting in a phase velocity approaching that of the deepest layer where \( V_{S2} = 0.7 \text{ km/sec} \) and \( p = 1/V_{S2} = 1.43 \text{ sec/km} \). As the frequency increases, the deeper layer becomes invisible to the surface waves, and the phase velocity approaches that of the shallow layer S-waves \( (V_{S1} = 0.2 \text{ km/sec} \text{ and } p = 1/V_{S1} = 5.0 \text{ sec/km}) \). The Rayleigh wave dispersion curves in Figure 34 were generated using an identical velocity model. The most significant difference is that the dispersion curves asymptotically approaches approximately 90% of the S-wave velocity of the shallow layer.

The remaining figures show the effects on the dispersion curves when the model parameters are varied. They demonstrate which geologic variables significantly influence the dispersion curves as a function of frequency. Each parameter in the two layer demonstration model has been independently varied -20%, -10%, 0%, +10%, and +20% with all 5 results plotted on a single graph.
FIGURE 34: Forward modeled dispersion curves for 2 layer model. Shallow shear wave velocity, compressional wave velocity, and density are 0.2 km/sec (p = 5.0 sec/km), 1.0 km/sec (p = 1.0 sec/km), and 2.0 gm/cc respectively. Deep shear wave velocity, compressional velocity, and density are 0.7 km/sec (p = 1.43 sec/km), 2.0 km/sec (p = 0.5 sec/km), and 2.2 gm/cc respectively. Depth to layer interface is 0.1 km.
Figure 35 shows the dispersion curve variation as a function of the shallow layer’s S-wave velocity. The substantial separation of the dispersion curves demonstrates that this is a significant variable. The uppermost curve in each mode was generated using a velocity of 0.16 km/sec, while the lowermost curve of each mode represents a velocity of 0.22 km/sec, with 3 intermediate values between. The Love and Rayleigh wave graphs show that at the lowest frequencies, variations in the shallow layer velocity have no effect. At the higher frequencies, or shorter wavelengths, variations in shallow velocities are clearly significant.

The deeper layer’s S-wave velocity is varied in Figure 36. It is apparent that nearly all resulting change appears in the lower frequencies of each mode. S-wave velocity variations again demonstrate a significant contribution to dispersion curve character.

The curves in Figure 37 were generated by varying the shallow layer’s P-wave velocity. Since the Love wave is a shear wave, one would not expect any variation in its dispersion curve, and none is observed. The variation seen in the fundamental mode of the Rayleigh curve implies that P-wave velocities may be measured and modeled to some extent with surface waves.

Figure 38 is associated with a variation in the P-wave velocity of the deeper layer while the upper layer remains constant. Once again, the Love wave dispersion curves show no effect. The Rayleigh wave curves show only a slight deviation mostly at the low frequency end of each mode. The previous two examples demonstrate that most of the P-wave information will be contained in the higher frequencies of the fundamental mode, and at the lower end of the higher modes. In order to invert for P- as well as S-wave velocities, Rayleigh wave dispersion curve data from these frequencies should be used.

The final two figures (39 and 40) demonstrate that dispersion curves are almost insensitive to the density variations of +/- 20%. Since density variations of this magnitude are not often encountered, inverting for densities from surface waves is considered impractical.
FIGURE 35: Dispersion curves for two layer model with shallow layer shear wave velocity varied by -20%, -10%, 0%, +10%, and +20%. Upper curve for each mode was computed using $V_{S1} = 0.16$ km/sec. Subsequent curves generated with $V_{S1} = 0.18, 0.20, 0.22$, and 0.24 km/sec ($p = 6.25, 5.56, 5.00, 4.55$, and 4.16 sec/km).
FIGURE 36: Two layer model dispersion curves generated with S-wave velocity variations in the deep layer up to +/- 20%: \( V_s^2 = 0.56, 0.63, 0.70, 0.77, \) and 0.84 km/sec (\( p = 1.79, 1.59, 1.43, 1.30, \) and 1.19 sec/km).
FIGURE 37: Shallow layer compressional velocity variations up to +/- 20% of the original value. $V_p = 0.8, 0.9, 1.0, 1.1, \text{ and } 1.2 \text{ km/sec}$. 
FIGURE 38: Dispersion curves for variations of deep layer compressional velocity up to +/- 20% of the original value. $V_p = 1.6, 1.8, 2.0, 2.2$, and $2.4$ km/sec.
FIGURE 39: Dispersion curves for variations of shallow layer density up to +/- 20% of the original value. \( \rho_i = 1.8, 1.9, 2.0, 2.1, \) and \( 2.2 \) gm/cc.
FIGURE 40: Dispersion curves for deep layer density variations up to +/- 20% of the original value. \( \rho_2 = 1.76, 1.98, 2.20, 2.42, \) and 2.64 gm/cc.
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