Communications

A Mathematical Basis for the Application of the Modified Geometric Method to Maximum Frequency Estimation

Kumari L. Fernando*, V. John Mathews, and Edward B. Clark

Abstract—**The application of ultrasound in assessing the fetal cardiovascular system often requires the accurate estimation of maximum blood flow velocity waveforms using Doppler measurements. The modified geometric method estimates the maximum Doppler frequency as the frequency at which the vertical distance between the integrated spectrum and the reference line that connects the origin to the maximum value of the integrated spectrum is the largest. This paper presents a mathematical formulation for a class of maximum blood flow velocity estimation algorithms that includes the modified geometric method. The analysis provides a rationale for the continued use of the modified geometric method for estimating the maximum frequency envelopes of Doppler signals. This paper also contains experimental results demonstrating the superiority of the modified geometric method over a commonly used threshold crossing method.**

Index Terms—**Integrated spectrum, maximum flow velocity, modified geometric method.**

I. Introduction

In Doppler ultrasonography, the frequency difference between the transmitted and backscattered waves, the Doppler shift, is directly proportional to the relative velocity between the reflective interface and the receiver. This velocity is estimated using the relationship [1]

$$\nu = \frac{cf}{2f_o \cos\theta} \tag{1}$$

where c is the velocity of ultrasound in the medium in meters (m)/s, f is the Doppler shift frequency in Hz, f_o is the frequency of the incident ultrasound beam in Hz, and θ is the angle of incidence in radians.

Doppler insonation of the blood vessel produces a spectrum of frequencies. The maximum frequency of the Doppler spectrum, $f_{\max}$ is proportional to the maximum blood flow velocity. In current practice, the maximum frequency is estimated as the largest frequency value at which the Doppler spectrum exceeds a preselected threshold [2]. In [3], Moraes *et al.* presented the modified geometric method in which a reference line was selected from the first point or origin of the integrated spectrum to its maximum point. The maximum frequency was then estimated as the frequency corresponding to the maximum vertical distance from the reference line to the integrated spectrum. However, no theoretical basis for the optimal performance of this method was given in [3]. In Section II, we provide a theoretical basis for the continued use of the modified geometric method [3]. For this, we analyze a more general integrated spectrum-based class of algorithms that includes the modified geometric method. Experimental results involving both simulated and real arterial Doppler measurements using the modified geometric method are presented in Section III. This section also contains a performance comparison of the modified geometric method with the conventional thresholding method in [2]. Finally, the concluding remarks are made in Section IV.

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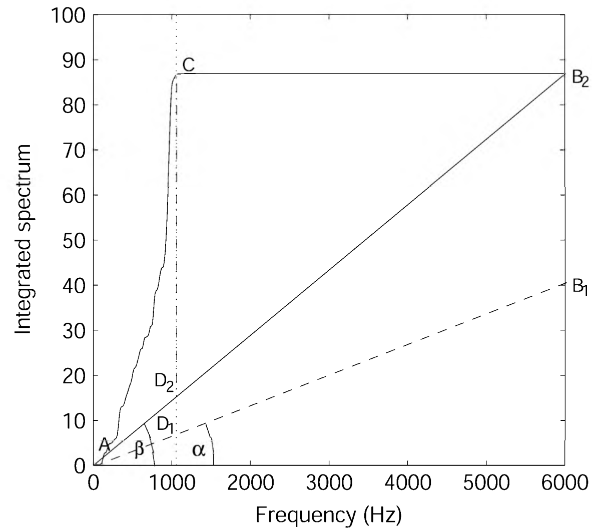

Fig. 1. Integrated spectrum of a portion of a Doppler signal recorded from the umbilical artery of an 18-week-old fetus.

II. Performance of the Modified Geometric Method

Let $y(n) = x(n) + \eta(n)$ represent the digitized Doppler signal at time n, where $x(n)$ is the signal of interest and $\eta(n)$is a zero-mean and white noise process with variance σ_η^2. Let f_N denote half the sampling frequency of the signal. We assume that $x(n)$ is bandlimited to the frequency $f_{\max}$, where $f_{\max} < f_N$. The integrated spectrum of the signal component $x(n)$ is given by

$$\phi_X(f) = \int_0^f S_X(\lambda)\, d\lambda \tag{2}$$

where $S_X(f)$ is its power spectrum at frequency f. The integrated spectrum of $y(n)$ is given by

$$\phi_Y(f) = \phi_X(f) + \sigma_\eta^2 f. \tag{3}$$

Since $x(n)$ is bandlimited to $f_{\max}$, $\phi_Y(f) = \phi_X(f_{\max}) + \sigma_\eta^2 f$ for $f > f_{\max}$. An example of the estimated integrated spectrum of a portion of a Doppler signal recorded from the umbilical artery of an 18-week-old fetus is depicted in Fig. 1.

We estimate the maximum Doppler frequency using the following method. In this method, we draw a straight line that subtends an angle α to the frequency axis and passes through the origin ($\mathtt{AB}_1$). The vertical distance between the integrated spectrum and the straight line $\mathtt{AB}_1$ at any frequency f is given by the rotated-integrated spectrum

$$\hat{\phi}_{\rm rot}(f) = \phi_Y(f) - f\tan\alpha. \tag{4}$$

The frequency at which $\hat{\phi}_{\rm rot}(f)$ is maximum ($\mathtt{CD}_1$) is the estimate of the maximum Doppler shift. The special case of the modified geometric method [3] is obtained when the straight line passes through the maximum value of the integrated spectrum (AB_2). In this case, we denote the angle between AB_2 and the frequency axis as β and is given by

$$\tan\beta = \frac{\varphi_Y(f_N)}{f_N} = \frac{\varphi_X(f_{\max})}{f_N} + \sigma_\eta^2. \tag{5}$$

In what follows, we prove that the maximum vertical distance occurs at the maximum Doppler shift for signals that satisfy the following smoothness constraint:

$$\frac{\varphi_X(f_{\max})}{f_N} < \frac{\varphi_X(f_{\max}) - \varphi_X(f)}{f_{\max} - f}; \quad 0 \le f \le f_{\max}. \tag{6}$$

This condition implies that the average spectrum of the input signal over the whole frequency range is smaller than the average spectrum computed in the interval $[f, f_{\max}]$ for any f within the signal bandwidth. This is a smoothness condition which constrains the signal spectrum to be relatively flat and not to exhibit significant dynamics over its bandwidth. For lowpass signals such that $f_N \gg f_{\max}$, this assumption is satisfied easily in most practical situations.

The maximum velocity reconstruction algorithm is based on the following lemma.

Lemma: For signal spectra satisfying (3) and (6), the maximum value of $\hat{\varphi}_{\mathrm{rot}}(f)$ defined in (4) occurs at the maximum Doppler shift for any angle of rotation α that satisfies

$$\sigma_\eta^2 < \tan\alpha \le \frac{\varphi_X(f_{\max})}{f_N} + \sigma_\eta^2. \tag{7}$$

The lower limit of $\tan\alpha$ is chosen such that it is greater than the noise density. The maximum limit of the angle of rotation is chosen such that the reference line passing through the origin and drawn at an angle α to the frequency axis does not intersect the integrated spectrum at any frequency value less than f_N.

Proof: We prove the lemma by contradiction. Let us assume that there exists a frequency $\tilde{f}(\neq f_{\max})$ at which the rotated-integrated spectrum is greater than its value at $f_{\max}$. That is

$$\hat{\varphi}_{\mathrm{rot}}(\tilde{f}) > \hat{\varphi}_{\mathrm{rot}}(f_{\max}) \tag{8}$$

at some $\tilde{f} \neq f_{\max}$.

Let us consider the two frequency ranges $\tilde{f} < f_{\max}$ and $\tilde{f} > f_{\max}$ separately.

Case I: $0 < \tilde{f} < f_{\max}$*:* Substituting for $\hat{\varphi}_{\mathrm{rot}}(\tilde{f})$ in (8) from (4) results in

$$\varphi_X(\tilde{f}) + \sigma_\eta^2\tilde{f} - \tilde{f}\tan\alpha > \varphi_X(f_{\max}) + \sigma_\eta^2 f_{\max} - f_{\max}\tan\alpha. \tag{9}$$

Manipulating this result further gives

$$\tan\alpha > \frac{\varphi_X(f_{\max}) - \varphi_X(\tilde{f})}{f_{\max} - \tilde{f}} + \sigma_\eta^2. \tag{10}$$

From (7), we can write

$$\tan\alpha = K\frac{\varphi_X(f_{\max})}{f_N} + \sigma_\eta^2 \tag{11}$$

where K is a constant such that $0 < K \le 1$. Substituting (11) in (10) yields

$$\frac{\varphi_X(f_{\max})}{f_N} > \frac{1}{K}\frac{\varphi_X(f_{\max}) - \varphi_X(\tilde{f})}{f_{\max} - \tilde{f}} \tag{12}$$

which contradicts the smoothness assumption in (6).

Case II: $f_{\max} < \tilde{f} < f_N$*:* Substituting for $\hat{\varphi}_{\mathrm{rot}}(\tilde{f})$ in (8) from (4) gives

$$\varphi_X(f_{\max}) + \sigma_\eta^2\tilde{f} - \tilde{f}\tan\alpha > \varphi_X(f_{\max}) + \sigma_\eta^2 f_{\max} - f_{\max}\tan\alpha. \tag{13}$$

Since $\tilde{f} > f_{\max}$, simplifying (13) yields

$$\sigma_\eta^2 > \tan\alpha, \tag{14}$$

which contradicts the inequality in (7).

Therefore, we conclude that $\hat{\varphi}_{\mathrm{rot}}(f)$ peaks at the maximum Doppler signal frequency. ■

Remark: As can be seen from (5), the modified geometric method [3] rotates the integrated spectrum by the maximum allowed angle in (7). Consequently, the frequency estimation using this approach will provide accurate results. This, combined with the ease of implementing the rotation, makes the maximum allowed rotation an excellent choice for maximum velocity estimation.

III. Results and Discussion

In this section, we present the results of experiments evaluating the capabilities of the modified geometric method for maximum frequency estimation using simulated and real Doppler signals.

A. Simulated Doppler Signals

First, the performance of the modified geometric method was evaluated as a function of the bandwidth and the signal-to-noise ratio (SNR) of the Doppler signal. For this experiment, synthetic Doppler signals were generated using the algorithm described by Sirmans, *et al.* in [4]. The spectrum of the signal was given by

$$S(f) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_f^2}} e^{-\frac{(f-\bar{f})^2}{2\sigma_f^2}}, & f_{\min} \le f \le f_{\max} \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

where $\bar{f}$ and σ_f were constant parameters of the model. The signal was generated using the parameters $\sigma_f = 300$ Hz and $\bar{f} = f_{\max} - 2.1\sigma$. The signal was then corrupted by additive white Gaussian noise with zero mean-value and variance σ_η^2. The sampling frequency was 12 kHz. The maximum frequency values were estimated using the modified geometric method and the conventional thresholding method described in [2]. The power spectrum was estimated as the periodogram of a Hann-windowed 512-sample segment of data. The fast Fourier tramsformsize was 1024 samples. The root-mean-square (rms) error was estimated from one thousand independent experiments at several SNR values ranging from 0 to 30 dB and bandwidth ranging from 500 Hz to 3200 Hz, and normalized with the actual maximum frequency.

Fig. 2(a) shows the distribution of the normalized rms error with both the SNR and the bandwidth for the modified geometric method. The normalized rms error is less than 4% of the actual maximum frequency at all SNR values and bandwidths. The decrease of the normalized rms error with the increase of bandwidth is due to the normalization process; that is, the rms errors at smaller bandwidths were normalized by a smaller frequency value compared to the normalization for higher bandwidth signals. The corresponding results for the threshold-based method [2] displayed in Fig. 2(b). Comparison of the two sets of results shows that the modified geometric method is relatively robust to variations in the SNR and the instantaneous bandwidth or the maximum frequency of the Doppler signal.

B. Real Doppler Signals

The Doppler ultrasound blood flow measurements were performed at the Department of Obstetric and Gynecology, Academic Hospital, Erasmus University, Rotterdam, the Netherlands. This study was approved by the Hospital Ethics Committee at the Erasmus University. Pulsed Doppler recordings from the umbilical artery were performed

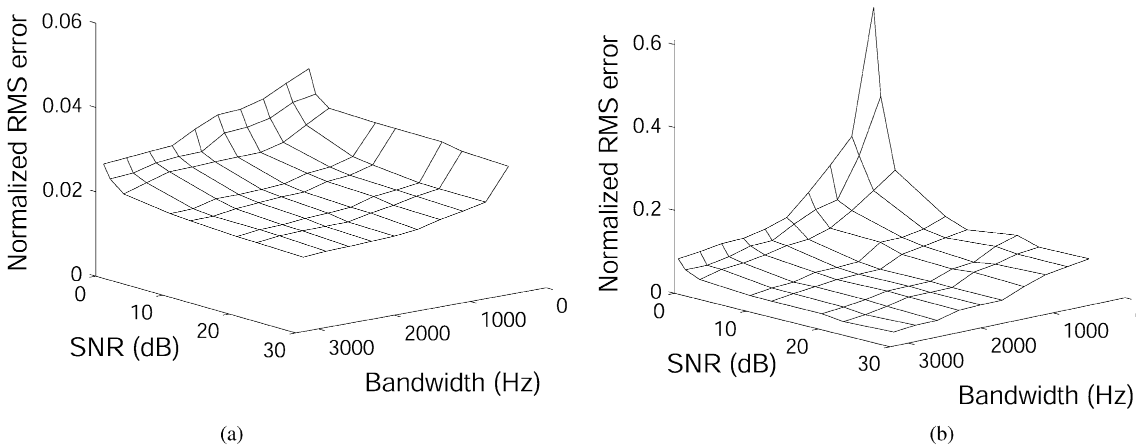

Fig. 2. Distribution of the normalized rms error with the SNR and the bandwidth of the Doppler spectrum. (Note that the scales are different for the two figures.)

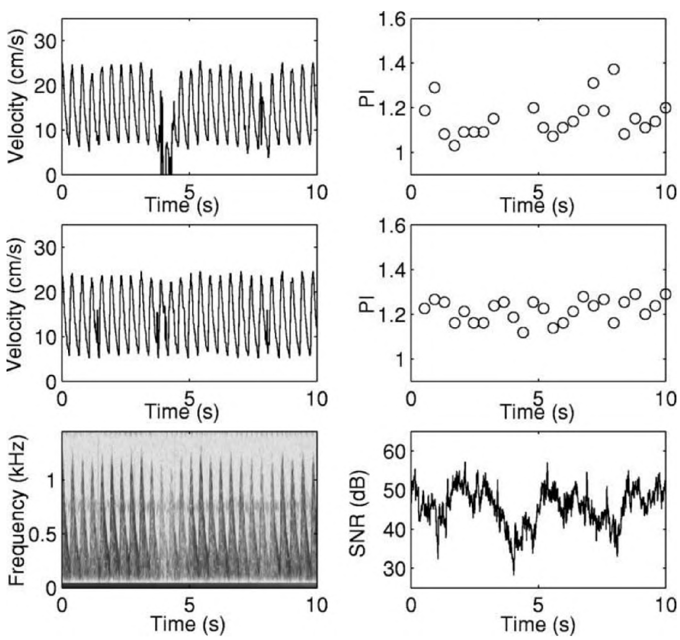

Fig. 3. Comparison of maximum blood velocity waveforms reconstructed from an umbilical arterial Doppler signal.

during fetal apnea for 18–50 s with women in a semi-recumbent position. The incident ultrasound frequency was 3.75 MHz and the angle of incidence was less than 15°. The sampling frequency was 12 kHz.

In order to reconstruct Doppler signals, we employed the modified geometric method with the following parameters. The digitized signals were segmented into smaller blocks of 512 samples with 75% overlap such that one frequency value was estimated for every 128 samples. The power spectrum was estimated using the periodogram method using 1024-point FFTs. A 512-point Hann window was employed to suppress spectral leakage. The conventional thresholding method also estimated the power spectrum using the same procedure. Finally, the results were smoothed using a 3-point moving average filter and scaled for blood velocity using (1) with $c = 1540$ m/s, $f_o = 3.75$ MHz and $\theta \approx 0$ radians.

Fig. 3 depicts the estimated maximum blood flow velocity waveforms corresponding to the Doppler signal whose integrated spectrum is given in Fig. 1. The waveforms on the left two upper panels were reconstructed using the threshold crossing method and the modified geometric method. Fig. 3 also displays the estimated spectrogram, the SNR, and the blood flow velocity parameter *pulsatility index* (PI) distributions with time. The PI is defined as $\mathrm{PI} = (v_{\mathrm{ps}} - v_{\mathrm{ed}}/v_{\mathrm{mean}})$, where $v_{\mathrm{ps}}, v_{\mathrm{ed}}$ and v_{mean} are respectively, the peak-systolic, end-diastolic and the mean blood velocities over a pulse interval. The PI is related to the vascular resistance and is widely used as an indicator for the intrauterine growth retardation [5]. In normal fetuses, the umbilical arterial PI decreases with advancing gestation indicating the decrease of the placental vascular resistance [6]. The noise density $(\hat{\sigma}_\eta^2)$ was estimated as the average gradient of the integrated spectrum at frequencies greater than 3000 Hz and the SNR was estimated as

$$\mathrm{SNR} = \frac{\phi_Y(\hat{f}_{\max}) - \hat{\sigma}_\eta^2 \hat{f}_{\max}}{\hat{\sigma}_\eta^2 f_N} \tag{16}$$

where $\hat{f}_{\max}$ is the estimated maximum frequency using the modified geometric method.

The estimated blood velocity waveforms in Fig. 3 indicate that the modified geometric method is relatively independent of the average power level of the input signal. In the conventional method, when the signal power drops significantly (for example, see the region around 4 s), the output of the frequency estimator decreased and even went to zero. In such situations, the conventional method was unable to estimate the PI. During the rest of the time, the instantaneous PIs estimated for the blood velocity waveform reconstructed using the threshold crossing method varied from 1.0 to 1.4 whereas those estimated for the blood velocity waveform reconstructed using the modified geometric method varied in a narrower range from 1.1 to 1.3. We can also see that the larger dynamics of the PI estimated for the thresholding method occur when the SNR is smaller. From these results, we conclude that the modified geometric method is more robust to the variation of instantaneous SNR.

IV. Conclusion

Reliable estimation of the frequency components are difficult using thresholding algorithms when the instantaneous SNR varies with the environment. The rotated-integrated spectrum-based maximum frequency estimation algorithm is adaptive to the environmental noise level and is less impacted by the changes in noise density than conventional methods. Furthermore, it is easier to implement such algorithms than methods that require *ad hoc* parameters for their operation. This paper provided a theoretical analysis indicating that the criterion employed by the modified geometric method [3] is appropriate for estimating the maximum Doppler frequency. Based on the theoretical analysis and the experimental results presented in this paper, we conclude that this method provides superior maximum velocity waveforms from Doppler signals when compared with conventional algorithms.

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