

Near-Horizon Virasoro Symmetry and the Entropy of de Sitter Space in Any Dimension

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Abstract

De Sitter spacetime is known to have a cosmological horizon that enjoys thermodynamic-like properties similar to those of a black hole horizon. In this note we show that a universal argument can be given for the entropy of de Sitter spacetime in arbitrary dimensions, by generalizing a recent near horizon symmetry plus conformal field theory argument of Carlip for black hole entropy. The implications of this argument are also discussed.

I. INTRODUCTION

In general relativity, gravitational collapse of massive stars and the expansion of the universe are two sides of the same coin. Both are manifestation of the instability of gravitation. The singularity theorem of Hawking and Penrose [1] applies to both. In its final stage, gravitational collapse always leads to formation of an event horizon, the black hole horizon, which is the boundary of a spacetime region which is not visible to an external observer. In cosmology the counterpart of the black hole horizon is the cosmological event horizon¹ in a de Sitter universe.

The de Sitter metric in d dimensions is given by

$$ds^2 = -\left(1 - \frac{r^2}{\ell^2}\right)dt^2 + \left(1 - \frac{r^2}{\ell^2}\right)dr^2 + r^2d\Omega^2, \quad (1.1)$$

where $d\Omega^2$ is the solid-angle element on $(d-2)$ -dimensional sphere, and the range $0 \leq r \leq \ell$ covers a portion of de Sitter space with the boundary at $r = \ell$. With $d = 4$, it was first discovered [2] as a vacuum solution to the Einstein equations with a repulsive cosmological constant $\Lambda = (d-1)(d-2)/2\ell^2$. On one hand, cosmological models with a repulsive cosmological constant which expands forever approach asymptotically de Sitter space at large times. On the other hand, the exponential expansion of our universe in the early inflationary period was driven by the vacuum energy of a scalar field, effectively acting as a positive cosmological constant, and thus can be described by the de Sitter metric.

It is well-known that a de Sitter universe expands so rapidly that for the geodesic observer at the origin, there is an cosmological horizon located at $r = \ell$, from beyond which light can never reach him/her. The area of this horizon may be regarded as a measure of his/her lack of knowledge about the rest of the universe beyond his/her ken. Thus, one expects that a cosmological horizon should have many similarities with a black hole horizon. Indeed in late seventies Gibbons and Hawking [3] have shown that in general relativity, both the black-hole

¹Hereafter we will use simply the term “horizon” in lieu of “event horizon”.

and the cosmological horizons share the same set of laws which are formally analogous to those of thermodynamics: In either case, the surface gravity at the horizon is proportional to the effective temperature, and the area A of the horizon is to the entropy S , as given by the Bekenstein-Hawking formula:

$$S = \frac{A}{4G}. \quad (1.2)$$

Furthermore, they showed that if the quantum effects of pair creation in curved spacetime are included, this similarity between the laws of horizons and thermodynamics is more than an analogy: An observer will detect a background of thermal radiation coming apparently from the cosmological horizon, in a manner similar to the Hawking radiation from a black hole horizon. Thus the close connection between horizons and thermodynamics has a wider validity than the ordinary black hole cases in which it was first discovered [3].

This lesson becomes particularly important in the wake of the recent progress in string theory in understanding the microscopic states that are responsible for the black hole entropy [4,5]. One is naturally led to search for a fundamental, microscopic mechanism for gravitational entropy that is *universally applicable to de Sitter entropy in arbitrary dimensions as well as to black hole entropy*. The method of counting D-brane states for black hole entropy does not seem to satisfy this universality requirement, since at present we do not know yet what are the D-brane states responsible for a cosmological horizon. Recently Maldacena and Strominger [6] has attacked the problem of de Sitter entropy in the particular case of $2 + 1$ dimensions. They explored the equivalence between Chern-Simons gauge theory and $2 + 1$ de Sitter gravity, and showed that the asymptotic symmetry group of the theory near the cosmological horizon contains a Virasoro subalgebra [7], with a central charge right to reproduce the de Sitter entropy via the Cardy formula [8,9]

$$\log(\rho(h, \bar{h})) = 2\pi \left(\sqrt{\frac{ch}{6}} + \sqrt{\frac{c\bar{h}}{6}} \right), \quad (1.3)$$

which counts the asymptotic density of states of the Hilbert space of a conformal field theory (CFT) labelled by the conformal weight (h, \bar{h}) and the (effective) central charge c .

However, it is hard to see how this argument could be generalized to quantum gravity in other dimensions, where no equivalent Chern-Simons theory is available.

Recently Carlip [10] has put forward a universal argument for black hole entropy in any dimension, exploring near-horizon symmetry and conformal field theory. In the present note we will show that his argument can be generalized to give a universal argument for the de Sitter entropy in arbitrary dimensions. Also a recent proposal in ref. [14] for a candidate CFT on black horizon by dimensional reduction of gravity will be shown to be applicable to the de Sitter case. This note not only adds weight to the universality of Carlip's argument, but also points to a profound connection between the microscopic origin of the gravitational entropy associated with any horizon and conformal field theory. Indeed for quite a while there have been suggestions on the close relationship between the black hole entropy and conformal field theory, see e.g. [11] and references therein. Our note confirms the existence of such a relationship for cosmological horizons.

II. NEAR-HORIZON VIRASORO SYMMETRY

Let us consider a “sector” in quantum gravity that, in the semiclassical limit, corresponds to fluctuations (of both geometry and coordinates) around the standard de Sitter metric (1.1):

$$ds^2 = -N^2 dt^2 + f^2(dr + N^r dt)^2 + \sigma_{\alpha\beta}(dx^\alpha + N^\alpha dt)(dx^\beta + N^\beta dt). \quad (2.1)$$

Here x^α are coordinates on a $(d - 2)$ -dimensional sphere. We have adopted the Arnowitt-Deser-Misner scheme, fixing the horizon to be located at $r = \ell$ and requiring that near the horizon the lapse function N behave as

$$N^2 = \frac{4\pi}{\beta}(\ell - r) + O(\ell - r)^2, \quad (2.2)$$

where $\beta \equiv 2\pi\ell$. We will treat the horizon as an outer boundary (i.e. $0 \leq r \leq \ell$) and require that the metric approach that of a standard de Sitter metric on this boundary. To define the theory more precisely, one may impose the following fall-off conditions near the boundary:

$$\begin{aligned}
 f &= O(N^{-1}), \quad N^r = O(N^2), \quad \sigma_{\alpha\beta} = O(1), \quad N^\alpha = O(N), \\
 (\partial_t - N^r \partial_r) g_{\mu\nu} &= O(N) g_{\mu\nu}, \quad \nabla_\alpha N_\beta + \nabla_\alpha N_\alpha = O(N^2).
 \end{aligned} \tag{2.3}$$

similar to those in ref. [10], with the only difference in the equations involving N_α , which in our case essentially require that the angular momentum be constantly vanishing on the horizon. Given this asymptotic behavior of the metric, it is easy to check that near the horizon, the extrinsic curvature of a slice of constant time behaves like

$$K_{rr} = O(N^{-3}), \quad K_{\alpha r} = O(N^{-2}), \quad K_{\alpha\beta} = O(1). \tag{2.4}$$

In the same spirit of the work of Brown and Henneaux [13] and of Carlip [10], we want to show that the gauge symmetries of this classical theory with boundary contains a Virasoro subalgebra with a central charge. In the Hamiltonian formulation of general relativity, the gauge symmetries are the so-called surface deformations² that preserve the fall-off conditions (2.3) near the horizon. The full generator for surface deformations is known to be given by

$$L[\hat{\xi}] = H[\hat{\xi}] + J[\hat{\xi}], \tag{2.5}$$

where the first term is the bulk term and the second the boundary term [13]:

$$\begin{aligned}
 H[\hat{\xi}] &= \int_{\Sigma} d^{d-1}x \hat{\xi}^\mu \mathcal{H}_\mu \\
 J[\hat{\xi}] &= \frac{1}{8\pi G} \int_{r=\ell} d^{d-2}x \left\{ n^a \nabla_a \hat{\xi}^t \sqrt{\sigma} + \hat{\xi}^a \pi_a^r + n_a \hat{\xi}^a K \sqrt{\sigma} \right\}.
 \end{aligned} \tag{2.6}$$

Here $\{\mathcal{H}_t, \mathcal{H}_a\}$ ($a = r, \alpha$) are the Hamiltonian and momentum constraints. The surface deformation parameters $\hat{\xi}^\mu$ are related to spacetime diffeomorphism parameters ξ^μ by $\hat{\xi}^t = N\xi^t$ and $\hat{\xi}^a = \xi^a + N^a \xi^t$. To preserve the fall-off conditions (2.3) it is required that near $N = 0$,

$$\hat{\xi}^t = O(N), \quad \hat{\xi}^r = O(N^2), \quad \text{and} \quad \hat{\xi}^\alpha = O(1). \tag{2.7}$$

² Readers who are unfamiliar with the formulation of the surface deformations can see [12] for details.

They satisfy a Lie algebra; the part relevant to our later analysis is

$$\{\hat{\xi}_m, \hat{\xi}_n\}_{SD}^t = \hat{\xi}_1^a \partial_a \hat{\xi}_2^t - \hat{\xi}_2^a \partial_a \hat{\xi}_1^t. \quad (2.8)$$

In the full generator (2.5), we need the boundary term $J[\hat{\xi}]$, whose variation cancels the unwanted surface term in the variation of the bulk term $H[\hat{\xi}]$, so that the functional derivative of $L[\hat{\xi}]$ is well defined. It is easy to verify that this is indeed true if we restrict our variations to those satisfying

$$\delta f/f = O(N), \quad \delta K_{rr}/K_{rr} = O(N). \quad (2.9)$$

Following [10], we consider a particular class of surface deformations as follows:

First, for simplicity, we specialize to the cases with $0 = N^r = N^\alpha = \partial_{t,\phi}(N^\phi) = \partial_{t,\phi}(N^2)$, ($\alpha \neq \phi$), where ϕ is a selected azimuthal angle such that N^ϕ is $O(N)$ but not zero. Later we will see that this will help us to distinguish the left and right circular modes of ξ^μ . To avoid the singular behavior on the horizon for the angular mode-decomposition of ξ^t , we introduce a spherical surface H_ϵ with distance ϵ to the horizon, and take N^ϕ to be constant on this surface, which tends to zero as we finally take $\epsilon \rightarrow 0$ at the end of the calculation.

Second we single out a particular class of surface deformations satisfying the $Diff(S^1)$ algebra by the following conditions:

- 1. In the conformal coordinates defined by $fdr = Ndr^*$, the red-shift effect makes the diffeomorphism ξ^t to be light-like, and the classical nature of the horizon allows only the outgoing one into the horizon, that is $(\partial_t + \partial_{r^*})\xi^t = 0$.
- 2. We restrict to the surface deformations which do not change the location of the horizon defined by the zero of the lapse function N^2 . We then impose $\delta_{\xi^\mu} g^{tt} = 0$, where $g^{tt} \equiv \frac{-1}{N^2}$. This leads to $(\partial_t - N^\phi \partial_\phi)\xi^t + [(N^2)_{,r}/2N^2]\xi^r = 0$ on H_ϵ . Then the right circular modes defined by $(\partial_t - N^\phi \partial_\phi)\xi_{(+)} = 0$ becomes zero, and the components of the left circular modes defined by $(\partial_t + N^\phi \partial_\phi)\xi_{(-)} = 0$ satisfy the relation

$$\xi_{(-)}^r = \frac{-2N^2}{(N^2)_{,r}} (\partial_t - N^\phi \partial_\phi)\xi_{(-)}^t = \frac{-4N^2}{(N^2)_{,r}} \partial_t \xi_{(-)}^t. \quad (2.10)$$

- 3. If we assume the parameter $\xi_{(-)}^t$ to be periodic in time, with period T when analytically continued to the Euclidean signature, then from (1) and (2) it can be decomposed into the Fourier modes

$$\xi_{(-)n}^t = a_n \exp\left\{\frac{2\pi i n}{T}\left(t - r^* - \frac{\phi}{N\phi}\right)\right\}. \quad (2.11)$$

Note that the angular decomposition would become singular on the horizon since $N\phi = 0$ there; however, in our treatment this is avoided by evaluating everything first on H_ϵ , not directly on the horizon.

- 4. From (1), (2) and (3), the surface deformation Lie algebra (2.8) becomes

$$\{\hat{\xi}_{(-)m}, \hat{\xi}_{(-)n}\}_{SD}^t = i(n-m)\hat{\xi}_{(-)m+n}^t + \hat{\xi}_m^\phi \partial_\phi \hat{\xi}_{(-)n}^t - \hat{\xi}_n^\phi \partial_\phi \hat{\xi}_{(-)m}^t, \quad (2.12)$$

if $a_n = T/4\pi$. Clearly, (2.12) reduces to $Diff(S^1)$ only if we impose $\hat{\xi}^\phi = 0$, which implies that we restrict the surface deformations to the $r-t$ plane. We emphasize (2.12) is exact without using any fall-off condition, so it is valid away from the horizon.

To show that the above class of surface deformations generate a Virasoro algebra, and to calculate its central charge, we invoke the well-known fact [13,10] that the Poisson brackets of generic surface deformations close to the Lie algebra of surface deformations, $\{\hat{\xi}_1, \hat{\xi}_2\}_{SD}$, with a possible central term $K[\hat{\xi}_1, \hat{\xi}_2]$:

$$\{L[\hat{\xi}_m], L[\hat{\xi}_n]\} = L[\{\hat{\xi}_m, \hat{\xi}_n\}_{SD}] + K[\hat{\xi}_m, \hat{\xi}_n] = i(n-m)L[\hat{\xi}_{m+n}] + K[\hat{\xi}_m, \hat{\xi}_n]. \quad (2.13)$$

In ref. [10], it has been shown that the right-hand side of (2.13) is given by the boundary variation of (2.5), (2.6), which in our case yields

$$\frac{1}{8\pi G} \int_{r=\ell-\epsilon} d^{d-2}x \sqrt{\sigma} \left\{ \frac{1}{f^2} \partial_r (f \hat{\xi}_n^r) \partial_r \hat{\xi}_m^t + \frac{1}{f} \partial_r (\hat{\xi}_m^r \partial_r \hat{\xi}_n^t) - (m \leftrightarrow n) \right\}. \quad (2.14)$$

To have a well-defined angular decomposition, this integral is defined on the hypersurface H_ϵ . The result turns out to be independent of ϵ , so it is safe to take the limit $\epsilon \rightarrow 0$ in

the final result. Also, there is an overall sign difference from [10] because of the reversed direction of the outward unit normal as compared to that of the black hole horizon.

Now let us substitute the modes (2.11) into (2.14) and evaluate it on shell at the de Sitter metric on H_ϵ , then (2.13) becomes

$$\left\{L[\hat{\xi}_{(-)m}], L[\hat{\xi}_{(-)n}]\right\} \Big|_{H[\hat{\xi}]=0} = i \frac{A}{8\pi G} \frac{\beta}{T} n^3 \delta_{m+n,0} = i (n-m) J[\hat{\xi}_{(-)m+n}] + K[\hat{\xi}_{(-)m}, \hat{\xi}_{(-)n}], \quad (2.15)$$

where A is the area of the cosmological horizon. From (2.6) one obtains

$$J[\hat{\xi}_{(-)m}] = \frac{A}{16\pi G} \frac{T}{\beta} \delta_{m,0}, \quad (2.16)$$

Thus,

$$K[\hat{\xi}_{(-)m}, \hat{\xi}_{(-)n}] = i \frac{A}{8\pi G} \frac{\beta}{T} \left(n^3 - n \frac{T^2}{\beta^2}\right) \delta_{m+n,0} \quad (2.17)$$

If we define $L_m = L[\hat{\xi}_{(-)m}]$, then eq. (2.13) gives us a Virasoro algebra with central charge³

$$c = \frac{3A}{2\pi G} \frac{\beta}{T}. \quad (2.18)$$

We view the above classical symmetry as resulting from that of the quantum theory of gravity. Thus, we infer that the latter should respect a (chiral copy of) Virasoro algebra with the central charge (2.18), and the quantum states characterizing the cosmological horizon must form a representation of this algebra with the conformal weight $h = (A/16\pi G)(T/\beta)$, read from (2.16). Then one applies Cardy's formula (1.3) to count the asymptotic density of states, and get the correct entropy (1.2) for the cosmological horizon.⁴

³ This formula is true for any value of the period T ; however, a preferred value is $T = \beta$, with which the horizon is free of conical singularity.

⁴To get the standard form of the central term in (2.17), one may shift L_0 , and therefore h , by $c/24 - h$; this also causes a shift in c by $24h - c$, which makes eq. (1.3) invariant.

III. CFT CANDIDATE BY DIMENSIONAL REDUCTION

In the last section we have used symmetry arguments to derive the behavior of any quantum mechanical theory of cosmological horizon states. But they did not answer the question of what are the specific degrees of freedom that account for the horizon states. Recently Solodukhin [14] has suggested one possible candidate for black hole horizon states by dimensional reduction. In this section, we will show that it is possible to adapt his procedure to the case of a cosmological horizon, yielding a conformal field theory (CFT) with the same entropy we have just derived.

Let us start with the Einstein-Hilbert action with a positive cosmological constant Λ :

$$S_{(d)} = -\frac{1}{16\pi G_d} \int_{M^d} d^d x \sqrt{-g_{(d)}} (R_{(d)} - 2\Lambda). \quad (3.1)$$

where G_d is the Newton constant in d -dimensional space-time. We assume spherical symmetry so that the metric is of the following form ($a, b = 0, 1$)

$$ds^2 = \gamma_{ab} dx^a dx^b + r^2(x_0, x_1) d\Omega^2, \quad (3.2)$$

where γ_{ab} is the metric of the destined 2-dimensional manifold and r^2 represents the degrees of freedom for spherical symmetric fluctuations of the $(d-2)$ -dimensional spheres. Moreover, the 2-dimensional part of the metric will have the following near horizon behavior as in the de Sitter background,

$$ds_{(2)}^2 = -N^2 dt^2 + \frac{dr^2}{N^2}, \quad (3.3)$$

where N^2 is the asymptotic lapse function (2.2).

After dimension reduction with the metric in the form of (3.2), the action is reduced to an effective 2-dimensional theory

$$S_{(2)} = - \int_{M^2} d^2 x \sqrt{-\gamma} \left\{ \frac{1}{2} (\partial\Phi)^2 + \frac{1}{8} \left(\frac{d-2}{d-3} \right) \Phi^2 (R_{(2)} - 2\Lambda + \left(\frac{C}{\Phi^2} \right)^{(d-2)} \Omega_{(d-2)}) \right\}. \quad (3.4)$$

Here $\Omega_{(d-2)} = (d-2)(d-3)$ is the scalar curvature of the $(d-2)$ -dimensional unit sphere, and

$$\Phi^2 = Cr^{d-2}, \quad C = \frac{\Sigma_{d-2}}{2\pi G_d} \left(\frac{d-3}{d-2} \right), \quad (3.5)$$

where Σ_{d-2} is the area of the unit sphere S^{d-2} .

Apply the following substitution

$$\gamma_{ab} \rightarrow \left(\frac{\phi_h}{\phi} \right)^{\frac{d-3}{d-2}} e^{\frac{1}{Q}\phi} \gamma_{ab}, \quad \Phi^2 \rightarrow 4 \left(\frac{d-3}{d-2} \right) Q\phi. \quad (3.6)$$

where ϕ_h is the value of ϕ at the horizon $r = \ell$. Then (3.4) is transformed into the familiar Liouville-type form

$$S_L = - \int_{M^2} d^2x \sqrt{-\gamma} \left\{ \frac{1}{2} (\partial\phi)^2 + \frac{1}{2} Q\phi R_{(2)} + U_d(\phi) \right\} \quad (3.7)$$

where the scalar potential

$$U_d(\phi) = \left\{ \left(\frac{\Sigma_{d-2}}{16\pi G_d} \right)^{\frac{2}{d-2}} \left(\frac{Q\phi}{2} \right)^{\frac{d-4}{d-2}} - Q\Lambda\phi \right\} \left(\frac{\phi_h}{\phi} \right)^{\frac{d-3}{d-2}} e^{\frac{1}{Q}\phi}. \quad (3.8)$$

Note that $U_d(\phi_h)$ is finite. Our U_d only differs from [14] by the term involved Λ , which is irrelevant in deriving the Virasoro algebra below.

The trace of the stress tensor derived from (3.7) by varying γ^{ab} is

$$T \equiv \gamma^{ab} T_{ab} = \frac{-1}{2} Q \square\phi + U_d(\phi) \quad (3.9)$$

It is obvious that the theory is not conformal because of the nonzero T . However, near the horizon, the red-shift effect will suppress the self interactions $U_d(\phi_h)$ [14], and the theory will become conformal in the following coordinates:

$$z = \int \frac{dr}{N^2(r)} = \frac{-\beta}{4\pi} \ln(\ell - r). \quad (3.10)$$

One remark here is that even after suppressing the scalar potential U_d , T will vanish only if the equation of motion of ϕ in the new coordinate is satisfied, so the CFT is purely classical. Because the incoming motion from the horizon is forbidden in the classical theory, one can only use the component $T_{++} = T_{tt} + T_{tz}$ of the stress tensor, not the component T_{--} , as the ‘‘physical’’ charge generating the conformal transformations. So the resultant CFT is chiral.

Note that this arguments for the chiral nature of CFT is different from the condition (2) used in Sec. 2.

Following [14], the Virasoro algebra generated by T_{++} can be shown to have the central charge $c = 12\pi Q^2$. If we write $Q = q\Phi_h/2$, then the central charge becomes

$$c = 6q^2 \left(\frac{d-3}{d-2} \right) \frac{A}{4G_d}. \quad (3.11)$$

The value of L_0 can be determined in a way similiar to [14] to be

$$h = \frac{1}{4\pi^2 q^2} \left(\frac{d-2}{d-3} \right) \frac{A}{4G_d}. \quad (3.12)$$

Using Cardy's formula this leads to the same entropy (1.2) for cosmological horizon, independent of the parameter q .

IV. DISCUSSIONS

Some remarks are in order.

1) We have shown that the close relationship between gravitational entropy associated with event horizon and conformal field theory that was found for black holes actually has a wider validity, i.e. it holds also for cosmological de Sitter-like horizon.

2) The central charge calculated above for the de Sitter horizon, either in Sec. 2 or in Sec. 3, is *classical in nature*. Namely it is the central charge in a Virasoro algebra in the classical theory of gravity.

3) Why the classical central charge gives us the correct entropy when we blindly apply the Cardy formula in quantum conformal field theory is still a mystery not well understood yet. The applicability of the Cardy formula for de Sitter entropy actually involves several key assumptions about the conformal field theory associated with quantum gravity. These assumptions are essentially the same as those involved in the conformal field theory arguments for black hole horizons, as metioned and discussed in the literatures [6,10,9,15]. We would not like to repeat them here.



4) However, in view of the fact that the value of the classical central charge gives the correct value for the gravitational entropy when combined with the Cardy formula, we naturally expect that the value of the classical central charge would not get modified in quantum gravity. Perhaps this suggests that the correct quantum theory of gravity should not be a theory that quantizes the classical theory of gravity. Rather the latter is a low-energy effective theory of the former.

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