Evolution of Time Preference by Natural Selection

By Alan R. Rogers*

This paper entertains the hypothesis that human time preferences are in evolutionary equilibrium (i.e. that no mutation changing time preferences could be favored by natural selection). This hypothesis implies that the marginal rate of substitution (MRS) holding Darwinian fitness constant must equal the MRS holding utility constant. Furthermore, in a market economy the latter must equal the MRS in exchange. Exploiting these principles, I find that the long-term real interest rate should equal ln(2) per generation (about 2 percent per year) and that young adults should discount the future more rapidly than their elders. (*JEL* D91, E43)

Preferences are central to economic theory, yet little is known about them. Because of this ignorance, economists have worked hard to make their assumptions about preferences as mild as possible, and economic theory has grown more abstract and general over the years. This has cushioned the theory against violations of assumptions but has not increased its power. Economics would gain much from a theory of preferences. The theory of evolutionary genetics may be useful in this regard. If genes affect preferences, then an evolutionary model may succeed in predicting them.[1]

The proposal that preferences have evolved by natural selection may strike some readers as strange. It is customary to think of selection acting on "biological" characters such as size, shape, and patterns of fertility, but less customary to view preferences in the same light. Yet there seems really to be no basis for this prejudice. Consider, for example, preferences concerning mates. Most individuals prefer to mate with opposite-sexed individuals of their own species. Presumably, this is because those who preferred otherwise left few offspring: they were selected against. How else—except by natural selection—can such preferences be explained? If selection affects mating preference, then why not time preference? Every animal that builds a nest or burrow is deferring immediate consumption

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[1] This proposal has been made previously by economists including Gary S. Becker (1976), Hirshleifer (1977), R. H. Coase, (1978), Paul H. Rubin and Cris W. Paul (1979), Heinrich W. Ursprung (1988), and Ingemar Hansson and Charles Stuart (1990). In addition, several authors have published evolutionary models of time preference. Some have used ideas from the theory of optimal foraging in an effort to understand how animals discount rewards over delays measnred in seconds or minutes (John H. Kagel et al., 1986; A. W. Logue, 1988). Others (Gordon Getty, 1989, 1991; Hansson and Stuart, 1990) have tried to explain time preference over intervals of years and decades. The model I present here deals with time preference on this longer time scale. It differs from Getty's in emphasizing preferences rather than production and in incorporating a more complex model of the life cycle. It differs from that of Hansson and Stuart in considering time preference within as well as between generations, and in allowing for sexual reproduction.

for a delayed reward. Thus, time preference has an ancient evolutionary history.

It might also be objected that time preference is shaped by learning and culture, not by natural selection. In response, one can argue that the capacity for culture is itself shaped by natural selection, so that we tend to learn what selection would have favored anyway. I have shown elsewhere (Rogers, 1988) that neither of these positions is entirely correct. A model of natural selection will predict more poorly where cultural transmission is important. However, the reduction in predictive power may be small. Thus, a model ignoring culture may prove useful even if patterns of time preference are transmitted culturally.

Below, Section I defines the marginal rates of substitution and of time preference and summarizes previous results from economics and evolutionary genetics; Section II combines these to derive new results concerning the intertemporal utility function and the real interest rate. These results all assume complete certainty about which individual will enjoy the returns from an investment. Section III relaxes this last assumption, allowing the returns to be directed toward an initially unspecified relative of the investor. In this context, the marginal rate of time preference turns out to vary both with the age of the investor and with the delay between investment and return on investment. Finally, the results are summarized and discussed in Section IV.

I. Preliminaries

A. *Marginal Rates of Substitution in Preference, in Exchange, and in Fitness*

Immediate benefits are ordinarily preferred over benefits that are delayed. Irving Fisher's (1930) graphical analysis of this matter is shown in Figure 1. The dotted indifference curves there connect points of equal utility. The absolute value of the slope of an indifference curve is called the *marginal rate of substitution* (MRS) *in preferences* and measures the rate at which present and future consumption can be substituted without affecting utility.

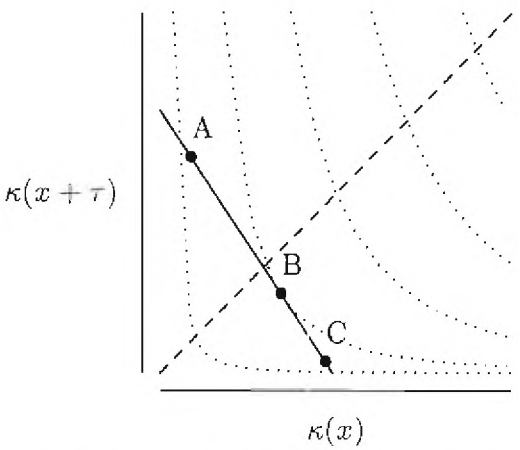

FIGURE 1. IRVING FISHER'S (1930) ANALYSIS OF INTERTEMPORAL CHOICE

Notes: $\kappa(x)$ and $\kappa(x+\tau)$ denote consumption at ages x and $x+\tau$, respectively. The dotted indifference curves show the contour map of a hypothetical utility function. The solid "market line" has slope $-e^{i\tau}$, where i is the interest rate. The dashed 45° line connects points at which $\kappa(x+\tau)=\kappa(x)$.

The solid "market line" in the figure connects points of equal wealth. By borrowing, the individual at A can move along the market line toward B. The move from C to B, on the other hand, is accomplished by lending. The absolute slope of the market line is called the *MRS in exchange*. The optimum B occurs where the MRS in preferences is equal to that in exchange.

Following Hansson and Stuart (1990), I define the *MRS in fitness* as the marginal rate at which two goods can be substituted without changing Darwinian fitness.[2] An

[2]In models with discrete generations, Darwinian fitness is the conditionally expected number of an individual's offspring, given its genotype. In models with overlapping generations, fitness is measured by R. A. Fisher's (1958) "Malthusian parameter," which measures the asymptotic rate of exponential increase in the numbers of one's descendants (Brian Charlesworth, 1980).

allele[3] that encourages substitution at this rate is neither favored nor disfavored by natural selection: it is selectively neutral. If preferences have been shaped by natural selection, then individuals should be indifferent to choices that do not affect fitness. Thus in evolutionary equilibrium the MRS in fitness must equal the MRS in preferences. I exploit this principle below by deriving an expression for the MRS in fitness and setting it equal first to the MRS in preferences and then to the MRS in exchange.

The analysis will employ two familiar representations of the MRS in preferences. First, I write it in terms of the *marginal rate of time preference* (MRTP), which measures the extent to which a unit increase in present consumption is preferred to a unit increase in future consumption. If I am indifferent between another apple today and $e^{\theta\tau}$ apples delayed by τ years, then θ is my MRTP for apples. The MRTP is related to the MRS in preferences by

$$(1) \qquad -\left.\frac{d\kappa(x+\tau)}{d\kappa(x)}\right|_{U \text{ constant}} \equiv e^{\theta\tau}$$

where $\kappa(x)$ is consumption at age x, U is utility, and the derivative is taken along a line of constant utility. The left side is, by definition, the MRS in preferences. In general, θ will be a function of age x, of the time-delay τ between investment and return on investment, and of the dated consumption levels $\kappa(x)$ and $\kappa(x+\tau)$.

A second familiar expression for the MRS in preferences is obtained by writing the utility of a consumption stream beginning at time t_0 and ending at t_1 as

$$(2) \qquad U = \int_{t_0}^{t_1} e^{-\eta(t-t_0)} u[\kappa(t)]\, dt$$

where $\kappa(t)$ is consumption at time t, $u[\kappa(t)]$ is the utility of this dated consumption, and η is the subjective rate of utility discount. The MRS in preferences between $\kappa(x+\tau)$ and $\kappa(x)$ is

$$(3) \qquad -\left.\frac{d\kappa(x+\tau)}{d\kappa(x)}\right|_{U \text{ constant}} = \frac{u_\kappa[\kappa(x)]}{e^{-\eta\tau} u_\kappa[\kappa(x+\tau)]}$$

where $u_\kappa[\kappa] = du[\kappa]/d\kappa$ is the marginal utility of dated increments to consumption (Hirshleifer, 1970 p. 127).

Finally, I will make use of the fact that the real interest rate i is related to the MRS in exchange by

$$(4) \qquad -\left.\frac{d\kappa(x+\tau)}{d\kappa(x)}\right|_{W \text{ constant}} \equiv e^{i\tau}$$

where W is wealth and the derivative is taken along a line of constant wealth, that is, along the solid market line in Figure 1. The left side is, by definition, the MRS in exchange.

At simultaneous evolutionary and market equilibrium, each of these marginal rates of substitution must equal the MRS in fitness. In deriving an expression for this latter quantity, I will need results from the evolutionary theory of kin selection.

B. *Results from the Theory of Kin Selection*

The theory of kin selection (W. D. Hamilton, 1964a, b) is about altruism toward relatives. It tells when natural selection will favor an "altruist allele," which predisposes its bearer toward some behavior that decreases its own Darwinian fitness but increases that of a relative. Those who provide these benefits are called "donors"; those who receive benefits are called "recipients."

Kin selection is relevant to economic problems of investment decision because the returns from an investment may affect the Darwinian fitness of the investor's relatives

[3] The word *allele* refers to a variant of a gene. Thus, a gene that occurs in two forms such as "altruist" and "nonaltruist" is a gene with two alleles.

TABLE 1—HOW CHANGES IN FERTILITY AND SURVIVAL AFFECT FITNESS

(i) Effect on:	(ii) Probability of act	(iii) Additive effect	(iv) Reproductive value	(v) Discount factor	(vi) Relationship to donor
Donor	$l_D(x_1)$	$\Delta P(x_1)$	$v_D(x_1+dx)$	$e^{-\rho(x_1+dx)}$	1
Recipient	$l_D(x_1)$	$\Delta P(x_1+\tau)$	$v_R(x_2+dx)$	$e^{-\rho(x_1+\tau+dx)}$	r

Notes: The altruist allele is favored (disfavored) by selection if the sum of row products is positive (negative). The notation is defined in the text. To simplify the table, I assume that the sex ratio at birth is unity, that effects on survival are brief, that these effects are small enough that second-order terms in ΔP can be ignored, and that a single recipient is affected by each act of altruism.
Source: Rogers (1993 appendix A).

rather than that of the investor himself. For example, benefits received by a woman of age 50 cannot increase the number of her own children but may increase that of her daughter. The "investor" of economic parlance becomes the "donor" in kin-selection theory. The "recipient" in kin-selection theory is the individual (or individuals) whose Darwinian fitness is increased by the returns from that investment. Often, the recipient will be the donor (investor) himself. I will continue to use the term "altruism" in this case, although this usage is nonstandard; in ordinary usage it is not altruism unless the recipient is someone else.

The fate of an altruist allele depends not only on the delay between investment and return, but also on the ages of the donor and recipient. I assume that the investment is made when the donor's age is x_1, and benefits the recipient τ years later when the recipient's age is x_2. The fate of the altruist allele depends also on the coefficient of relationship,[4] r, between donor and recipient. If the donor and recipient are the same individual, then $r = 1$ and $x_2 = x_1 + \tau$.

Subscripts m and f will denote males and females, respectively. In the absence of altruism, an individual of sex g will survive from age x to $x + dx$ with probability $P_g(x)$. He or she will survive from birth to age x with probability $l_g(x)$. An individual of sex g who reaches age x can expect to produce $m_g(x)$ offspring of sex g before age $x + dx$. These age-specific rates of survival and fertility are assumed to be constant, which implies, under mild additional assumptions,[5] that the population will eventually converge to a stable age distribution. Thereafter, its growth is exponential. I denote by ρ the rate of exponential population growth in the absence of altruism.

In general, the fate of the altruist allele will depend on how it affects survival and fertility. In this paper, I ignore effects on fertility and consider effects on survival alone. The effect of a given interaction is assumed to be brief, of duration dx. Altruism changes survival probabilities at age x_1 in the donor and age x_2 in the recipient, adding $\Delta P(x_1)$ to the survival of the donor and $\Delta P(x_1 + \tau)$ to that of the recipient. In the usual case, the interaction harms the donor and benefits the recipient, so $\Delta P(x_1) < 0$ and $\Delta P(x_1 + \tau) > 0$. For mathematical convenience, I assume that these effects are small enough that second-order terms can be ignored.

I have derived elsewhere (Rogers, 1993) the conditions under which the altruist allele will increase in frequency, and these are summarized in Table 1. In the table, subscripts D and R indicate the sex of the donor and the recipient. The *reproductive*

[4] Wright's coefficient of relationship (J. F. Crow and M. Kimura, 1970 pp. 69, 137–38) can be interpreted as the fraction of their genes that two individuals can expect to hold in common. It equals 1 if the donor and recipient are the same individual, $\frac{1}{2}$ if the recipient is an offspring, $\frac{1}{4}$ if a grandchild, and so forth.

[5] It is sufficient that there be positive fertility in two adjacent age classes (J. H. Pollard, 1973 pp. 42–44).

value (R. A. Fisher, 1958) is defined by

$$v_g(x) = \frac{\sum_{y=x}^{\infty} e^{-\rho y} l_g(y) m_g(y)}{e^{-\rho x} l_g(x)} \quad (5)$$

and can be interpreted as the expected present value of an individual's future contributions to the gene pool. The equivalence of $v_g(x)$ to the expected present value follows from the following observations. First, the probability of surviving from age x to age y is $l_g(y)/l_g(x)$. Thus, $m_g(y)l_g(y)/l_g(x)$ is the expected number of offspring that an individual of age x will produce at age y. The reproductive value is a sum of these contributions, discounted exponentially at rate ρ.

The rationale for this discounting can be understood as follows. The birth of an altruist at time y increases the frequency of the altruist allele by $1/[2N(y)]$, where $N(y)$ is the size of the population at time y. Thus, one must weight each birth by $1/[2N(y)]$. This is equivalent to weighting by $e^{-\rho y}$ since my assumptions imply that the population is increasing exponentially at rate ρ. In $v_g(x)$, births at age $y > x$ are discounted by

$$e^{-\rho(y-x)} = \frac{1/[2N(y)]}{1/[2N(x)]}.$$

In effect, this takes births at age x as a numeraire, or unit of value. The value of a birth at age y is measured by the number of births at age x that would be needed to produce the same effect on allele frequency.

The fate of the altruist allele is determined by the sum of the products of the entries in the two rows of Table 1. The allele will increase in frequency when rare[6] if this sum is positive, and decrease if it is negative. The allele is selectively neutral if the sum is zero. Elsewhere (Rogers, 1993), I justify this result in two ways, one rigorous and the other heuristic. Here, I paraphrase the heuristic justification.

The altruist allele is expressed only if the donor survives to perform an altruistic act at age x_1. Thus, column (ii) shows that the allele's effect is proportional to the probability $l_D(x_1)$ of this event. It is also proportional to the additive effects on the age-specific survival of donor and recipient, as shown in column (iii). Multiplying by reproductive value in column (iv) expresses these effects on survival in terms of future births discounted back to age $x_1 + dx$ in the case of the donor, and to $x_2 + dx$ in the case of the recipient. The additional discount factors in column (v) discount these future births back to the time when the donor's age was zero, thus expressing all births in a common currency. The product of columns (ii)–(v) gives the expected effect of an altruist gene on births, appropriately discounted. However, I am not interested in all births, but only in the births of new altruist individuals. Thus, column (vi) multiplies by the coefficient of relationship r, which can be interpreted as the probability that the recipient has the altruist allele given that the donor does.[7]

Having justified the entries in Table 1, I will now examine its implications.

II. The MRS in Fitness when Recipients are Known with Certainty

In this section I show how the MRS in fitness can be expressed in terms of demographic parameters, the degree of kinship between donor and recipient, and the overall rate of population growth. Throughout this section, I assume that the individuals involved in an interaction are known with certainty.

[6] The theory holds not only when the altruist allele is rare (with frequency near zero), but also when its frequency is near unity. There is no theory for the case of an allele of intermediate frequency.

[7] This interpretation is valid when the inheritance system is diploid and the altruist allele is rare.

A. The MRS in Fitness and Irving Fisher's Indifference Diagram

The altruist allele is selectively neutral if the sum of row products in Table 1 is zero, that is, if

$$v_D(x_1+dx)\,\Delta P(x_1) + e^{-\rho\tau} r v_R(x_2+dx)\,\Delta P(x_1+\tau) = 0.$$

Rearranging this expression yields the MRS in fitness between increments to the survival of the altruist and survival of the recipient:

$$(6)\quad \mathrm{MRS}_P \equiv -\frac{\Delta P(x_1+\tau)}{\Delta P(x_1)} = \frac{v_D(x_1+dx)}{re^{-\rho\tau}v_R(x_2+dx)} \approx \frac{v_D(x_1)}{re^{-\rho\tau}v_R(x_2)}$$

where the approximation assumes that the duration dx of effects on survival is small. MRS_P specifies the rate at which the survival of the donor can be substituted for the (delayed) survival of the recipient without affecting Darwinian fitness.

I now use this result to construct an analogue of the indifference diagram in Figure 1. The dotted lines in Figure 2 connect points of equal fitness and may thus be called *fitness isograms*. As the graph shows, the fitness isograms are convex to the origin.[8]

[8] In deriving this result, I take dx as the time unit so that $v(x+dx) \equiv v(x+1)$. Since ρ is constant along a fitness isogram, equation (6) implies that

$$\frac{d\ln \mathrm{MRS}_P}{dP(x_1)} = \frac{1}{v(x_1+1)}\left[\frac{\partial v(x_1+1)}{\partial P(x_1)} - \frac{\partial v(x_1+1)}{\partial P(x_2)}\mathrm{MRS}_P\right] - \frac{1}{v(x_2+1)}\left[\frac{\partial v(x_2+1)}{\partial P(x_1)} - \frac{\partial v(x_2+1)}{\partial P(x_2)}\mathrm{MRS}_P\right]$$

Now substitute

$$\frac{\partial v(x)}{\partial P(y)} = \begin{cases} 0 & \text{if } y < x \\ \dfrac{v(y+1)e^{-\rho(y+1)}l(y)}{e^{-\rho x}l(x)} & \text{if } y \geq x \end{cases}$$

to obtain

$$\frac{d\ln \mathrm{MRS}_P}{dP(x_1)} = -\frac{\partial v(x_1+1)}{\partial P(x_2)}\mathrm{MRS}_P = -\frac{e^{-\rho(x_2-x_1-\tau)}l(x_2+1)}{rl(x_1+1)}$$

which is always negative. Thus, MRS_P decreases with increasing values of $P(x_1)$, and the fitness isogram is convex.

To relate this result to economics, I assume that P is not only a function of age, but is also a differentiable function of consumption. Thus, $P = P[x, \kappa(x)]$ where $\kappa(x)$ is consumption at age x. Further suppose that the mortality trade-off discussed above is caused by a trade-off in consumption. The donor forgos some consumption at age x_1 in return for a delayed increase in the consumption of the recipient, who is then of age x_2. Specifically, the donor's consumption at age x_1 changes from $\kappa(x_1)$ to $\kappa(x_1)+\Delta\kappa(x_1)$, while that of the recipient at age x_2 changes from $\kappa(x_2)$ to $\kappa(x_2)+\Delta\kappa(x_1+\tau)$. If these changes in consumption are small, then

$$\Delta P(x_1) \approx P_\kappa[x_1, \kappa(x_1)]\,\Delta\kappa(x_1)$$

$$\Delta P(x_1+\tau) \approx P_\kappa[x_2, \kappa(x_2)]\,\Delta\kappa(x_1+\tau)$$

where $P_\kappa = \partial P/\partial\kappa$. Substituting these into equation (6) gives the MRS in fitness between the consumption of the donor and the delayed consumption of the recipient:

$$(7)\quad \mathrm{MRS}_\kappa \equiv -\frac{\Delta\kappa(x_1+\tau)}{\Delta\kappa(x_1)} = \mathrm{MRS}_P\left\{\frac{P_\kappa[x_1,\kappa(x_1)]}{P_\kappa[x_2,\kappa(x_2)]}\right\}.$$

MRS_κ is obtained by multiplying MRS_P by the ratio of marginal effects of consumption on survival of the donor and recipient. If P

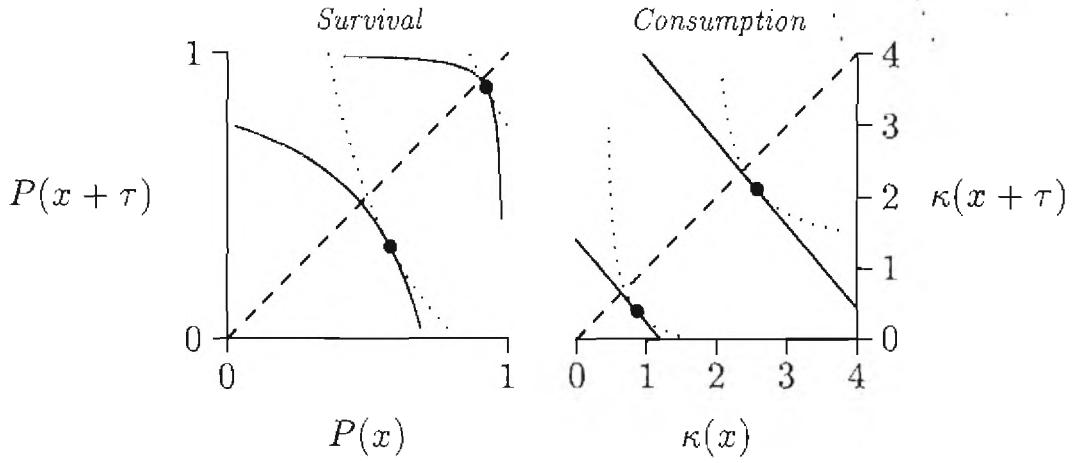

FIGURE 2. INDIFFERENCE DIAGRAMS ON SURVIVAL AND CONSUMPTION AXES

Notes: The dotted lines in each panel are fitness isograms, which are equivalent to indifference curves under evolutionary equilibrium. The solid market lines define opportunities for lending and borrowing. The left panel refers to effects on survival and exhibits the indifference curves implied by equation (6). The image of these curves in the right panel is even more convex, because of the diminishing marginal effect of consumption on survival.

has the properties that are usually attributed to utility—increasing with consumption, but at a decreasing rate—then the transformation from survival to consumption axes will increase the convexity of the fitness isograms (John Hicks and Roy G. D. Allen, 1934), as shown in the right-hand panel of Figure 2.

At evolutionary equilibrium, the MRS in fitness must equal that in preferences, and the fitness isograms in Figure 2 become indifference curves. To recover the familiar indifference diagram of economics, we need only introduce a market line, which is drawn as a solid line in both panels of Figure 2. As usual, the market line defines opportunities for lending or borrowing and is straight when drawn on consumption axes. In the left panel, however, it is concave to the origin because of the nonlinear transformation from consumption axes to survival axes.

In adding the market line, one need not assume that credit markets have been important throughout human evolution. The hypothesis of evolutionary equilibrium merely specifies the form of the indifference curves. This equilibrium may have been reached long before the advent of credit markets. Given these preferences, Irving Fisher's (1930) argument (Fig. 1) explains how modern humans can maximize utility by lending and borrowing. The hypothesis of evolutionary equilibrium modifies Fisher's argument only by giving definite form to indifference curves that were arbitrary in Fisher's analysis.

Equations (6) and (7) identify several factors that contribute to the preference for immediate rather than delayed consumption. First, if the time path of consumption is increasing and the benefits of investment will go to the investor himself, then $P_\kappa[x_1, \kappa(x_1)] > P_\kappa[x_2, \kappa(x_2)]$, thus inflating MRS_κ. This is the familiar effect of diminishing marginal utility on indifference curves, which accounts for the increased convexity of indifference curves drawn on consumption axes. But this is not the whole story. MRS_κ is further inflated by MRS_P, the product of $e^{\mu\tau}$, r^{-1}, and $v_{\mathrm{D}}(x_1)/v_{\mathrm{R}}(x_2)$. The first two of these factors are never less than unity. The third exceeds unity when the donor is adult and is also the recipient. Thus, MRS_P will usually exceed unity for adult donors. This implies that, for such donors, MRS_κ will usually exceed unity even along the 45° line, where present and future consumption are equal. The excep-

tions will involve transfers from aged adults with low reproductive value to their offspring with higher reproductive value. This provides an evolutionary explanation for positive time preference.

B. *The Marginal Rate of Utility Discount and the Marginal Utility of Consumption*

The procedure of equating the MRS in fitness with that in preferences also provides a variety of algebraic results. Substituting (6) into (7) and setting the result equal to (3) gives

$$(8)\quad \frac{u_\kappa[\kappa(x)]}{e^{-\eta\tau}u_\kappa[\kappa(x+\tau)]} = \frac{v_D(x_1)}{re^{-\rho\tau}v_R(x_2)}\left\{\frac{P_\kappa[x_1,\kappa(x_1)]}{P_\kappa[x_2,\kappa(x_2)]}\right\}.$$

This requires that the marginal utility of consumption be a function of age x as well as consumption κ, and that it be proportional to the reproductive value $v(x)$. For example, one might define

$$(9)\quad u_\kappa[\kappa] = u_\kappa[x,\kappa] \propto P_\kappa[x,\kappa]v(x)$$

$$(10)\quad \eta = -(\ln r)/\tau + \rho$$

where for simplicity I have suppressed subscripts denoting sex.[9] Economic models typically assume the function u to be independent of age. It seems impossible to reconcile this assumption with the hypothesis of evolutionary equilibrium.

Thus, the evolutionary perspective restricts the range of permissible formulations of intertemporal utility. It also broadens their scope to include consumption by relatives, weighted by the coefficient of relationship. This suggests a restriction on the "dynastic utility function" of Becker and Robert J. Barro (1988), which assigns arbitrary weights to the consumption of descendant generations. In evolutionary equilibrium those weights are no longer arbitrary.

Equation (10) relates the subjective rate η of utility discount to r, τ, and ρ. In the special case when recipient and donor are the same individual, $r = 1$ and therefore $\eta = \rho$. The relevant value of ρ is not the current rate of population growth, but that which prevailed on average during some long but unknown period of evolutionary history. Apart from the last couple of centuries, ρ must have been on average very near zero (Colin McEvedy and Richard Jones, 1978). Thus, equation (10) implies that $\eta \approx 0$ when donor and recipient are the same individual. When u satisfies equation (9), there is no additional discounting due to length of delay. Consequently, η is needed only in models that allow intergenerational transfers.

When intergenerational transfers do occur, equation (10) says that the subjective rate of utility discount is $\eta \approx -(\ln r)/\tau$. For a given degree of relatedness r between donor and recipient, η is inversely proportional to the delay τ between cost (investment) and benefit (return on investment). However, this conclusion may not hold in general, since r is likely to vary as a function of τ [see footnote (9)].

C. *The Long-Term Real Interest Rate*

An expression for the long-term interest rate can be obtained by setting

$$(11)\quad \mathrm{MRS}_\kappa = e^{i\tau}$$

a procedure justified under evolutionary equilibrium by equation (4). To simplify the result, I restrict attention to the case of an investment that benefits the investor's daughter after exactly one generation. By assumption, the mother and daughter are affected at the same age, so that $v_D(x_1) = v_R(x_2)$, and $\mathrm{MRS}_\rho = r^{-1}e^{\rho\tau}$, where $r = \frac{1}{2}$ is

[9] This is not the only definition of u_κ and η that is consistent with (8). I have included r in the expression for η because it will vary with the length of delay, provided that benefits arriving after a delay of one generation go to offspring, those arriving after two generations go to grandchildren, and so on.

the coefficient of relationship between mother and daughter, and τ equals T, the generation length.[10] In stationary equilibrium, the mother and daughter will have equal wealth at this common age, so that $MRS_\kappa = MRS_\rho$. I am thus restricting attention to points along the dashed 45° line in Figure 2. Equation (11) becomes $2e^{\rho T} = e^{iT}$, or

$$(12) \qquad i = (\ln 2)/T + \rho.$$

The generation time T is usually a little less than 30 years in human populations. For example, $T = 28.9$ in the 1906 population of Taiwan (Hamilton, 1966). Thus, if $\rho \approx 0$, selection should favor long-term interest rates that average $(\ln 2)/28.9 = 0.024$ per year,[11] which is in reasonable agreement with observation.

Time-productivity does not appear explicitly in this analysis. However, in equilibrium the intergenerational MRS_κ must also equal the marginal time-productivity of intergenerational investment, for any discrepancy between the two would trigger a capital accumulation or decumulation process eventually restoring the equilibrium (see Hirshleifer, 1970 p. 172). In a Malthusian interpretation, the accumulation process would take the form of a population surge, rather than a capital surge, but would lead in either case to an equilibrium along the 45° line.[12]

These conclusions require qualification. In equating marginal rates of substitution in preferences and in exchange, I have implicitly assumed that maximization of utility requires some nonzero transfer of consumption from donor to recipient. This is not necessarily so: the donor will not dispense benefits to *all* potential recipients. Furthermore, I have assumed that at the time an individual contemplates some altruistic act, he or she somehow knows in advance both the age and the relatedness of the recipient. I have not allowed for the possibility that there will be no recipient at all. To deal with these complications, one needs a statistical model incorporating uncertainty about which relative, if any, will eventually receive the benefit.

III. Uncertainty Concerning Recipients

I now assume that the identity of the recipient is initially uncertain. Uncertainty might be incorporated in several ways, and I have tried to choose the simplest. I assume that when a benefit arrives it will be allocated among potential recipients (including the donor herself) so as to maximize its discounted value to the donor. Since the benefits in my model are very small, I rule out the possibility of distributing a benefit among several recipients. Below, Subsection III-A will incorporate uncertainty into the theory, Subsection III-B will show how the parameters of this extended theory can be estimated, and Subsection III-C will use demographic data from natural-fertility populations to draw inferences about time preference.

[10] There are several measures of generation length in demography (Ansley J. Coale, 1972). The most appropriate here is the mean age of childbearing (i.e., the average age of mothers of newborn daughters): $T = \sum_x x e^{-nx} l_f(x) m_f(x)$.

[11] This is similar to Getty's (1989, 1991) finding that $i = 1/T$. The similarity is probably mere coincidence, however, as Paul Romer has pointed out to me. Getty assumes that i must match the rate of production of aggregate fitness (i.e., the rate at which new individuals are recruited into the population). Since the population replaces itself once per generation, the recruitment rate R must satisfy $RT = 1$ in stationary equilibrium. Setting $R = i$ produces Getty's conclusion that $i = 1/T$.

[12] More explicitly, the Malthusian interpretation assumes that: (i) A population will grow whenever its members find that marginal time-productivity of intergenerational investment exceeds MRS_κ. (The productivity pays for larger families.) (ii) Marginal time-productivity is, however, a declining function of population size. (This must be so under any form of density-dependent population regulation.) Thus, (iii) at stationary equilibrium, MRS_κ must equal the marginal time-productivity of intergenerational investment. (iv) This same equilibrium also requires that the consumption of a daughter at age 25 equals that which her mother enjoyed at age 25; consequently $MRS_\kappa = MRS_\rho$. I thank Jack Hirshleifer for this Malthusian interpretation.

A. *The Evolutionary Discount Function*

As before, I assume that an altruist allele adds $\Delta P(x_1)$ to the donor's survival at age x_1, and $\Delta P(x_1+\tau)$ to the recipient's when the donor's age is $x_1+\tau$. Now however, the recipient's age, reproductive value, and coefficient of relationship are all random variables. This requires that the row-products in Table 1 be replaced by their expectations. The altruist allele is selectively neutral when the sum of these expected row-products is zero. If the duration dx of effects is small, this sum is approximately

$$v_D(x_1)\,\Delta P(x_1) + e^{-\rho\tau}E\{rv_R(x_2)\}\Delta P(x_1+\tau)=0$$

where E denotes the expectation. In taking this expectation, I define $v_R(x_2)=0$ when there is no recipient at all. Rearranging gives

$$(13)\qquad \mathrm{MRS}_\rho \equiv -\frac{\Delta P(x_1+\tau)}{\Delta P(x_1)} = \frac{v_D(x_1)}{E\{re^{-\rho\tau}v_R(x_2)\}}.$$

To facilitate presentation of numerical results, I define an *evolutionary discount function* λ, which satisfies

$$(14)\quad \mathrm{MRS}_\rho \equiv \exp\left[\int_x^{x+\tau}\lambda(x,w)\,dw\right].$$

This formulation is general enough to accommodate almost any pattern of discounting. For example, when λ is a constant, future survival benefits are discounted exponentially at a constant rate. On the other hand, hyperbolic discounting (George Ainslie and Nick Haslam, 1992) implies that λ is a hyperbolic function of τ.[13]

[13]For example, suppose future benefits are valued at $(1+\alpha\tau)^{-1}$ times their nominal value. Equating this to $\exp[-\int_x^{x+\tau}\lambda(x,y)\,dy]$ shows that $\lambda(x,x+\tau)=\alpha/(1+\alpha\tau)$.

In evolutionary equilibrium, the MRS in fitness must equal that in preferences. Combining equations (1) and (14) gives

$$(15)\quad \theta=\theta(x,x+\tau)=\frac{1}{\tau}\int_x^{x+\tau}\lambda(x,w)\,dw.$$

The average value of λ over any interval predicts θ, the MRTP over that interval. Note however, that this rate refers to effects on survival rather than on consumption. Consequently, λ is more closely related to the marginal rate of utility discount [η in equation (2)] than to the MRTP for consumption.

To define λ, note that equations (13) and (14) imply that

$$(16)\quad E\{Z(x,x+\tau)\} \equiv \exp\left[-\int_x^{x+\tau}\lambda(x,w)\,dw\right]$$

where $Z(x,x+\tau)=e^{-\rho\tau}rv_R(x_2)/v_D(x_1)$. Z measures the beneficial effect of a unit increase in the recipient's survival relative to a unit increase in the donor's. To calculate $E\{Z\}$, one must make some assumption about the process by which the recipient is selected. There are several possibilities worth investigating, but I will consider only one: I assume that the benefit is awarded to the single individual whose value of Z is largest. The problem thus is to take the expectation of Z under this assumption.

This expectation depends on the joint distribution of the numbers of relatives of each age and degree of relatedness who are alive at particular times. I assume that the number of same-sexed offspring produced at age x by a parent of sex g is a Poisson random variable with mean $m_g(x)$, and that offspring of sex g survive independently to age a with probability $l_g(a)$. The Appendix describes an algorithm for calculating $E\{Z\}$ and $\lambda(x,y)$ under these assumptions.

When costs and benefits affect consumption, Z is replaced by

$$V=\frac{e^{-\rho\tau}rP_\kappa[x_2,\kappa(x_2)]v_R(x_2)}{P_\kappa[x_1,\kappa(x_1)]v_D(x_1)}.$$

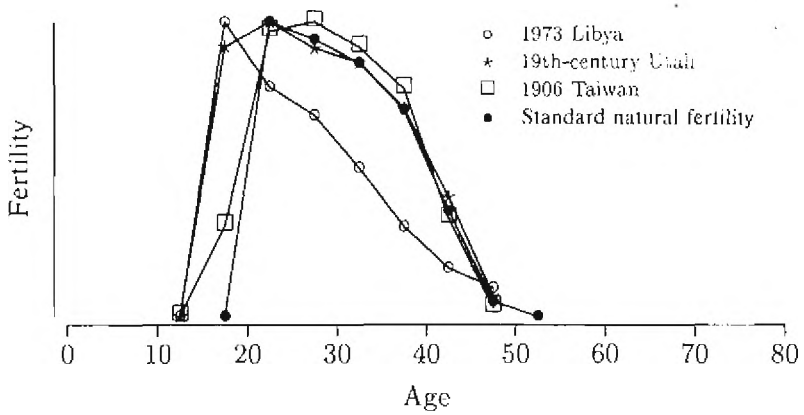

FIGURE 3. AGE-SPECIFIC FERTILITY (FEMALES)

Notes: These schedules are normalized so that all have the same maximum value. *Sources:* 1906, Taiwan (Hamilton, 1966); standard natural fertility (Coale and T. J. Trussell, 1974); 1973 Libya and 19th-century Utah (Mahjoub A. El-Faedy and Lee L. Bean, 1987).

The rest of the analysis proceeds as before, except that it is necessary to know the form of the function $P_\kappa[x, \kappa]$ and to take the expectation with respect to the joint distribution of wealth, age, and relatedness among potential recipients. This task is not attempted here.

B. *Methods for Estimating the Evolutionary Discount Function*

The evolutionary discount function depends on the population's growth rate ρ, on male and female fertility schedules $[m_m(x)$ and $m_f(x)]$ and survivorship functions $[l_m(x)$ and $l_f(x)]$. The relevant values of these parameters are not those of contemporary populations, but those which prevailed during recent evolutionary history. These past values are, of course, unknown. Fortunately, age-specific rates of human birth and death obey certain regularities that allow strong inferences about λ.

The first of these regularities concerns the shape of the age-specific (female) fertility schedule in so-called "natural-fertility populations."[14] Although such populations vary greatly in overall rates of fertility, there is little variation in the shape of the fertility schedule: the ratio of fertilities at any two ages is relatively constant from population to population, as is illustrated by Figure 3. Thus, one can be fairly confident that age-specific fertilities in ancient human populations were roughly proportional to those in contemporary natural-fertility populations.

Less is known about male paternity schedules. I have found appropriate data for only three natural-fertility populations: those shown in Figure 4. The paternity schedules illustrated there are broadly similar, but they vary more than do fertility schedules of females. Among 19th-century Mormons, paternity peaked when men were in their mid-twenties. In contrast, the peak comes at least a decade later among Turkmen and Libyans.

Human survival schedules are also broadly similar, notwithstanding the emphasis that is often placed on variation among them. Coale and Paul Demeny (1983) define four families of model life tables, of which the

[14] A natural-fertility population is one in which birth control is either absent or else is applied independently of the number of a woman's existing children. In natural-fertility populations, women may use birth control to space births, but they do not use it to achieve a target family size (Elizabeth A. Cashdan, 1985).

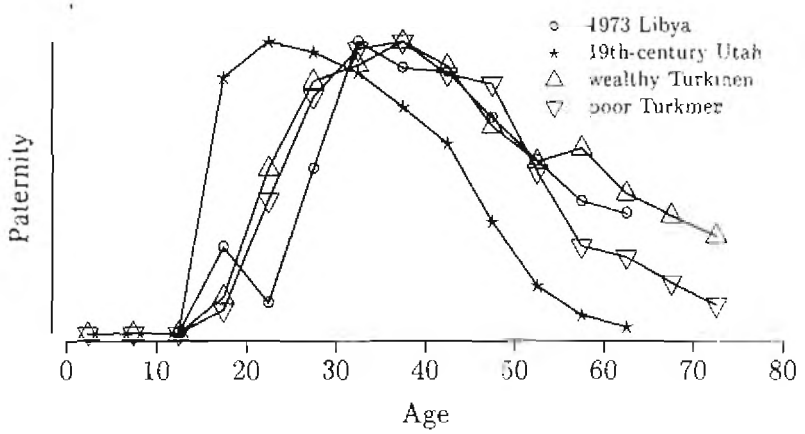

FIGURE 4. AGE-SPECIFIC PATERNITY (MALES)

Notes: These schedules are normalized so that all have the same maximum value. *Sources:* Libya and frontier Utah (El-Faedy and Bean, 1987); Turkmen of Persia (William Irons, 1979).

"West" family seems most reasonable as a model for prehistoric human mortality. Within this family, one can choose survivorship schedules for a great variety of levels of overall mortality, as measured by e_0^0, the expectation of life at birth.

Finally, I must make an assumption about the rate ρ of population growth during prehistory. Although the human population has been growing for several centuries, its rate of growth prior to this must on average have been very low (McEvedy and Jones, 1978). Thus, I assume that $\rho = 0$.[15]

C. *Evidence on Discounting: Inference from Natural-Fertility Populations*

Figure 5 presents the evolutionary discount function implied by the fertility and paternity data of 19th-century Utah (El-Faedy and Bean, 1987) and the Model West life table with mortality level 12 (Coale and Demeny, 1983 p. 47). This mortality level implies that the expectation of life at birth, e_0^0, is approximately 45 years. In the figure, "age at investment" refers to the age at which a decision is made between an immediate and a delayed benefit. Ages beyond the age at investment are "future ages." Thus, the line marked by open circles shows the discount function pertaining to some investment that might be undertaken by newborn infants,[16] whereas the line marked by stars pertains to investments by young adults.

To understand what these curves mean, consider a hypothetical 20-year-old woman who has been offered some survival benefit that will not arrive until she is 40. Since she is female and is now age 20, the starred curve in the upper panel of Figure 5 applies. It indicates that the average discount

[15] This implies both that $1 = \sum_x l_m(x) m_m(x)$ and that $1 = \sum_x l_f(x) m_f(x)$. To satisfy these requirements, fertility and paternity schedules are rescaled as follows. Let $m_f^*(x)$ denote the schedule of female fertility as given by the data. Prior to entering this schedule into the analysis, I rescale it as $m_f(x) = m_f^*(x) / \sum_y l_f(y) m_f^*(y)$. An analogous rescaling is used for male paternity schedules.

[16] The notion of investment by newborns is not quite as preposterous as it sounds. An infant who demanded less of its parents, for example, by crying less or eating less, would allow resources to be diverted from itself to other siblings. To an evolutionist, this constitutes an investment in these siblings. Nonetheless, I do *not* mean to argue that such investments are of any great importance.

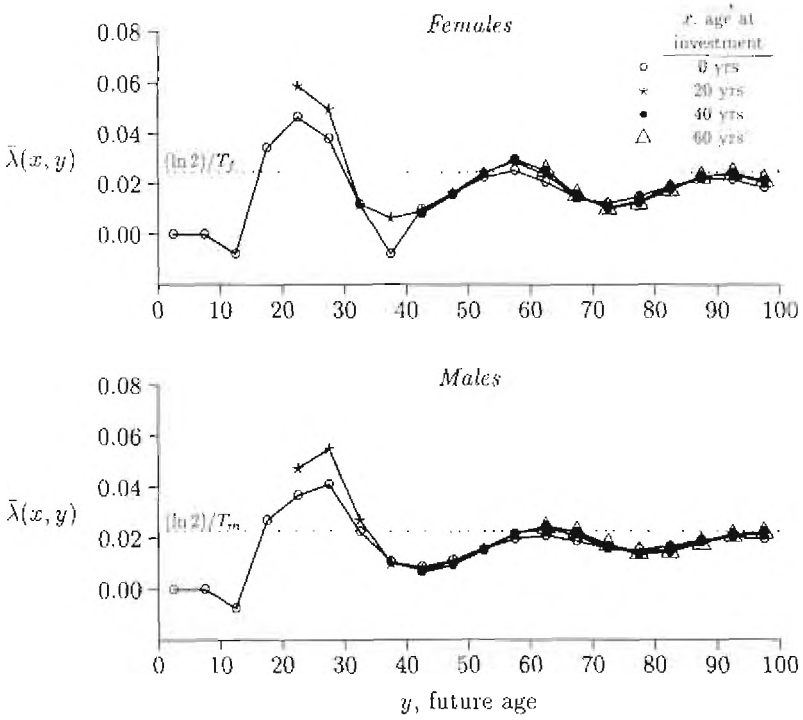

FIGURE 5. EVOLUTIONARY DISCOUNT FUNCTION

Notes: $\bar{\lambda}(x, y)$ is the average evolutionary discount rate within a five-year age interval. "Age at investment," x, refers to the age at which a decision is made between an immediate and a delayed benefit. "Future age," y, refers to ages beyond the age at investment. The dotted lines show the rate of interest predicted by equation (12), where the generation time is $T_f = 27.98$ for females and $T_m = 30.45$ for males. The figure is based on male and female fertility of 19th-century Utah Mormons (El-Faedy and Bean 1987) and on the Model West life table (mortality level 12, $e_0^0 = 47.5$ for females and 44.5 for males) (Coale and Demeny, 1983 p. 47).

rates within the four five-year intervals spanning ages 20–40 are 0.059, 0.050, 0.012, and 0.007, respectively. The average of these is 0.032, and this implies that the future benefit should be discounted by a factor of $\exp[-20 \times 0.032] = 0.529$.[17] The 20-year-old, therefore, should value this delayed benefit at only about half of its nominal value. In general, one applies a MRTP that is an average of λ over the relevant interval.

[17] Equation (15) says that this average is equal to $\theta = 0.032$, and equation (1) implies that the future benefit is discounted by a factor of $e^{-\theta\tau}$, where $\tau = 20$ is the time delay.

This same female should discount a shorter delay at a higher rate. For example, she should discount a delay of five years at an average rate of 0.059, nearly twice the average rate required for a 20-year delay.

Figure 5 predicts strong age differences in the MRTP applied to delays of 5–10 years. Such benefits should be discounted only slowly by preteen-age children, very rapidly by young adults, and at intermediate rates by older adults.

Figure 5 also suggests that individuals of all ages will apply roughly the same discount rate (about 2 percent per year) to benefits that are delayed more than 50 years. This is consistent with equation (12) and

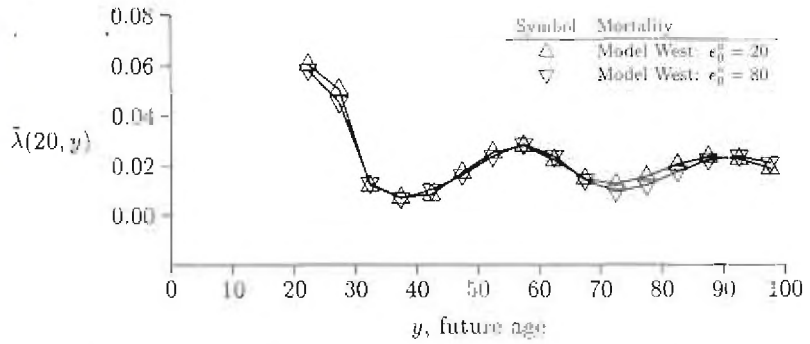

FIGURE 6. EFFECT OF MORTALITY LEVEL ON THE EVOLUTIONARY DISCOUNT FUNCTION

Notes: Shown are evolutionary discount functions for females calculated using the 19th-century Utah fertility and paternity schedules and the Model West mortality schedule, with two extreme levels of mortality. The curve indicated by "△" symbols has extremely high mortality: the expectation of life at birth (e_0^0) is only 20 years for females and 18 for males. The curve indicated by "▽" symbols has extremely low mortality ($e_0^0 = 80$ for females, 76.6 for males).

shows that the approximations underlying that equation introduce no serious error.

It is interesting that the curves for various ages of altruism lie very nearly atop one another, so that $\lambda(x, y) \approx \lambda^*(y)$. The pattern of time preference predicted here is well approximated by a discount function that depends on a single argument. This has an important implication: it precludes reversals of preference. Reversals of preference occur when an individual changes his mind about the relative value of two alternative rewards as the time of their arrival approaches. For example, I may plan on Monday to diet on Tuesday, but when Tuesday arrives I may succumb instead to the temptation of a chocolate cake. Empirical work on this issue has relied on theoretical work showing that preference reversals must occur unless future benefits are discounted exponentially at a constant rate (Robert Strotz, 1956; Ainslie, 1975). However, an important limitation of this analysis seems to have gone unnoticed: it assumes that the discount function depends on delay, but not on age. The present model provides a counterexample to the conclusion of Strotz and of Ainslie. To see why, consider two benefits that will arrive at ages y and $y + \tau$, respectively. At time $x < y$ the two benefits will be discounted by factors of $\exp[-\int_x^y \lambda^*(z)\,dz]$ and $\exp[-\int_x^{y+\tau} \lambda^*(z)\,dz]$, respectively. The ratio of these discount factors is always

$$\exp\left[-\int_y^{y+\tau} \lambda^*(z)\,dz\right]$$

regardless of the value of x. Thus, there can be no reversal of preference even when λ^* is far from constant. This implies that evidence for nonconstant rates of discount cannot by itself account for reversals of preference. To apply the Strotz-Ainslie analysis it is necessary also to show that the pattern of time preference is age-independent.

The value of the results in Figure 5 depends on their sensitivity to the underlying demographic parameters. If each set of demographic data implies a markedly different evolutionary discount function, then the particular results shown in Figure 5 are of little interest. In Figure 5 the curves for different ages at investment are quite similar. Thus, it is probably sufficient to compare populations using a single age at investment, say, 20. This is done in Figure 6, which uses the 19th-century Utah fertility

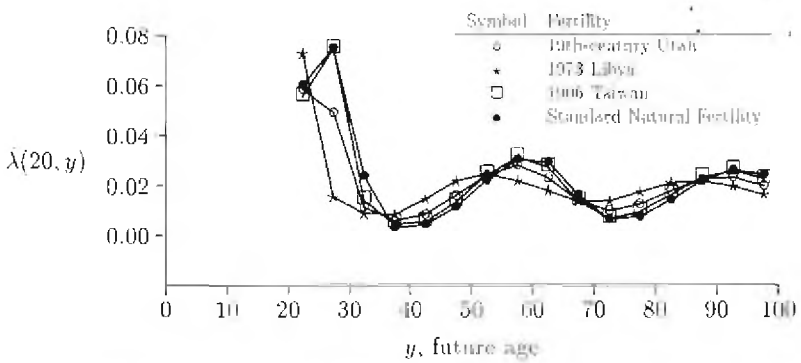

FIGURE 7. EFFECT OF FERTILITY SCHEDULE ON THE EVOLUTIONARY DISCOUNT FUNCTION OF FEMALES

Notes: Each curve uses a different schedule of female fertility. All curves use the Model West mortality schedule (level 12) and the 19th-century Utah Mormon paternity schedule.

and paternity schedules together with two variants of the Model West mortality schedule. One schedule assumes extremely high mortality ($e_0^o = 20$ for females, 18 for males), while the other assumes extremely low mortality ($e_0^o = 80$ for females, 76.6 for males). As the figure shows, mortality has only a minor effect on the evolutionary discount function. The similarity of these curves is remarkable in view of the extremely different conditions that they represent. The mortality experience of all real populations can be expected to fall within the extremes represented.

The small effect of differences in mortality rates is surprising. Intuitively, one might expect discounting to be rapid when mortality is high, since the chances of living to enjoy a delayed benefit are then decreased. However, the assumption of zero population growth implies that populations with high mortality must also have high fertility. This elevated fertility increases the delayed benefit's effect on fitness. Apparently, these opposing effects balance almost exactly.

Figure 7 shows the effect of various fertility schedules on the female evolutionary discount function. The three curves there use identical schedules of survival and of paternity. All three are similar in implying a high MRTP among young adults followed by damped oscillations that approach a long-term rate of about 2 percent. The main differences are in the timing of the peak. The fertility data for Libya and Utah imply that discounting should peak in the late teens or early twenties, while the Taiwan and standard natural-fertility schedules generate a peak in the late twenties. These differences mirror those in the fertility schedules (Fig. 3).

Figure 8 shows the effect of various paternity schedules on the evolutionary discount functions of females and of males. All the curves employ identical schedules of survival and of female fertility. They show that, whereas the evolutionary discount functions of females are insensitive to the paternity schedule, those of males are somewhat more sensitive. The Utah Mormon paternity schedule implies that discount rates should peak when males are in their twenties, while the other schedules imply a somewhat later peak. Without knowing more about the paternity schedules that characterized human evolutionary history, it is difficult to choose between these alternatives. In spite of their differences, the predicted male discount functions are broadly similar. All imply high rates of time preference by young adults and lower rates among their elders. All converge toward rates of roughly 2 percent in old age.

In each figure, rates oscillate up and down with a period of roughly a generation. To understand these oscillations, consider a

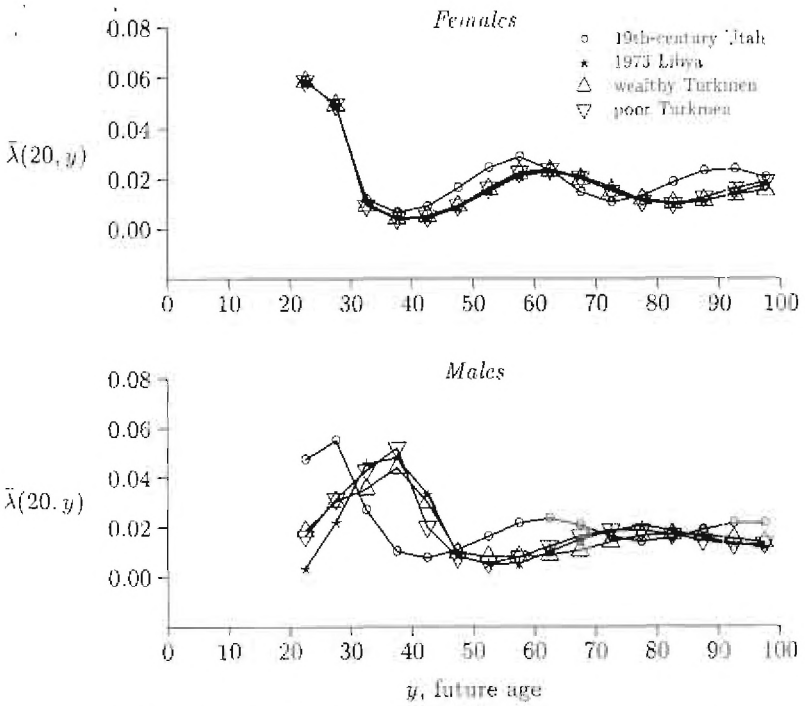

FIGURE 8. EFFECT OF PATERNITY SCHEDULE ON THE EVOLUTIONARY DISCOUNT FUNCTION

Notes: Each curve uses a different schedule of paternity. All curves use the Model West mortality schedule (level 12) and the 19th-century Utah Mormon fertility schedule.

hypothetical woman (Ego), who has a daughter at age 25, a granddaughter at age 50, and a great-granddaughter at age 75. Table 1 indicates that the benefits that Ego provides for herself and for these descendants increase her own expected reproductive success by an amount proportional to $rv_R(x)$, where r is the coefficient of relationship, and $v_R(x)$ is the reproductive value of the recipient. The reproductive-value curves, discounted by the relatedness r of each individual to Ego, are shown in Figure 9 in log scale. At each age, Ego should direct benefits to the individual whose curve is highest. Thus, she will keep for herself any benefits that arrive between ages 20 and 31. Since her own reproductive value is falling, these benefits are worth more the earlier they arrive, and her age-specific discount rates are therefore high. Benefits that arrive after age 31, on the other hand, will go to her daughter. Since the daughter's reproductive value is rising at first, these benefits are worth more if delayed. Consequently, the mother's discount rate is negative. As the daughter's reproductive value begins to fall, the process repeats itself, giving rise to oscillations similar to those seen in the other figures.

Finally, I return to the question of what these results imply about the interest rate. Figures 5–8 show that, in all cases, λ is close to 2 percent for individuals in their thirties and beyond, that is, when $x > 30$. For such individuals, the MRS in fitness for survival effects [equation (14)] is approximately

$$\mathrm{MRS}_p \approx e^{0.02t}.$$

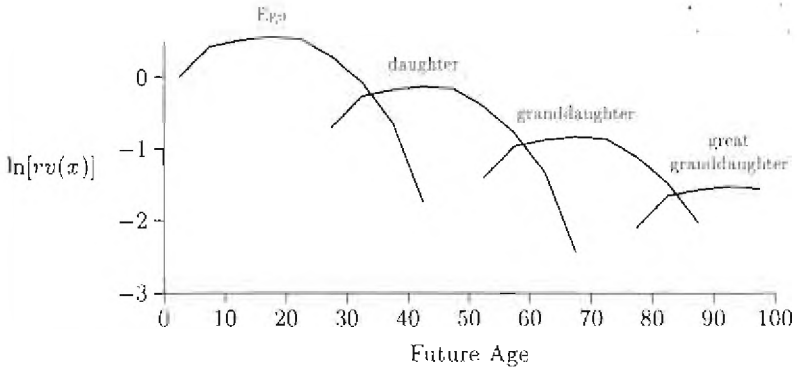

FIGURE 9. VALUES TO EGO OF SELF AND OF DESCENDANTS

Notes: This shows the log of contributions to Ego's reproductive success of herself, a child, a grandchild, and a great-grandchild, assuming that individuals of each generation are born 25 years apart (calculated from demographic data for Taiwan, 1906).

In the special case when the marginal effect of consumption on survival is the same for the donor and all potential recipients, this is equivalent to the MRS in fitness for consumption. Equating the marginal rates of substitution in fitness and in exchange [equation (4)] shows that the real interest rate should be near 2 percent. As before, the result is in good agreement with reality.

IV. Discussion and Summary

This paper has explored the potential of a new method of economic analysis, that of setting the MRS in fitness equal to those in preferences and in exchange, as first suggested by Hansson and Stuart (1990). This procedure is justified by the hypothesis that human preferences are at evolutionary equilibrium: that no mutation changing preferences could be favored by natural selection.

The quantitative results derived from this hypothesis all depend on demographic parameters. The relevant parameter values are those which prevailed on average during humans' recent evolutionary history. Since these are unknown, I have instead relied on demographic statistics from modern natural-fertility populations, whose vital rates are thought to resemble those of preindustrial populations. It would be unwise, however, to take any single modern population as the examplar of our unknown ancestors. We do not know whether prehistoric human demography was more similar to that of 19th-century Taiwan, or that of 19th-century Utah, to name just two possibilities. Nonetheless, it seems likely that species-wide mean demographic parameters have for a very long time fallen within the range spanned by modern natural-fertility populations. And as I have shown, the results are hardly changed regardless of which natural-fertility population one examines. Thus, the results are probably not compromised by ignorance about prehistoric demography.

The results may be summarized as follows:

(i) The hypothesis of evolutionary equilibrium gives definite form to indifference curves that are arbitrary in the classical Fisherian analysis. In addition to the classical indifference diagram, which is drawn on consumption axes, I introduce a complementary diagram (Fig. 2) that displays the same information on survival axes. A market line that is linear on consumption axes becomes

concave on survival axes. In evolutionary equilibrium, indifference curves are more convex on consumption axes than on survival axes, but they are convex in either case.

(ii) Algebraic results are obtained by equating two expressions for the MRS in preferences: one derived from the hypothesis of evolutionary equilibrium, and the other from a familiar additively separable intertemporal utility function. The result implies that the marginal utility of dated consumption depends on age as well as on level of consumption. I suggest a formulation in which the subjective rate η of utility discount is zero when an investment benefits the investor himself. In the general case, $\eta = -(\ln r)/\tau$ where τ is the delay between investment and return on investment and r is the coefficient of relationship between the investor and the recipient. Thus, for a fixed degree of relatedness $r < 1$, the subjective rate of utility discount is inversely proportional to τ.

(iii) The analysis confirms some classical ideas about the causes of time preference, but it also suggests revisions. It identifies three factors that lead humans to prefer immediate over delayed benefits: (a) an expectation of rising consumption, (b) declining reproductive value, and (c) the possibility that delayed benefits may accrue to children or other descendants rather than to the investor. Factor (a) is well known to economists (Fisher, 1930 pp. 73–75). Factor (b) combines a familiar and a novel effect. After reproductive maturity, reproductive value declines both because of a steady decrease in the expectation of future life and because of a decline with age in fertility. The first of these effects is well known (Fisher, 1930 pp. 84–85), but the effect of declining fertility has apparently not been discussed. Its implications are important, however, for it underlies all of the age effects discussed below. Finally, factor (c) makes definite what has heretofore been arbitrary: the strength of the bequest motive. Survival benefits to descendants are discounted by a factor of 2^{-d}, where d is 1 for offspring, 2 for grandchildren, and so forth.

(iv) The hypothesis of evolutionary equilibrium implies that the marginal rate of time preference (MRTP) will vary both with age and with the length of delay. Young adults should discount at a higher rate than their elders, especially over short delays. The predicted age effect receives support from the common observation that young adults seem to live as if there were no tomorrow, and from the fact that crime rates are high among this age group[18] (T. Hirschi and M. Gottfredson, 1983; Walter R. Gove, 1985; Martin Daly and Margo Wilson, 1990). It may also be relevant that empirical studies of college students have concluded that humans discount rapidly (Richard Thaler, 1981; John K. Horowitz, 1991; Gordon C. Winston and Richard G. Woodbury, 1991), while studies of individuals nearing retirement have reached the opposite conclusion (L. J. Kotlikoff et al., 1982; Alan Blinder et al., 1983).

(v) The predicted discount functions imply that young adults should discount longer delays at progressively lower rates. This pattern is often reported in empirical studies (Thaler, 1981; Winston and Woodbury, 1991) and has been used to explain the phenomenon of preference reversal. Yet no reversals of preference are implied under the present model. As shown above, nonconstant discount rates need not imply preference reversals unless the pattern of time preference is age-independent.

(vi) The hypothesis of evolutionary equilibrium implies that the interest rate

[18] It is plausible to suppose that a high MRTP does, in fact, encourage crime: the benefits of crime are often immediate, and its costs are delayed. This point of view is often expressed in the criminological literature (James Q. Wilson and Richard J. Herrnstein, 1985 p. 205), and is supported by at least one empirical study (F. Buss, 1964).

should be close to

$$(17) \qquad i = (\ln 2)/T + \rho$$

where T is the generation time and ρ is the rate of population growth [see equation (12)]. The idea that the real interest rate should depend on the rate of population growth goes back at least to Paul Samuelson (1958), who found an equilibrium at $i = \rho$. However, this equilibrium is dynamically unstable in Samuelson's model until money is introduced. Hansson and Stuart (1990) find that this equilibrium is stable after all, when preferences evolve by natural selection. Their result differs from equation (17) because they assume (unrealistically) that parents and offspring are genetically identical. The ln(2) in my result is really $-\ln r$, where $r = \frac{1}{2}$ is the genetic correlation between parent and offspring. When offspring are genetically identical to parents, $r = 1$ and the first term in (17) disappears. This term measures the discounting that occurs because human offspring are not genetic replicas of their parents.

The two terms on the right side of (17) change on vastly different time scales. The correlation between parents and offspring does not change at all, and large changes in ρ produce only modest changes in T.[19] A large change in T would require evolutionary change in some parameter such as age at maturity or the rate of reproductive senescence. Thus, important changes in T happen only on an evolutionary time scale. On the other hand, ρ can change dramatically from one decade to the next. Thus, one might expect the interest rate to track the rate of population growth over short time scales.

However, I would argue otherwise. Population growth enters into the present analysis only via its effect on the evolution of preferences. If preferences respond via genetic evolution, then this response will also be slow. This led me to argue above that the relevant value of ρ is a long-term average, which will be near zero if human populations grew only slowly prior to the last couple of centuries. The real interest rate should therefore remain relatively constant, near ln(2) per generation or roughly 2 percent per year, regardless of the current rate of population growth. If this view is correct, then models relating the interest rate to population growth will not be very helpful. The more important effect is that arising from the fact that offspring are imperfect genetic replicas of their parents.

[19] See, for example, Nathan Keyfitz's (1968 table 7.4) tabulation of the mean age of childbearing.

Appendix: An Algorithm for Calculating $\lambda(x, y)$

I assume that the distribution of the number of offspring of sex g born to an individual of sex g at age x (given survival to that age) is Poisson, with mean $m_g(x)$, and that each offspring of sex g survives to age a independently with probability $l_g(a)$. I also assume a sex ratio of unity, which implies that $m_g(x)$ is also the expected number of opposite-sex births. These assumptions imply that the numbers of descendants of each later generation born when the ancestor's age is y are also Poisson random variables. Let $h_i(y \mid x)$ denote the expected number of female descendants of generation i born when the ancestor's age is y, given that the ancestor lives at least to age x. Since the sex ratio is unity, $h_i(y \mid x)$ is also the conditionally expected number of male descendants of generation i born at this time. Here, children are in generation 1, grandchildren in generation 2, and so forth.

Let D denote the sex of the donor, where $D \in \{\mathrm{m}, \mathrm{f}\}$. The conditionally expected number of daughters born when the donor is of age y is

$$h_1(y \mid x) = \begin{cases} m_D(y) & \text{if } y \le x \\ k_D(y)/l_D(x) & \text{if } y > x \end{cases}$$

where $k_D(x) = l_D(x)m_D(x)$. An equal number of sons is also expected at age y.

Generations 2,3,... refer to grandchildren, great-grandchildren, and so forth. Female descendants of generation $i+1$ are produced by both males and females of generation i. Thus,

$$h_{i+1}(y \mid x) = \int_0^y h_i(z \mid x)[k_m(y-z) + k_f(y-z)]\,dz \qquad i = 1,2,\ldots.$$

Consider first the expected numbers of descendants of generations 1,2,... (i.e., of genuine descendants excluding the ancestor herself). The number of descendants of sex g, age z, and generation i who are alive when the ancestor's age is y is also Poisson with mean

$$u_j = h_i(y-z \mid x) l_g(z) \qquad i > 0$$

where the subscript j indexes the various feasible values of g, i, and z, given x and y. There are only two feasible values of g: m for male descendants and f for females. Since descendants cannot be older than their ancestor, $0 \le z \le y$. I consider only generations $i = 1,2,\ldots,5$ since I am calculating the discount function only up to an ancestral age of about 100. This is sufficient since children begin to appear only when the ancestor is about 20, grandchildren when she is about 40, and so forth.

None of this applies to the donor herself. The probability that she will be alive to receive the benefit at age y is not Poisson, but Bernoulli, with mean $l_D(y)/l_D(x)$.

The coefficient of relationship between an ancestor and a descendant in generation i is 2^{-i}. Thus, equation (16) implies that a benefit accruing to a descendant of sex g, generation i, and age z when the donor is of age y would be discounted by a factor of

$$Z_j = \frac{e^{-\rho(y-z)} v_g(z)}{2^i v_D(x)}$$

where j, as above, indexes all the feasible values of i and z, for given values of x and y. Let $\mathbf{Z}$ denote the vector whose jth entry is Z_j, and $\mathbf{u}$ the vector whose jth entry is u_j.

The expectation of Z_j is calculated by the following algorithm:

(i) Insert into $\mathbf{Z}$ the discount factor for the donor herself: $v(y)/v(x)$, and insert a zero in the corresponding position of $\mathbf{u}$.
(ii) Sort $\mathbf{Z}$ and $\mathbf{u}$ by order of decreasing values of Z_j. Let j^* represent the position of the donor in the sorted vectors.
(iii) Z_j is the largest available value with probability

$$P_j = \begin{cases} \exp\left[-\sum_{i=1}^{j-1} u_i\right](1-e^{-u_j}) & \text{if } j < j^* \\ \exp\left[-\sum_{i=1}^{j^*-1} u_i\right] l(z)/l(x) & \text{if } j = j^* \\ \exp\left[-\sum_{i=1}^{j-1} u_i\right]\left(1-\frac{l(z)}{l(x)}\right)(1-e^{-u_j}) & \text{if } j > j^*. \end{cases}$$

The expression for the case where $j < j^*$ follows because the distribution of the number of individuals of type i (where $i < j^*$) is Poisson with mean u_i. Consequently, the probability of zero individuals of type i is e^{-u_i}, the probability of zero individuals of types $1,2,\ldots,j-1$ is $\exp[-\sum_{i=1}^{j-1} u_i]$, and the probability that at least one individual of type j is present is $1-e^{-u_j}$. The other cases follow from analogous arguments. Taking the expectation over this distribution gives the discount function,

$$E\{Z(x,y)\} = \sum_j P_j Z_j.$$

This completes the algorithm.

This algorithm calculates the discount function favored by natural selection, assuming that deaths and births are the only factors influencing discounting. Once this discount function has been calculated, it remains to calculate the discount-rate function, λ. Typically, demographic data are grouped in five-year age classes, and it is only possible to calculate the mean value of λ within these intervals. The mean discount

rate within a five-year age class is

$$(A1)\quad \bar{\lambda}(x,y) \equiv \frac{1}{5}\int_{y}^{y+5}\lambda(x,w)\,dw$$

$$= -\ln\left[E\{Z(x,y+5)\}/E\{Z(x,y)\}\right]/5.$$

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