EVALUATION OF THE ASPHALT MIXTURE PERFORMANCE TESTER (AMPT)

by

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Master of Science

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ABSTRACT

Between 2007 and 2008, the Utah Department of Transportation (UDOT) has been collecting field material and testing it using the Asphalt Mixture Performance Tester (AMPT). Thirty-four field mixtures were evaluated as part of this study. The mixtures were produced following UDOT standard volumetric specifications and two binder grades, PG 64-34 and PG 70-28, from three suppliers.

Based on the evaluation of the results obtained from these tests, it was found that the AMPT produced data with a coefficient of variation below 15%, indicating good repeatability. Mixtures prepared with different binder grades were easily separated. The dynamic modulus master curves of mixtures prepared using the same binder grade were essentially the same regardless of the aggregate source (same type of mixture was used). Given these results, it was shown that it is possible to obtain asphalt mixture level 1 MEPDG input parameters based on historic data, knowledge of the binder grade used and testing at only 1 temperature. The single temperature test can be used for quality control and to verify that the mixture is of the same kind as the historical data available. This approach can significantly reduce the time and effort required to obtain AMPT data without any loss in performance prediction capacity, thus making adoption of this device more appealing to state DOT's.
# TABLE OF CONTENTS

**ABSTRACT** .................................................................................................................. iii

**LIST OF TABLES** .......................................................................................................... vi

**LIST OF FIGURES** ......................................................................................................... vii

**ACKNOWLEDGMENTS** ................................................................................................. viii

**INTRODUCTION** ......................................................................................................... 1

  Background .................................................................................................................. 1
  Test Description ........................................................................................................... 2
  Sample Preparation ...................................................................................................... 5
  Data Obtained ............................................................................................................... 8

**EVALUATION OF THE ASPHALT MIXTURE PERFORMANCE TESTER** ................. 9

  Abstract ......................................................................................................................... 9
  Introduction ................................................................................................................... 10
  Materials and Protocols ............................................................................................... 11
  Variability .................................................................................................................... 13
  Characterization .......................................................................................................... 15
  Parameters ................................................................................................................... 17
  Discussion .................................................................................................................... 20
  MEPDG .......................................................................................................................... 22
  Application ................................................................................................................... 25
  Conclusions .................................................................................................................. 26
  Recommendations ...................................................................................................... 27
# LIST OF TABLES

1. Test Specimen Dimensional Tolerances ............................................................. 5
2. Recommended Testing Temperatures and Loading Frequencies ..................... 7
3. Data Quality Requirements ................................................................................ 8
4. Project Information ............................................................................................ 12
5. CV Values for 34 Projects ................................................................................ 14
6. Parameter Ranges for Each Type of Binder ..................................................... 18
7. Wisconsin Parameter Ranges ........................................................................... 20
LIST OF FIGURES

1  IPC Global AMPT Machine ................................................................. 2
2  Schematic of Dynamic Modulus Test ................................................. 4
3  Dynamic Modulus Master Curve with Shifted Values ......................... 4
4  Specimen Coring (Left) and Cutting (right) ......................................... 5
5  Specimen with LVDT's attached in the Environmental Chamber ............ 6
6  CV Values for 20 °C and 1 Hz ............................................................. 15
7  Collection of Dynamic Modulus Master Curves ..................................... 16
8  Dynamic Modulus Master Curves, Sorted by Binder ............................. 17
9  Aggregate Gradation of Utah (Solid) and Wisconsin (Dashed) ............... 22
10 MEPDG Rutting Over Life of Pavement ............................................... 23
11 MEPDG Rutting Over Life of Pavement ............................................... 24
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CHAPTER 1

INTRODUCTION

Background

The Asphalt Mixture Performance Tester (AMPT), also known as the Simple Performance Tester (SPT), was developed explicitly for testing and evaluating asphalt mixtures. The parameters obtained from this test are used as inputs for the structural design of pavements using the Mechanistic-Empirical Pavement Design Guide (MEPDG) software. The AMPT (Figure 1 - IPC Global AMPT Machine) was developed as part of the National Cooperative Highway Research program (NCHRP) Project 9-29 for testing stiffness and permanent deformation properties of asphalt concrete. Three tests were developed: dynamic modulus, flow number, and flow time. However, at the time of this writing, only the dynamic modulus and flow number tests have been applied in pavement design and asphalt concrete mixture analysis. The flow time test was initially designed to be a quicker version of the flow number test but the flow number test is still preferred. Presently there is a significant research effort aimed at developing testing protocols, data analysis, as well as the equipment. The AMPT began being evaluated by leading state agencies, including UDOT, in 2002. Although the AMPT is capable of evaluating the dynamic modulus and flow number tests, this thesis focuses primarily on the dynamic modulus test.
Test Description

To obtain the dynamic modulus, a haversine axial compressive stress is applied to a cylindrical asphalt concrete specimen at a specified temperature and different loading frequencies. The applied stress and the resulting axial strain of the specimen are measured and used to calculate the dynamic modulus and phase angle. The dynamic modulus is defined as the peak stress divided by the peak strain at a specific frequency and temperature combination. This is the overall stiffness of the asphalt concrete mixture at the given conditions. The phase angle is defined as the angle, in degrees or radians between a haversine applied peak stress, and the resulting peak strain in a controlled stress test. This is shown schematically in Figure 2. Once the dynamic modulus values are measured over a range of
temperatures and loading frequencies, they can be combined or shifted into a single curve as shown in Figure 3. This curve is known as the dynamic modulus master curve. The master curve is created using an equation developed as part of the NCHRP Project 9-29. This is Equation 1 shown below.

The master curve along with the shift factors, provides information about the mechanical response of the specific asphalt mixture at any given load frequency and temperature. The values obtained from this master curve can be used for performance prediction and analysis. The values of dynamic modulus and phase angle can also be used as performance criteria for control/quality assurance during Hot Mix Asphalt (HMA) design and construction.

\[
log|E^*| = E_{\min} + \frac{(E_{\max}-E_{\min})}{1+e^{\beta+\gamma \left( \log_{10} \omega + \frac{\Delta E_a}{19.14T^4} \left( \frac{1}{T} - \frac{1}{T_r} \right) \right)}}
\] (1)

where:
- \(E_{\min}\) = Minimum Modulus used as a fitting parameter, kPa (psi);
- \(E_{\max}\) = Maximum Modulus obtained from volumetrics, kPa (psi);
- \(\beta\) = Fitting Parameter;
- \(\gamma\) = Fitting Parameter;
- \(\Delta E_a\) = Activation energy, fitting parameter;
- \(\omega\) = Testing frequency, Hz;
- \(T\) = Testing temperature, °C;
- \(T_r\) = Reference temperature, °C;
Figure 2 - Schematic of Dynamic Modulus Test

Figure 3 - Dynamic Modulus Master Curve with Shifted Values
Sample Preparation

Preparation of the asphalt concrete specimens used for this test consists of making a Superpave Gyratory Compactor (SGC) sample that meets the requirements of AASHTO T 312. The SGC sample is then cored to a nominal 102 mm diameter using a water-cooled diamond bit core drill. The specimen is then cut at both ends of the core to obtain a nominal 150 mm tall test specimen. This process is shown in Figure 4. Actions are taken to secure and support the specimen during coring and cutting to make sure it meets the requirements presented in Table 1.

Figure 4 - Specimen Coring (Left) and Cutting (right)

Table 1 - Test Specimen Dimensional Tolerances

<table>
<thead>
<tr>
<th>Item</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Diameter</td>
<td>100 mm to 104 mm</td>
</tr>
<tr>
<td>Standard Deviation of Diameter</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Height</td>
<td>147.5 to 152.5 mm</td>
</tr>
<tr>
<td>End Flatness</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>End Perpendicularity</td>
<td>1.0 mm</td>
</tr>
</tbody>
</table>
Once the specimens have been cored and cut and meet the specifications presented in Table 1, they are ready to start the testing procedure for the AMPT. As shown in Figure 5, six gauge points are attached to the specimen using a standard epoxy. These six gauge points will house the three Linear Variable Differential Transformers (LVDT’s) that measure the axial deformations during testing. The specimens are then placed in an environmental chamber to condition them for the determined testing temperature. A dummy specimen with a mounted thermocouple at the center is also placed in the environmental chamber for temperature verification. The chamber of the AMPT is allowed to equilibrate at the testing temperature for at least one hour before testing begins.

Figure 5 - Specimen with LVDT's attached in the Environmental Chamber
When the dummy specimen and the testing chamber reach the target temperature, the chamber is opened and the specimen is set in place. Friction reducers are placed on the top and the bottom of the specimen to mitigate the friction effects of the loading platen on the specimen. The LVDT's are then installed between the gauge points. Two compensating springs are attached on the LVDT's to counteract the force generated by the LVDT. The LVDT's are then tared and checked to confirm that they are within their calibrated range. After everything has been installed, the chamber is closed and allowed to equalize to the testing temperature.

While the chamber is returning to the testing temperature, the required identification and control information is entered into the Dynamic Modulus software. The frequencies and temperatures chosen to evaluate the specimens come from suggested values in a pending standard from the American Association of State Highway and Transportation Officials (AASHTO). These temperatures and corresponding frequencies can be seen in Table 2.

<table>
<thead>
<tr>
<th>Test Temperature °C</th>
<th>Loading Frequencies (Hz)</th>
<th>Test Temperature °C</th>
<th>Loading Frequencies (Hz)</th>
<th>Test Temperature °C</th>
<th>Loading Frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10, 1, 0.1</td>
<td>4</td>
<td>10, 1, 0.1</td>
<td>4</td>
<td>10, 1, 0.1</td>
</tr>
<tr>
<td>20</td>
<td>10, 1, 0.1</td>
<td>20</td>
<td>10, 1, 0.1</td>
<td>20</td>
<td>10, 1, 0.1</td>
</tr>
<tr>
<td>35</td>
<td>10, 1, 0.1, and 0.01</td>
<td>40</td>
<td>10, 1, 0.1, and 0.01</td>
<td>45</td>
<td>10, 1, 0.1, and 0.01</td>
</tr>
</tbody>
</table>

Table 2 - Recommended Testing Temperatures and Loading Frequencies
After running the test at the specified temperatures and frequencies, the software included in the machine calculates needed information such as the dynamic modulus, phase angle, and data quality statistics. These data are presented by the program and can be exported into a comma separated values (.csv) file for further analysis.

Data Obtained

Once the data are obtained they are initially evaluated for quality based on the requirements presented in Table 3. The dynamic modulus software uses a standard procedure for calculating needed information. The initial analysis of the data, as well as the calculations for determining the dynamic modulus, phase angle, and data quality statistics are shown in the Appendix.

Table 3 - Data Quality Requirements

<table>
<thead>
<tr>
<th>Data Quality Statistic</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformation Drift</td>
<td>In direction of applied load</td>
</tr>
<tr>
<td>Peak to Peak Strain</td>
<td>75 to 125 micro strain</td>
</tr>
<tr>
<td>Load standard error</td>
<td>10%</td>
</tr>
<tr>
<td>Deformation standard error</td>
<td>10%</td>
</tr>
<tr>
<td>Deformation uniformity</td>
<td>30%</td>
</tr>
<tr>
<td>Phase uniformity</td>
<td>3 degrees</td>
</tr>
</tbody>
</table>
CHAPTER 2

EVALUATION OF THE ASPHALT MIXTURE PERFORMANCE TESTER (AMPT): UTAH EXPERIENCE

Abstract

For over 5 years, Utah Department of Transportation (UDOT) has been collecting field material and testing it using the Asphalt Mixture Performance Tester (AMPT). Thirty four field mixtures were evaluated as part of this study. The mixtures were produced following UDOT standard volumetric specifications and two binder grades, PG 64-34 and PG 70-28, from three suppliers.

Based on the evaluation of the results obtained from these tests, it was found that the AMPT produced data with a coefficient of variation below 15%, indicating good repeatability. Mixtures prepared with different binder grades were easily separated. The dynamic modulus master curves of mixtures prepared using the same binder grade were essentially the same regardless of the aggregate source (same type of mixture was used). Given these results, it was shown that it is possible to obtain asphalt mixture level 1 MEPDG input parameters based on historic data, knowledge of the binder grade used and testing at only one temperature. The single temperature test can be used for quality control and to verify that the mixture is of the same kind as the historical data available. This approach can significantly reduce the time and effort required to obtain AMPT data without any loss in
performance prediction capacity, thus making adoption of this device more appealing to state DOT's.

Introduction

The Asphalt Mixture Performance Tester (AMPT), also known as the Simple Performance Tester (SPT), was developed for testing and evaluating asphalt mixtures. The AMPT was developed as part of the National Cooperative Highway Research program (NCHRP) Project 9-29 [1]. It is used to determine the dynamic modulus of asphalt mixtures both for the purpose of characterization and to provide input values to the Mechanistic-Empirical Pavement Design (MEPDG) software. As with any new technology there is a process of evaluation that is needed prior to adoption and implementation. Such processes involve significant training of the staff and some experimenting regarding the application to specific local conditions. The State of Utah is in the process of implementing this device and, as such, they have collected data for the last 5 years. This paper discusses the findings of such endeavor.

Other states (e.g., Wisconsin, Hawaii) have also evaluated this device and found that:

- For a given aggregate source, mixtures have similar dynamic modulus values when the variability of the testing is considered.
- At high temperatures, the AMPT appears to have the ability to distinguish aggregate source.
- Differences in binder grade do not appear to affect the dynamic modulus values that are produced by the AMPT.
Materials and Protocols

This study is based on 34 field mixes that were produced between 2007 and 2010 and placed on the roads. Material from the back of the paver was collected and sent to the laboratory where the mix was compacted into cylindrical samples using the Superpave Gyratory Compactor (SGC). All samples were compacted to a design of 100 gyrations in accordance to UDOT's specifications [2]. These samples were cored and cut to obtain testing specimens with the final dimensions of 102 mm diameter by 150 mm height according to the requirements set forth in AASHTO PP 60-09. The characteristics of the mixtures used in this study are shown in Table 4.

For each project, four specimens were created. Once the specimens were prepared and evaluated for their requirements of: diameter, height, end flatness and end perpendicularity, they were tested using the AMPT. Three test temperatures were used each with three or four frequencies, as recommended in AASHTO PP 61-09. All the tests were run without any confinement. This resulted in 408 tests with a total of 4,080 data points.

The raw measurements were converted to dynamic modulus values using the software developed by the manufacturer of the AMPT. This software also performed the quality checks on the data to ensure they meet the accepted standards. The dynamic modulus data obtained from the AMPT was imported into a Microsoft Excel file that contained a macro that created the master curve and provided the parameters: $\beta$, $\Gamma$, $E_{\text{min}}$, $E_{\text{max}}$, and $\Delta E_A$. This macro was developed by Dr. Ramon Bonaquist [3].
<table>
<thead>
<tr>
<th>Project</th>
<th>Binder</th>
<th>RAP (%)</th>
<th>Asphalt Content (%)</th>
<th>Air Voids (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US-6 MP 218.7 to Emma Park (Field Mix #2)</td>
<td>C 64-34</td>
<td>15</td>
<td>4.65</td>
<td>3.5</td>
</tr>
<tr>
<td>US-6 MP 218.7 to Emma Park (Field Mix #1)</td>
<td>C 64-34</td>
<td>15</td>
<td>4.65</td>
<td>3.5</td>
</tr>
<tr>
<td>US-6 MP 218.7 to Emma Park Road (Lab #2)</td>
<td>C 64-34</td>
<td>15</td>
<td>4.6</td>
<td>3.5</td>
</tr>
<tr>
<td>US-6 MP 218.7 to Emma Park (Lab Mix #1)</td>
<td>C 64-34</td>
<td>15</td>
<td>4.65</td>
<td>3.5</td>
</tr>
<tr>
<td>Legacy Segment #2 (Field)</td>
<td>A 70-28</td>
<td>15</td>
<td>4.6</td>
<td>3.5</td>
</tr>
<tr>
<td>US-491, Monticello to MP 7 (Field Mix)</td>
<td>E 64-34</td>
<td>0</td>
<td>4.8</td>
<td>3.6</td>
</tr>
<tr>
<td>I-80, Wahsatch to Wyoming State Line (Lab)</td>
<td>D 64-34</td>
<td>0</td>
<td>4.75</td>
<td>3.1</td>
</tr>
<tr>
<td>US-491, Monticello to MP 7 (Lab Mix)</td>
<td>E 64-34</td>
<td>0</td>
<td>4.8</td>
<td>3.6</td>
</tr>
<tr>
<td>Legacy Segment #1 (Field)</td>
<td>B 70-28</td>
<td>15</td>
<td>4.6</td>
<td>3.5</td>
</tr>
<tr>
<td>US-40, Clegg Canyon to Strawberry (Lab)</td>
<td>D 64-34</td>
<td>15</td>
<td>4.6</td>
<td>3.3</td>
</tr>
<tr>
<td>Legacy Segment #2 (Lab)</td>
<td>B 70-28</td>
<td>15</td>
<td>4.6</td>
<td>3.5</td>
</tr>
<tr>
<td>Legacy Segment #1 (Lab)</td>
<td>B 70-28</td>
<td>15</td>
<td>4.6</td>
<td>3.5</td>
</tr>
<tr>
<td>I-15, Arizona St. Ln. to Bluff Street (Field #2)</td>
<td>B 70-28</td>
<td>0</td>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>I-15, Arizona St. Ln. to Bluff Street (Field #1)</td>
<td>B 70-28</td>
<td>0</td>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>I-15, Arizona St. Ln. to Bluff Street (Lab)</td>
<td>B 70-28</td>
<td>0</td>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>Fort Pierce #2</td>
<td>B 70-28</td>
<td>0</td>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>US-40 (Field Mix #4)</td>
<td>D 64-34</td>
<td>15</td>
<td>4.6</td>
<td>3.3</td>
</tr>
<tr>
<td>Fort Pierce #1</td>
<td>B 70-28</td>
<td>0</td>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>US-40 (Field Mix #3)</td>
<td>D 64-34</td>
<td>15</td>
<td>4.6</td>
<td>3.3</td>
</tr>
<tr>
<td>US-40 (Field Mix #2)</td>
<td>D 64-34</td>
<td>15</td>
<td>4.6</td>
<td>3.3</td>
</tr>
<tr>
<td>US-40 (Field Mix #1)</td>
<td>D 64-34</td>
<td>15</td>
<td>4.6</td>
<td>3.3</td>
</tr>
<tr>
<td>I-80, Wahsatch to Wyoming (Field)</td>
<td>D 64-34</td>
<td>0</td>
<td>4.75</td>
<td>3.1</td>
</tr>
<tr>
<td>I-80, Wahsatch to Wyoming (Field)</td>
<td>D 64-34</td>
<td>0</td>
<td>4.75</td>
<td>3.1</td>
</tr>
<tr>
<td>I-80, Wahsatch to Wyoming (Field)</td>
<td>D 64-34</td>
<td>0</td>
<td>4.75</td>
<td>3.1</td>
</tr>
<tr>
<td>I-80, Wahsatch to Wyoming (Field)</td>
<td>D 64-34</td>
<td>0</td>
<td>4.75</td>
<td>3.1</td>
</tr>
<tr>
<td>I-80, Wahsatch to Wyoming (Lab)</td>
<td>D 64-34</td>
<td>0</td>
<td>4.75</td>
<td>3.1</td>
</tr>
<tr>
<td>SPT #1</td>
<td>F 64-34</td>
<td>No Info</td>
<td>No Info</td>
<td>No Info</td>
</tr>
<tr>
<td>SPT #2</td>
<td>F 64-34</td>
<td>No Info</td>
<td>No Info</td>
<td>No Info</td>
</tr>
<tr>
<td>Geneva W-Pioneer</td>
<td>F 64-34</td>
<td>No Info</td>
<td>No Info</td>
<td>No Info</td>
</tr>
<tr>
<td>Cox W-Crown</td>
<td>F 64-34</td>
<td>No Info</td>
<td>No Info</td>
<td>No Info</td>
</tr>
<tr>
<td>Cox Pit W-Idaho (Field Mix #1)</td>
<td>F 64-34</td>
<td>No Info</td>
<td>No Info</td>
<td>No Info</td>
</tr>
<tr>
<td>Cox Pit W-Pioneer</td>
<td>F 64-34</td>
<td>No Info</td>
<td>No Info</td>
<td>No Info</td>
</tr>
<tr>
<td>Cox Pit W-Idaho (Field Mix #2)</td>
<td>F 64-34</td>
<td>No Info</td>
<td>No Info</td>
<td>No Info</td>
</tr>
<tr>
<td>Echo TLA 2002 Samples</td>
<td>F 64-34</td>
<td>No Info</td>
<td>No Info</td>
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</table>

<table>
<thead>
<tr>
<th>Binder Reference</th>
<th>Supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binder A PG 70-28</td>
<td>Supplier 1</td>
</tr>
<tr>
<td>Binder B PG 70-28</td>
<td>Supplier 2</td>
</tr>
<tr>
<td>Binder C PG 64-34</td>
<td>Supplier 1</td>
</tr>
<tr>
<td>Binder D PG 64-34</td>
<td>Supplier 2</td>
</tr>
<tr>
<td>Binder E PG 64-34</td>
<td>Supplier 3</td>
</tr>
<tr>
<td>Binder F PG 64-34</td>
<td>Assumed</td>
</tr>
</tbody>
</table>
Variability

The first analysis conducted from the data was the within lab variability of the test. The variability from specimen to specimen in the dynamic modulus test data can be the result of many discrepancies in the long testing process. While care was taken to ensure that the specimens were as similar as possible, it can be assumed that there is a fraction of the testing error attributed to the mixing and compaction process, coring and cutting of the specimen and mounting the studs for the LVDT's. This process has been discussed elsewhere and will not be directly evaluated in this paper [5-9].

The AMPT used in this study used spring loaded LVDT's. The LVDT's themselves create a spring force on the studs. This force is counteracted by compensating springs on either side of the LVDT's. As part of this study the forces between the LVDT's and the compensating springs was measured [4]. It was determined that the springs were not providing enough force to counteract the force caused by the LVDT's; furthermore, the magnitude of the force exerted by the springs was not constant across different LVDT's. This inconsistency and lack of compensating force caused the LVDT's to report inaccurate data, especially at high temperatures where the specimen is at its weakest state. On some occasions the LVDT force can slowly tear the studs off of the specimen.

All of the factors listed above, when combined, results in the observed variability. If this variability is too high, then the reliability of the results is questionable. The within lab variability within specimens of the same project was compared directly from the data produced by the AMPT for each specimen. For each temperature and frequency combination the standard deviation was calculated and compared to the mean; thus the variability expressed as a coefficient of variation
(CV) was investigated. Table 5 shows that there is a general increase in the CV value for both a decrease in frequency and an increase in temperature. These increases are representative of all of the data that was collected as part of this study.

To evaluate the CV for each project, a temperature of 20 °C and frequency of 1 Hz was selected and is shown in Figure 6. All four specimens from each project were used in the computation of the statistic; outliers were not excluded from the analysis. As seen in Figure 6, most of the values are below 15%. There is an irregularly high CV value of 24%. This individual project was examined and an obvious outlier in the data was found. If this single data point is removed from the computation of the statistics, a CV value of 8% is obtained for this project. Due to the rarity of this outlier, it is believe that it was caused by operator error rather than variability in the AMPT testing process.

Table 5 - CV Values for 34 projects

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Frequency (Hz)</th>
<th>CV (avg) %</th>
<th>Highest CV Value</th>
<th>Lowest CV Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10</td>
<td>5.8</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>6.6</td>
<td>16.5</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>8</td>
<td>19.9</td>
<td>2.4</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>7.8</td>
<td>17.3</td>
<td>1.2</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>10.2</td>
<td>24</td>
<td>2.7</td>
</tr>
<tr>
<td>20</td>
<td>0.1</td>
<td>12.5</td>
<td>29.9</td>
<td>5</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
<td>12.5</td>
<td>26.2</td>
<td>5.1</td>
</tr>
<tr>
<td>40</td>
<td>1</td>
<td>14.8</td>
<td>27.8</td>
<td>1</td>
</tr>
<tr>
<td>40</td>
<td>0.1</td>
<td>14.9</td>
<td>26.3</td>
<td>4</td>
</tr>
<tr>
<td>40</td>
<td>0.01</td>
<td>13.6</td>
<td>24.3</td>
<td>6</td>
</tr>
</tbody>
</table>
Based on 34 projects evaluated, it was found that the CV of this test at 20°C and 1 Hz is, on average, 10.2%. As the temperature increases, this value also increases but, on average, the CV was always below 15%.

Characterization

Once the variability of the test device was determined, the next step was to evaluate the difference in the result between projects. Projects that have different mix designs were compared through the dynamic modulus master curves they created [5]. Figure 7 shows a compilation of all 34 projects evaluated. The curves were examined and it was found that there is a distinct separation of the curves. When the curves are coded for binder used in the mix design, the separation becomes clear (Figure 8). The curves are separated by binder grade and, as expected, the mixtures prepared with a PG 70 binder resulted in higher dynamic modulus as compared with mixtures prepared with a PG 64 binder, regardless of supplier. The
AMPT appears to have the ability to group projects of the same binder grade together. Of course, it would be of interested to determine if the test is capable of grouping mixtures based on aggregate or volumetric differences. However, all mixtures used as part of this study had to meet the requirements set forth by the Utah Department of Transportation. Thus all mixtures have similar design volumetrics with low absorption quartzite used as the predominant aggregate, making it impossible to assess the tests ability to evaluate different aggregate structures. Other researchers have looked into this issue and found that the AMPT does have the ability to differentiate different aggregate sources in the high temperature testing [13,14].

Figure 7 - Collection of Dynamic Modulus Master Curves
Figure 8 - Dynamic Modulus Master Curves, Sorted by Binder

**Parameters**

In order to evaluate the dynamic modulus, a master curve is usually created. The shape of the master curve is characterized by five parameters. The details of how these parameters are obtained and the meaning of them can be found in reference [1]. These parameters are: $\beta$, $\Gamma$, $E_{\min}$, $E_{\max}$, and $\Delta E_A$. The parameter $E_{\max}$ is calculated directly from the volumetrics of the mix so this is not considered a fitted parameter and was not evaluated. The rest of the parameters defined the master curve; thus it is of interest to look at any possible relations in these parameters as a function of mixture components (at least binder grade). By finding relations between these parameters and mixture components, the fitting process can be simplified. Table 6 shows the ranges of these parameters based on all of the data collected in this project.
Table 6 - Parameter Ranges for Each Type of Binder

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PG 70-28</th>
<th></th>
<th></th>
<th>PG 64-34</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Average</td>
<td>Min</td>
<td>Max</td>
<td>Average</td>
</tr>
<tr>
<td>Max E* (ksi)</td>
<td>3384</td>
<td>3419</td>
<td>3409</td>
<td>3317</td>
<td>3438</td>
<td>3376</td>
</tr>
<tr>
<td>Min E* (ksi)</td>
<td>2.3</td>
<td>5.3</td>
<td>3.1</td>
<td>0.6</td>
<td>19.3</td>
<td>4.7</td>
</tr>
<tr>
<td>β</td>
<td>-1.43</td>
<td>-0.83</td>
<td>-1.25</td>
<td>-1.02</td>
<td>-0.29</td>
<td>-0.76</td>
</tr>
<tr>
<td>Γ</td>
<td>-0.52</td>
<td>-0.50</td>
<td>-0.51</td>
<td>-0.59</td>
<td>-0.40</td>
<td>-0.50</td>
</tr>
<tr>
<td>ΔEA</td>
<td>197113</td>
<td>211628</td>
<td>201180</td>
<td>183761</td>
<td>205113</td>
<td>195287</td>
</tr>
</tbody>
</table>

After examining the data it can be said that beta has the most sensitivity to binder grades with an average of -1.25 for PG 70-28 and -0.76 for PG 64-34. The average value of gamma is essentially the same for both binder grades; however, a larger range of data values is observed for the PG 64-34 binder. The parameter ΔEA, which is used to generate the time-temperature shift factors, shows very little variation in values within all of the mixtures evaluated of the same binder grade. This implies that there is very little variation in the time-temperature shift factor of the mixtures evaluated in this study even though three different suppliers were
used. This means that a value of 195,287 could be used for mixtures using a PG 64-34 binder and a value of 201,180 could be used for mixtures using a PG 70-28 without much loss in predictive capabilities. The practical results of the values shown in Table 6 are discussed next.

Using the ranges shown on Table 6, quality control checks could be performed on any given mixture. If a given mixture results in parameters outside those shown in Table 6, it should be further investigated since it probably does not have the same properties as the mixtures previously tested by UDOT (i.e., different binder or different volumetrics as those typically specified in Utah). The parameter values obtained as part of this work were compared to the values reported as part of a similar study in Wisconsin [6].

Table 7 shows that the average value from Wisconsin falls within the ranges obtained in Utah. The most noticeable differences would be that Min E* is significantly higher for the mixtures in Wisconsin as compared to the mixtures in Utah. This has implications at high temperature and low speed performance predictions. The value of beta also seems to be higher for the Wisconsin mixtures as compared to the Utah mixtures. Some of the Wisconsin mixtures contained PG 58-34 binders; as shown in Table 6, high beta values imply lower high temperature binder grade. It should be noted that UDOT specifications have additional requirements for asphalt binders from those listed on the AASHTO M320. UDOT requires that the asphalt binders have a minimum value of phase angle. This means that a PG 64-28 graded according to UDOT specifications might result in a higher grade (i.e., PG 70-28) if only AASHTO M320 requirements are followed.
Table 7 - Wisconsin Parameter Ranges

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Wisconsin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max $E^*$ (ksi)</td>
<td>3118 3225 3165</td>
</tr>
<tr>
<td>Min $E^*$ (ksi)</td>
<td>6.1 33.7 17.1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.53 -0.15 -0.29</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>-0.62 -0.51 -0.55</td>
</tr>
<tr>
<td>$\Delta E_A$</td>
<td>185084 214463 196303</td>
</tr>
</tbody>
</table>

Discussion

Based on the data analyzed, it is evident that the master curves obtained from the AMPT fall almost exclusively along the binder grade. This would indicate that the AMPT has the ability to differentiate binder grades but it is unknown whether or not the AMPT can detect changes in the mix design as all mixtures evaluated had similar design volumetrics. The lack of variation in the mix designs may be the reason why binder grade was the only parameter that affected the dynamic modulus values measured by the AMPT.

The Wisconsin data use PG 58-28 and PG 70-28 binders and two separate mix designs for different traffic scenarios. The objective of that study was to determine if aggregate source has an impact on the results of the AMPT. It was determined in that study that the AMPT, at high temperatures, could differentiate one aggregate source from another. It was also determined that the binder grades
along with the traffic scenarios did not statistically impact the dynamic modulus values. Therefore, unlike what was observed in Utah, the master curves for the Wisconsin data do not fall exclusively along binder grades.

Of interest is the fact that when the master curves of the Wisconsin data and the Utah data are compared, the Wisconsin mixes appear to have higher dynamic modulus values at lower frequencies. This is opposite of the trend that would be expected due to the lower binder grade used in 2/3 of the Wisconsin mixes. The reasons for this are not known but the implications are that a higher modulus would result in better performance at high temperatures or lower speeds.

Initially, it was assumed that from the parameters, data trends could be determined that would correspond to the different master curves created by the different binder grades. However, it appears that the parameters are not independent of each other. There may also be other factors of the mix design that play an active role in determining the parameters. One of these factors may be aggregate properties and gradation. After comparing the data collected from Wisconsin with the data from Utah, a distinct separation of the master curves occurs at lower frequencies. When aggregate gradation is compared between the two (Figure 9), an obvious difference is observed with mixtures from Utah having a coarser gradation. This difference may play a role in the separation of the two groups of master curves as some gradations might be more sensitive to changes in binder grade than others. Unfortunately, with the data available such statements cannot be verified.
One of the main objectives for the development and use of the AMPT is to provide input data for the Mechanistic Empirical Design Guide (MEPDG) [7]. The data consist of dynamic modulus values over a range of temperatures and frequencies.

When AMPT data are not available, the MEPDG analysis is done using level 2 for asphalt properties which uses aggregate gradation along with binder properties to estimate the asphalt properties. To compare the difference between the predictions obtained using levels 1 and 2, a project previously run using level 2 data was compared with level 1 data. The results of total rut depth over the life of the pavement can be seen in Figure 10. This presents a visual of the effect of level 1 asphalt properties over a level 2.
With nearly a 50% decrease in predicted rut depth over the life of this pavement it can be stated that there is a significant impact when using the dynamic modulus values in level 1 over the aggregate gradation of level 2 for the asphalt properties. These results should come as no surprise to most users; however, the complexities and time requirements of running the AMPT device and analyzing the data produced create a valid reason for using a level 2 analysis. Thus, it would be desirable to simplify the process by reducing testing time. In light of the results shown in Table 6, it was questioned whether or not using three temperatures (4°C, 20°C, 40°C) when running the AMPT is necessary, or if a single temperature (20 ºC) is sufficient. Using 20°C would drastically reduce the time needed to run the AMPT.
and evaluate the data. However, by only using 20°C data no shift factors would be obtained so an average of the ΔEA values would have to be used (alternatively, binder tests can be used to help in this process). As was previously discussed, ΔEA does not change significantly within the same binder grade, so using only one test temperature is still reasonable. When using only the 20°C data collected from the AMPT and the resulting dynamic modulus values for the MEPDG, a rutting performance over the life of the pavement was evaluated and compared to the original level 1 rutting performances. Several projects were evaluated and after examination of Figure 11, it is apparent that there is almost no difference in the predicted rut depth of the pavement over the design life.

Figure 11 - MEPDG Rutting over the life of the pavement
After reviewing Figure 11 it is clear that using only one temperature (20°C) as an alternative to using three temperatures (4°C, 20°C, 40°C) provides almost the same predicted rut depth over the design life of the pavement. Note that the project US40 has a binder grade of PG 64-34. This project resulted in a greater rut depth as compared to the projects using a PG 70-28 binder. This is expected and shows the AMPT’s and MEPDG’s ability to distinguish binder grades. Except for the fact that the shift factors need to be estimated, there is no difference in performance prediction when only one temperature (20°C) was used.

Application

One of the most important considerations is how the AMPT will be used within the mixture design and verification process.

In general, mixture properties are selected in consideration of the following performance requirements:

1 – Stability at high temperatures. (rutting resistance)
2 – Durability (moisture susceptibility)
3 – Low temperature flexibility (resists cracking)
4 – Fatigue

Requirements 1 and 2 are currently evaluated by Utah DOT using the Hamburg Wheel Tracking Device [8]. For requirement 3 the test temperatures of the AMPT are not low enough at this point. Furthermore, an alternative low temperature testing is being implemented in the form of the Bending Beam Rheometer [9]. For requirement 4, the intermediate temperatures (4 to 20 ºC) currently used in the AMPT seem ideal to evaluate this distress. Work by Shenoy
and Romero [10] has suggested that perhaps dynamic data can provide information regarding fatigue of asphalt mixtures; unfortunately, the work has been mostly theoretical. At this point, more work needs to be done to evaluate fatigue and predict mixture performance.

Given these considerations, the application of the AMPT needs to be carefully evaluated. One added application that needs to be further explored is to use the AMPT to back calculate the effective binder grade of a mix design. While these data do not allow the determining of the effect of recycled asphalt pavement (RAP) or recycled asphalt shingles (RAS) on the aggregate properties, they should allow for an assessment of the contribution of RAP or RAS to the binder characteristics. Further work is underway to evaluate this statement.

Conclusions

After evaluation of the testing results and the resulting dynamic modulus master curves, the AMPT appears to be repeatable and has the ability to generate precise data. Based on these observations, the AMPT appears to be a viable control device for asphalt concrete pavements.

It was found that the AMPT has the ability to separate mixtures based on binder performance grades. Mixtures prepared using binders of the same grade result in essentially the same dynamic modulus master curve, regardless of aggregate source. Thus, the same performance predictions will likely be produced for a given structure and loading condition. Within the constraints of the available field data it is concluded that all binders with the same grade will result in essentially the same master curve, the parameters: $E_{max}$, $\beta$, $\Gamma$, $E_{min}$, and $\Delta EA$ that define the master curve fall within a narrow range presented in Table 6.
Considering that the test does not detect variations in mixture aggregate source and given that most mixtures are designed based on established volumetric requirements, after some initial work one can simply select the parameters normally obtained from testing done on the AMPT based on an existing database. Obviously, this can be said for Utah mixtures due to very similar design characteristics between projects but it is suspected that other states will run into this situation.

Due to the process of master curve formation (overlap), a considerable amount of the data is obtained from tests at 20°C. This implies that a simplification can be made so that the required number of temperatures is reduced from three to one. Comparisons were done using one temperature versus three temperatures and the resulting performance predictions of the MEPDG show no difference.

Recommendations

For those highway agencies currently not implementing the AMPT device, it is recommended that a database be collected that characterizes the typical mixtures used throughout the state. Once enough data have been collected, input parameters for the MEPDG can be easily selected. From that point on, the AMPT could be used as a quality control/check at 20°C by adding two more frequencies for a total of five (.01, .1, 1, 10, 25)Hz at 20°C. This will extend the range of the 20°C data and ensure more accurate predictions. As long as the results are similar to the database, performance would be the same.
Acknowledgements

The authors are grateful for the assistance provided by Clark Allen and Bin Shi, from UDOT. The analysis of the data was done using software provided by Dr. Ricardo Archilla from the University of Hawaii, Manoa; and Dr. Ramon Bonaquist from Advanced Asphalt Technologies. Their willingness to share their work is greatly appreciated.

Disclaimer

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CHAPTER 3

CLOSURE

Summary

Based on the results from this work, it was recommended that:

- The input parameters needed for a level 1 structural pavement design be obtained from the database presented in this work
- AMPT tests be run at a single temperature of 20 ºC to ensure that new asphalt mixtures have the same properties as those evaluated in this work
- If needed, the same data obtained at 20 ºC could be used as quality assurance that the properties of the mixture placed in the field are consistent to those used during the design process.

Conclusion

As part of this work, the AMPT was evaluated as a test method for asphalt mixtures. It is concluded that the information gained from this test can provide valuable insight into the properties of asphalt mixture; this can help in making longer lasting roads at a significant benefit to the traveling public.

Future Work

This research considers the dynamic modulus test, which is only one of the three tests that the AMPT is capable of performing. The flow time and flow number
tests are currently unstandardized and have not been heavily researched. These remaining tests would be of interest for future research and analysis.
The standard error of the applied load is a measure of the difference between the measured load data and the best fit sinusoid. The standard error of the load is defined in Equation 12.

\[
se(P) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n-4} \left(\frac{100\%}{\hat{x}_0}\right)}
\]  

(1)

where:

\(se(P)\) = standard error of the applied load,

\(x_i\) = measured load at point I,

\(\hat{x}_i\) = predicted load at point I from the best fit sinusoid,

\(n\) = total number of data points collected during test, and

\(\hat{x}_0\) = amplitude of the best fit sinusoid.

**Analyze Stress Data**

The first step in the analysis is to analyze the data in the stress array. The data analysis is performed on centered stress data, which are computed from the raw stress data by subtracting the average stress. Equation 2 defines the average stress.
where:

\[ \bar{\sigma} = \frac{\sum_{i=1}^{n} \sigma_i}{n} \] (2)

\[
\tau = \text{average stress,} \\
\sigma_i = \text{raw stress point } i \text{ in the data array, and} \\
\sigma_i = \text{number of points in the data array.}
\]

The centered stresses are then computed by subtracting the average stress from each of the stress measurements.

\[
\sigma_i' = \sigma_i - \bar{\sigma} \tag{3}
\]

where:

\[
\sigma_i' = \text{centered stress at point } i \text{ in the data array,} \\
\sigma_i = \text{raw stress point } i \text{ in the data array, and} \\
\bar{\sigma} = \text{average stress}
\]

From the centered stress data, the three stress coefficients offset, in-phase magnitude, and out-of-phase magnitude are computed using the following equations.

\[
A_{\sigma_0} = \sum_{i=1}^{n} \sigma_i' \tag{4}
\]
\[ A_{\sigma 1} = \frac{2}{n} \sum_{i=1}^{n} \sigma'_i \cos(\omega_0 t_i) \quad (5) \]

\[ B_{\sigma 1} = \frac{2}{n} \sum_{i=1}^{n} \sigma'_i \sin(\omega_0 t_i) \quad (6) \]

where:

- \( A_{\sigma 0} \) = stress offset coefficient, kPa (psi);
- \( \sigma'_i \) = centered stress at point i in the data array;
- \( A_{\sigma 1} \) = stress in-phase magnitude coefficient, kPa (psi);
- \( \omega_0 \) = frequency of applied stress, rad/sec;
- \( t_i \) = time at point i in the data array, sec; and
- \( B_{\sigma 1} \) = stress out-of-phase magnitude coefficient, kPa (psi).

From the stress coefficients, the stress magnitude and the stress phase angle are computed using the following equations.

\[ |\sigma^*| = \sqrt{A_{\sigma 1}^2 + B_{\sigma 1}^2} \quad (7) \]

\[ \theta_a = \arctan\left(-\frac{B_{\sigma 1}}{A_{\sigma 1}}\right) \quad (8) \]

where:

- \(|\sigma^*|\) = stress magnitude, kPa (psi);
- \(A_{\sigma 1}\) = stress in-phase magnitude coefficient, kPa (psi);
An array of predicted centered stresses and the standard error of the applied stress are computed using the following equations.

\[ \hat{\sigma}'_i = A_{\sigma_0} + A_{\sigma_1} \cos(\omega_0 t_i) + B_{\sigma_1} \sin(\omega_0 t_i) \]  

\[ se(\sigma) = \sqrt{\frac{\Sigma_{i=1}^{n} (\hat{\sigma}'_i - \sigma'_i)^2}{n-4}} \left( \frac{100\%}{|\sigma^*|} \right) \]

where:

- \( \hat{\sigma}'_i \) = predicted centered stress at point i, kPa (psi);
- \( A_{\sigma_0} \) = stress offset coefficient, kPa (psi)
- \( A_{\sigma_1} \) = stress in-phase magnitude coefficient, kPa (psi);
- \( \omega_0 \) = frequency of applied stress, rad/sec;
- \( t_i \) = time at point i in the data array, sec;
- \( B_{\sigma_1} \) = stress out-of-phase magnitude coefficient, kPa (psi);
- \( se(\sigma) \) = standard error for the applied stress, percent;
- \( \sigma'_i \) = centered stress at point i in the data array;
- \( n \) = number of points in data array; and
- \( |\sigma^*| \) = stress magnitude, kPa (psi).
Analyze Strain Data

The second step in the analysis is to perform a similar analysis on the data from each of the strain transducers. However, in this case the data are corrected for drift caused by permanent deformation during the test, and centered data based on the average strain for the transducer.

The average strain for each transducer is computed using the following equation.

\[
\bar{\varepsilon}_j = \frac{\sum_{i=1}^{n} \varepsilon_{ji}}{n}
\]  

(11)

where:

\( \bar{\varepsilon}_j \) = average strain for transducer \( j \),

\( \varepsilon_{ji} \) = raw strain for transducer \( j \) at point \( i \) in data array, and

\( n \) = number of points in the data array.

The strain data are corrected and centered for each transducer by subtracting from the measured strains the rate of drift times the loading time and also subtracting the average strain for the transducer. The following equation illustrates this concept.

\[
\varepsilon_{ji}' = \varepsilon_{ji} - D_j t_i - \bar{\varepsilon}_j
\]  

(12)
where:

\( \varepsilon_{ji}^{\prime} \) = corrected and centered strain for transducer j at point i in data array,

\( \varepsilon_{ji} \) = raw strain for transducer j at point i in data array,

\( D_j \) = rate of drift for transducer j,

\( t_i \) = time for point i in data array,

\( \bar{\varepsilon}_j \) = average strain for transducer j, and

From the corrected and centered strain data for each strain transducer, three strain coefficients are computed: offset, in-phase magnitude, and out-of-phase magnitude.

\[
A_{\varepsilon j0} = \frac{\sum_{i=1}^{n} \varepsilon_{ji}^{\prime}}{n} \tag{13}
\]

\[
A_{\varepsilon j1} = \frac{2}{n} \sum_{i=1}^{n} \varepsilon_{ji}^{\prime} \cos(\omega_0 t_i) \tag{14}
\]

\[
B_{\varepsilon j1} = \frac{2}{n} \sum_{i=1}^{n} \varepsilon_{ji}^{\prime} \sin(\omega_0 t_i) \tag{15}
\]

where:

\( A_{\varepsilon j0} \) = offset coefficient for strain transducer j;

\( \varepsilon_{ji}^{\prime} \) = corrected and centered strain for transducer j at point I in data array;

\( A_{\varepsilon j1} \) = in-phase magnitude coefficient for strain transducer j;

\( \omega_0 \) = frequency of applied stress, rad/sec;
\[ t_i = \text{time for point I in data array, sec}; \text{ and} \]

\[ B_{\varepsilon j} = \text{out of phase magnitude coefficient for strain transducer } j. \]

From the strain coefficients, the strain magnitude and the strain phase angle for each transducer are computed using the following equations.

\[ |\varepsilon_j^*| = \sqrt{A_{\varepsilon j}^2 + B_{\varepsilon j}^2} \quad (16) \]

\[ \theta_{\varepsilon j} = \arctan \left( -\frac{B_{\varepsilon j}}{A_{\varepsilon j}} \right) \quad (17) \]

where:

\[ |\varepsilon_j^*| = \text{strain magnitude for strain transducer } j; \]

\[ A_{\varepsilon j} = \text{in-phase magnitude coefficient for strain transducer } j; \]

\[ B_{\varepsilon j} = \text{out-of-phase magnitude coefficient for strain transducer } j; \text{ and} \]

\[ \theta_{\varepsilon j} = \text{phase angle for strain transducer } j, \text{ degrees.} \]

For each strain transducer, an array of predicted corrected and centered strains and the standard error of the strain data is computed using the following equations.

\[ \hat{\varepsilon}_{j i} = A_{\varepsilon j 0} + A_{\varepsilon j} \cos(\omega_0 t_i) + B_{\varepsilon j} \sin(\omega_0 t_i) \quad (18) \]
\[
se(\epsilon_j) = \sqrt{\frac{\sum_{i=1}^{n}(\tilde{\epsilon}_{ji} - \epsilon_{ji})^2}{n-4} \left(\frac{100\%}{|\epsilon_j|}\right)}
\] (19)

where:

\(\tilde{\epsilon}_{ji}'\) = predicted corrected and centered strain for strain transducer \(j\) at point \(i\);  
\(A_{\epsilon_{j0}}\) = offset coefficient for strain transducer \(j\);  
\(A_{\epsilon_{ji1}}\) = in-phase magnitude coefficient for strain transducer \(j\);  
\(\omega_0\) = frequency of applied stress, rad/sec;  
\(t_i\) = time for point \(i\) in data array, sec;  
\(B_{\epsilon_{ji1}}\) = out-of-phase magnitude coefficient for strain transducer \(j\) response, percent;  
\(se(\epsilon_j)\) = standard error for strain transducer \(j\) response, percent;  
\(\epsilon_{ji}'\) = corrected and centered strain for transducer \(j\) at point \(I\) in data array;  
\(n\) = number of points in data array; and  
\(|\epsilon_j^*|\) = strain magnitude for strain transducer \(j\).

The average phase angle, strain magnitude, and standard error for all \(m\) strain transducers, along with two uniformity coefficients representing the variation among the strain transducers are calculated as follows.

\[
\bar{\theta}_e = \frac{\sum_{j=1}^{m}\theta_{\epsilon_j}}{m}
\] (20)

\[
|\bar{\epsilon}^*| = \frac{\sum_{j=1}^{m}|\epsilon_j^*|}{m}
\] (21)
\[ se(\varepsilon) = \frac{\sum_{j=1}^{m} se(\varepsilon_j)}{m} \]  \hspace{1cm} (22) \\

\[ U_\varepsilon = \sqrt{\frac{\sum_{j=1}^{m} (|\varepsilon_j^*| - |\varepsilon^*|)^2}{m-1} \left( \frac{100}{|\varepsilon^*|} \right)} \]  \hspace{1cm} (23) \\

\[ U_\theta = \sqrt{\frac{\sum_{j=1}^{m} (\theta_{\varepsilon_j} - \bar{\theta}_\varepsilon)^2}{m-1}} \]  \hspace{1cm} (24) \\

where:
- \( \bar{\theta}_\varepsilon \) = average phase angle for all strain transducers, degrees;
- \( m \) = number of strain transducers;
- \( |\varepsilon^*| \) = average strain magnitude;
- \( se(\varepsilon) \) = average standard error for all strain transducers, percent;
- \( U_\varepsilon \) = uniformity coefficient for strain transducers, percent; and
- \( U_\theta \) = uniformity coefficient for phase angle, degrees.
BIBLIOGRAPHY


