Tricritical Point in Random-Field Ising Model

The numerical investigation of the random-field Ising model by Houghton, Khurana, and Seco[1] has revealed a number of interesting features concerning the *three-dimensional Ising model* in a random external field B_i. Earlier, Aharony[2] had already found that even in a *mean-field approximation* the bimodal $[B_i = \pm B]$ and Gaussian $[P(B_i) \approx \exp(-B_i^2/2B^2)]$ distributions led to different phase diagrams. (As one increases T at fixed B, the bimodal distribution has a second-order phase transition for B in the range $0 < B < 0.44$ which becomes first order for B in the range $0.44 \leqslant B < 0.5$. By way of contrast, in the Gaussian case, in the range of fields where the latter has a phase transition this transition is always second order.) In short, the bimodal distribution has a tricritical point at $T = 0.67$, $B = 0.44$, whereas the Gaussian does not. This is surprising, in that it is generally believed that only the average and the variance of random distributions are significant. Additionally, the detailed studies of Houghton, Khurana, and Seco[1] show the importance of dimensionality as well as of the shape of the random distribution on the thermodynamic properties of these special random systems.

In this note, I consider the *tri*modal distribution: Each spin is in a field $B_i = 0, \pm B$. The probability of $B_i = 0$ is P. If we assume that $\pm B$ are equally likely, the probability of each is $(1-P)/2$. (This distribution allows a physical interpretation as a diluted bimodal distribution in which a fraction P of the spins are not exposed to the external field.) Our model mimics the salient feature of the Gaussian distribution, for which a significant fraction of the spins are in weak external fields. Assuming that it is this feature of the Gaussian which leads to a behavior different from the bimodal distribution, we may expect the trimodal distribution also to lose the tricritical point for P greater than some P_c to be determined.

The principal advantage of the discrete trimodal over the continuous Gaussian is simplicity. Here, at least, the mean-field equation of state for the order parameter σ can be obtained conveniently in closed form. It is

$$\sigma = [\tanh\beta\sigma][P + (1-P)/(1+R^2)], \tag{1}$$

where

$$R \equiv \sinh^2\beta B/\cosh^2\beta\sigma. \tag{2}$$

When $P = 0$, one recovers the bimodal model originally solved by Aharony.[2] (It has the above-mentioned tricritical point.) At $P = 1$, arbitrary B, we recover the usual (zero-field !) mean-field equation, the prototype of a second-order phase transition.[3] It remains to determine P_c separating the two behaviors.

I have approached the problem from the following point of view[4]: Letting $\sigma \to 0$ in (1) and (2) yields a generally two-valued, continuous function $T_c(B)$, given implicitly by the equation

$$T_c = P + (1-P)/\cosh^2(B/T_c) \tag{3}$$

(with $k_B = 1$). Such a function must have at least one point at which $dT_c/dB = \infty$. Denote it by (T_0, B). It is a matter of simple algebra to obtain

$$(T_0/2)\{1 + (T_0 - P)/[1 - T_0 + (1-P)^{1/2}(1-T_0)^{1/2}]\} = (T_0 - P)\ln\{[(1-P)^{1/2} + (1-T_0)^{1/2}]/(T_0-P)^{1/2}\}, \tag{4}$$

and a matter of simple numerics to analyze it. The result: Solutions exist only for P within the range $0 \leqslant P < 0.25$. At $P_c = 0.25$ the vestigial tricritical point hangs up at $T_c = 0.44$ and $B_c = 0.57$. [For $P \geqslant 0.25$ the phase transition becomes second order at temperature $T_c(B)$, a rapidly decreasing function of B which vanishes once B exceeds a critical value depending on P.] Because a good approximation to the Gaussian has $P \approx \frac{1}{3}$ we believe that our analysis provides independent confirmation of the lack of tricritical points for nearest-neighbor Gaussian models in $d \geqslant 6$ and in mean-field theory.

Finally, in light of Ref. 1, it would be interesting to see whether for a nearest-neighbor model in 3D, the relevant P_c has been boosted to a value $\gtrsim \frac{1}{3}$. This suggests a need for numerical studies of the T,P,B phase diagram of the dilute bimodal, which we have called the *trimodal*, distribution in 3D.

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[2] A. Aharony, Phys. Rev. B **18**, 3318 (1978).

[3] D. Mattis, *The Theory of Magnetism* (Springer-Verlag, Berlin, 1985), Vol. 2, pp. 17 ff.

[4] See also S. Galam and J. L. Birman, Phys. Rev. B **28**, 5322 (1983); S. Galam, Phys. Rev. B **31**, 7274 (1985).

