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Scaling tests of the improved Kogut-Susskind quark action

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Improved lattice actions for Kogut-Susskind quarks have been shown to improve rotational symmetry and flavor symmetry. In this work we find improved scaling behavior of the ρ and nucleon masses expressed in units of a length scale obtained from the static quark potential, and better behavior of the Dirac operator in instanton backgrounds.

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Because of their large computational requirements, full QCD lattice simulations profit greatly from the use of improved actions, which allow better physics to be extracted from simulations at moderate lattice spacings. The Kogut-Susskind (KS) formulation of lattice fermions is attractive for full QCD simulations because the thinning of the degrees of freedom reduces the computational effort and, more importantly, because the residual unbroken chiral symmetry prevents additive renormalization of the quark mass, and hence eliminates problems with exceptional configurations.

The lattice artifacts in the Kogut-Susskind formulation are order a^2, unlike the Wilson formulation which has an order a artifact, which can be canceled by the ''clover'' improvement. A third nearest neighbor coupling introduced by Naik cancels order a^2 violations of rotational symmetry in the free quark propagator [1], and this has been shown to lead to an improvement in the rotational symmetry of meson propagators [2]. Another lattice artifact is the breaking of flavor symmetry, signaled by the fact that only one of the pions produced from the four flavors of quarks has an exactly vanishing mass at zero quark mass. This flavor symmetry breaking can be understood as a scattering of a quark from one corner of the Brillouin zone to another through the exchange of a gluon with momentum near π/a [3]. Roughly speaking, the cure for this problem consists of introducing a form factor for the quark-gluon interaction by smearing out, or ''fattening'' the gauge connection in the quark action. In our previous works we have investigated the effects of different fat link actions on the flavor symmetry violation seen in the pion mass spectrum [4–6]. (See also Ref. [7].) A theoretically attractive action is the ''Asqtad'' action studied in Ref. [6], which includes the Naik correction to improve rotational symmetry, fattening of the nearest neighbor coupling to cancel couplings to gluons with any momentum component equal to π/a, and a term introduced by Lepage [8] which corrects order a^2 errors at low momentum introduced by the form factor. This ''Asqtad'' action cancels all tree level order a^2 errors [8].

Our previous works tested improvements in physical quantities that were directly related to the improvements in the action. That is, the Naik term is introduced to improve rotational symmetry in the quark propagator and was seen to improve rotational symmetry in the meson propagators. The fat link term was designed to reduce flavor symmetry breaking and was seen to reduce the flavor symmetry breaking in the pion mass spectrum. It is perhaps less obvious that other quantities will be improved, and such tests are the subject of this Rapid Communication. We have calculated hadron masses and the static quark potential at different lattice spacings, and we can compare the scaling of these quantities with results with other actions. We have also investigated the physics of instantons on the lattice by computing eigenvalues of the improved and conventional Kogut-Susskind Dirac operators on smooth instanton configurations and on semirealistic ''noisy'' configurations containing an instanton.

We test scaling by computing hadron masses using a length scale determined from the static quark potential. In particular, we will use a variant of the Sommer parameter defined by $r_1^2 F(r_1)=1.00$, where the commonly used r_0 is defined by $r_0^2 F(r_0)=1.65$. (The advantage of r_1 is that it is determined at a shorter length scale, where the potential can be determined more accurately, and so lattice spacings in simulations with different parameters can be matched more accurately using r_1.) For the quenched potential, this is related to r_0 and σ by $r_1/r_0=0.725$ and $r_1\sqrt{\sigma}=0.85$. For quenched simulations with the single plaquette gauge action, we take the lattice spacing from the interpolating formula of Guagnelli, Sommer and Wittig [9]:

$$\log(a/r_1) = -1.3589 - 1.7139(\beta-6) + 0.8155(\beta-6)^2 - 0.6667(\beta-6)^3, \tag{1}$$

while for quenched simulations with the Symanzik improved gauge action we fit a similar formula to the string tension results of Collins *et al.* [10] and our own results at $\beta = 10/g^2 = 8.0$,

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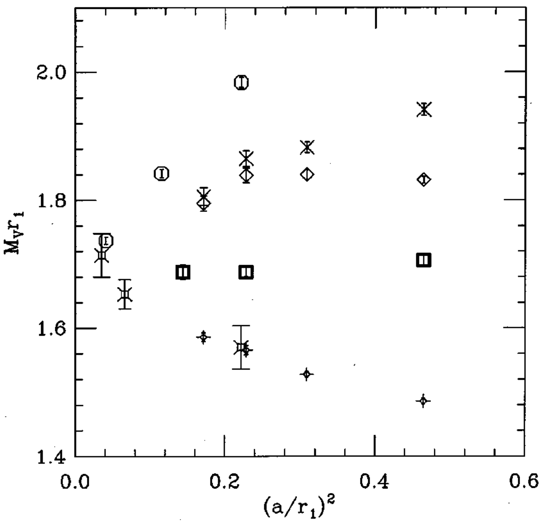

FIG. 1. Rho masses in units of r_1. The meaning of the symbols is described in the text.

$$\log(a/r_1) = -0.970 - 0.840(\beta-8) + 0.413(\beta-8)^2 + 0.304(\beta-8)^3, \quad (2)$$

with an error of around 1%. We use the Goldstone pion mass to adjust the quark mass in the various simulations, interpolating to the point where $m_\pi r_1 = 0.778$. (This somewhat arbitrary value corresponds to $am_q = 0.02$ at $10/g^2 = 8.0$, and has $m_\pi/m_\rho \approx 0.46$.)

We calculated the quenched hadron spectrum using the improved quark action and Symanzik improved gauge action at $10/g^2 = 7.4$ and 7.75 using $16^3 \times 32$ lattices archived in our earlier work [11], and on 20^3 lattices at $10/g^2 = 8.0$ generated in our current project, which have a lattice spacing of $a \approx 0.14$ fm. In Fig. 1 we plot the vector meson mass in units of r_1 versus the squared lattice spacing for several combinations of gauge and quark actions. The bold squares are from our improved quark action, with the Symanzik improved gauge action. The octagons are from a simulation with the conventional staggered action, and the simple one-plaquette gauge action [12]. The diamonds are the conventional staggered quark action, with a Symanzik improved gauge action, and the crosses are the Symanzik improved gauge action with a quark action containing only the Naik improvement [2,11]. We also show some results for tadpole improved clover Wilson quark actions. The plusses are from the SCRI Collaboration [10], using Symanzik improved gauge action, and the fancy squares from the UKQCD Collaboration [13], using the one-plaquette gauge action. Figure 2 is a similar plot of the nucleon masses in units of r_1. In both of these plots the "Asqtad" action shows scaling behavior that is dramatically better than the other actions tested.

Instantons play an important role in the Euclidean description of QCD. They provide the solution to the $U(1)$ problem [14], explain chiral symmetry breaking [15], and at least at a qualitative level reproduce the low mass hadron correlators [16,17]. Thus it is important that the lattice actions we use to simulate QCD approximate the continuum behavior at finite lattice spacing. Here we study the topological aspects of the improved Kogut-Susskind actions and compare with the standard formulations. The effect of topology on the spectrum of the Dirac matrix has been also studied in [18], and in [19]. In [18], the microscopic spectral density was computed and it was found that Kogut-Susskind fermions are insensitive to topology on coarse lattice spacings ($a \sim 0.5$ fm). In Ref. [19] it was shown that at lattice spacing about 0.07 fm there is a clear separation between topological and non-topological modes.

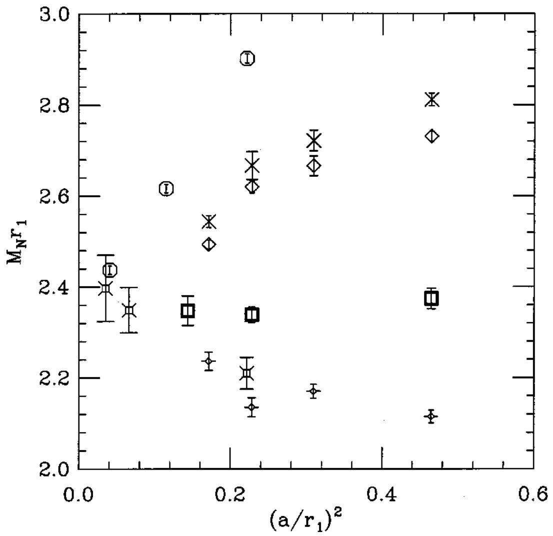

FIG. 2. Nucleon masses in units of r_1. The symbols are the same as in Fig. 1.

We first studied the behavior of the low eigenvalues of the Dirac matrix in the background of an instanton by computing eigenvalues and eigenvectors of $-\not{D}_e^2$ using the Ritz functional technique [20]. $\not{D}_e^2$ is the squared Dirac operator restricted to even lattice sites. In an instanton background $\not{D}_e^2$ should have two chiral eigenvectors with small eigenvalues, each corresponding to two eigenvectors of $\not{D}$. We computed the eight lowest eigenvalues of $-\not{D}^2$ on smooth instantons. The instantons are put on the lattice by discretizing the continuum formula for the instanton gauge field A_μ [21], and the resulting gauge configuration was smoothed twenty times by the APE Collaboration smearing technique. We calculated eigenvectors on lattices ranging in size from 4^4 to 16^4 containing an instanton with radius ρ equal to one fourth the lattice size, $\rho = L/4$. Since there is no QCD dynamics to define a length scale on these smoothed lattices, changing ρ/a can equally well be considered to be varying the size of the instanton at fixed lattice spacing or varying the lattice spacing for a fixed instanton size. We computed the eight lowest eigenvalues of $-\not{D}^2$ for the standard KS action, the Naik action, the "Fat7" action (all couplings to gluons with momentum π/a set to zero), the "Fat7" with the Naik term, and finally the Asqtad action (with $u_0 = 1$ since tadpole improvement has no meaning on smooth backgrounds) [6]. As expected, we found two small eigenvalues, which approach zero as the lattice spacing goes to zero. These two eigenvalues were degenerate to the accuracy of the computation. In Fig. 3 we plot the small eigenvalue λ of $\not{D}$ for all the actions tested as a function of ρ/a. For all the actions, the small

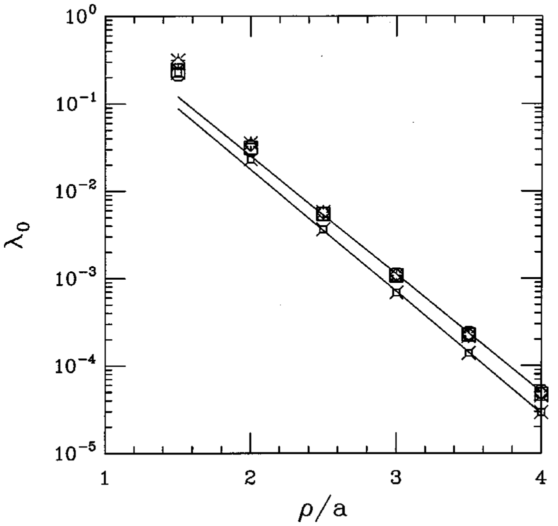

FIG. 3. The small eigenvalue of the Dirac operator as a function of instanton size for several actions. The octagons are the one link action, the squares are the Naik action, the diamonds are the ''Fat7'' action, the bursts are the ''Fat7 Naik,'' and the fancy squares are the Asqtad action.

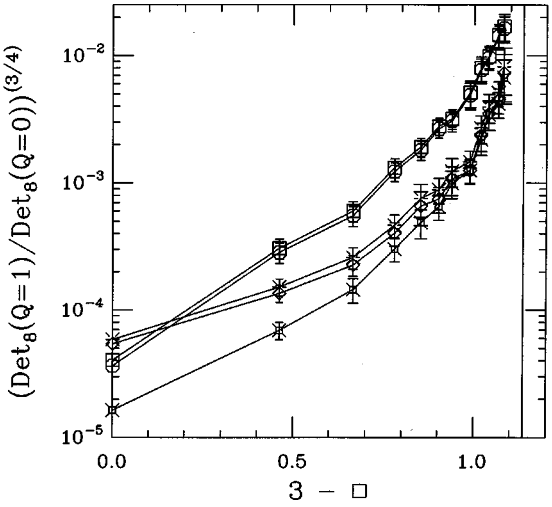

FIG. 4. The approximate instanton suppression factor for three quark flavors as a function of the amount of ''heating'' applied to the instanton. The meaning of the symbols is the same as in Fig. 3.

eigenvalues go to zero exponentially as a/ρ decreases. Neither the Naik term nor the fattening of the link term affects the eigenvalue significantly. (This is not surprising — there is no point in smoothing out the link when you are already working on a smooth background.) However, the Asqtad action, which differs from the Fat7 action in that it contains the small momentum correction introduced by Lepage, has smaller eigenvalues than the other actions. We also measured the chiralities $\chi=\psi^\dagger\gamma_5\psi$ of the eigenvectors associated with the small eigenvalues. Although χ differs significantly from -1, it differs from -1 by an amount proportional to a^2/ρ^2.

To see if these small eigenvalues persist when the QCD dynamics is turned on, we ''heated'' the $\rho/a=2$ instanton on the 8^4 lattice. This was done by short quenched molecular dynamics trajectories, using the tadpole improved one loop Symanzik gauge action with $\beta=8.0$. Since we want to introduce short distance structure without disturbing the long distance topological structure of the initial lattice, we used short trajectories—ten molecular dynamics steps of size 0.02. We ran for 20 such trajectories, saving the lattice at the end of each trajectory. The resulting sequence of 20 lattices was discarded if any of the lattices had topological charge different from one. In total we produced 31 different heating runs in which the instanton survived. At the end of the 20th trajectory the average plaquette was 1.92, similar to the average 1.86 of thermalized $\beta=8.0$ quenched lattices. For comparison, we did 24 runs starting from smooth lattices containing no instantons. In Fig. 4 we plot the 3/4 power of the averaged product of the eight smallest eigenvalues of $\not{D}_e^2$ for the $Q=1$ heating runs, divided by the same quantity from the $Q=0$ runs. For all of the quark actions, u_0 was kept at one during these heating runs. This quantity, which we call $\mathrm{Det}_8^{3/4}$, is an approximation to the factor by which three flavors of massless dynamical quarks would suppress such instanton configurations in a full QCD simulation. The abscissa in Fig. 4 is $3-\frac{1}{6}\langle\Sigma_{\mu<\nu}\mathrm{Tr}\,P_{\mu\nu}\rangle$, where $P_{\mu\nu}$ is the plaquette, which is related to the amount of disorder induced by the heating. The vertical line marks the average plaquette of a thermalized lattice. As the amount of disorder increases, the size of the small eigenvalues in the $Q=1$ runs increases, but the eigenvalues of the fat link $\not{D}$ rise less than those of the Naik and standard KS actions. Roughly speaking, Fig. 4 shows that at $a\approx 0.14$ fm three dynamical quarks with fat link actions suppress instantons by more than a factor of two relative to the conventional action. We have also looked at the chirality of the would-be zero modes. Again, as the disorder increases the deviation from -1 also increases, but the improved action holds up better. After 20 heating trajectories with the $\rho=2a$ instanton, the chirality for the fat actions is about -0.4, while for the standard KS action (and the Naik) it is -0.2. Finally, the degeneracy of the two would-be zero modes of $-\not{D}^2$ is lifted as disorder is added. The splitting of these modes is related to the flavor symmetry breaking and, as expected from our previous studies [6], is about a factor of three smaller for the fat actions than for the standard Kogut-Susskind action.

Lastly, we looked at the eight low eigenvalues of $-\not{D}_e^2$ for the Asqtad and standard Kogut-Susskind actions on 424 thermalized 8^4 quenched $\beta=8.0$ lattices, which have a lattice spacing of about 0.14 fm. At this volume we have 1 to 2 topological objects of average size $\rho=2a$ to $3a$ per lattice, based on the instanton distribution measurements in [21]. Typically, when the topological charge is non-zero we find small eigenvalues with high absolute value of the chirality (0.2 to 0.3) for the Asqtad action. In a chirality versus eigenvalue scatter plot the topological eigenvalues clearly separate from the non-topological. Similar but weaker separation is also observed for the standard Kogut-Susskind action. We have also checked the validity of the index theorem. For the Kogut-Susskind fermions which represent 4 flavors it takes the form $4\,Q=N_- - N_+$, where N_- are the number of left-handed modes, N_+ the number of right-handed modes, and Q is the topological charge. On the lattice this relation does not hold exactly for staggered fermions, but the quantities $4Q$ and $N_{-}-N_{+}$ are strongly correlated. For the Asqtad action this correlation is 90%, while for the standard Kogut-Susskind it is 83%. In both cases the topological charge was measured after 200 APE smearings.

The code used for generating smooth instanton configurations and for measuring the topological charge was written by Anna Hasenfratz and Tamas Kovacs. Computations were done on the T3E and PC cluster at NERSC, on the T3E and SP2 at SDSC, on the NT cluster and Origin 2000 at NCSA, and on the Origin 2000 at BU. This work was supported by the U.S. Department of Energy under contracts DOE–DE-FG02-91ER-40628, DOE–DE-FG03-95ER-40894, DOE–DE-FG02-91ER-40661, DOE–DE-FG05-96ER-40979 and DOE–DE-FG03-95ER-40906 and National Science Foundation grants NSF–PHY99-70701 and NSF–PHY97-22022.

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