Zero-bias anomaly in two-dimensional electron layers and multiwall nanotubes

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The zero-bias anomaly in the dependence of the tunneling density of states $\nu(\epsilon)$ on the energy ϵ of the tunneling particle for two- and one-dimensional multilayered structures is studied. We show that for a ballistic two-dimensional (2D) system the first-order interaction correction to density of states due to the plasmon excitations studied by Khveshchenko and Reizer is partly compensated by the contribution of electron-hole pairs, which is twice as small and has the opposite sign. For multilayered systems the total correction to the density of states near the Fermi energy has the form $\delta\nu/\nu_0 = \max(|\epsilon|,\epsilon^*)/4\epsilon_F$, where ϵ^* is the plasmon energy gap of the multilayered 2D system. For a 2D system with finite-range interaction the particle-hole contribution precisely cancels with the contribution of the zero-sound mode, in agreement with the Fermi liquid theory. In the case of one-dimensional conductors we study multiwall nanotubes with the elastic mean free path exceeding the radius of the nanotube. The dependence of the tunneling density-of-states energy, temperature and on the number of shells is found.

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I. INTRODUCTION

Electron-electron interaction results in a singular suppression of the tunneling (single-particle) density of states at the Fermi surface of low-dimensional metallic systems.[1] The effect, known as the zero-bias anomaly, was first discussed by Altshuler and Aronov[2] for diffusive systems with a short-range interaction, and by Altshuler, Aronov, and Lee[3] for the Coulomb interaction. In the two-dimensional (2D) case the correction to the density of states (DOS) is double logarithmic,[3] $\delta\nu/\nu_0 \sim (\epsilon_F\tau)^{-1}\ln(|\epsilon|\tau)\ln(\epsilon_F^4\tau^3/\epsilon)$, where τ is the impurity scattering time, ϵ is the energy of the tunneling electron measured from the Fermi energy ϵ_F. Zuzin[4] showed that the second logarithm in this formula is cut off at low energies for the experimental setup of a 2D electron plane screened by a metal shield. Rudin, Aleiner, and Glazman[5] generalized the theory of the zero-bias anomaly to incorporate the ballistic energies $\epsilon > 1/\tau$, and argued that the correction actually has the form, $\delta\nu/\nu_0 \sim -(\epsilon_F\tau)^{-1}\ln(\epsilon_F/|\epsilon|)\ln(\epsilon_F^4\tau^3/\epsilon)$. Khveshchenko and Reizer[6] analyzed the contribution of the collective electron excitations, 2D plasmons, to the tunneling DOS and obtained an additional correction $\delta\nu/\nu_0 = (|\epsilon| - \epsilon_F)/2\epsilon_F$. This correction is less singular near the Fermi surface but is dominant in the wide range of energies and is present even in the absence of disorder.

In the present paper we consider the interaction correction to the tunneling DOS of a different system, consisting of a number of identical periodically spaced two-dimensional electron layers. We also neglect the possibility of electron tunneling between the layers. While analyzing such a setup we primarily have in mind high-T_c materials, which are attracting considerable interest with respect to the properties of electron-electron interactions. The electron transport in these materials is extremely anisotropic, and the interlayer tunneling amplitude in some crystals can be as weak as 0.05–2 K.[7] Even though the motion of electrons tunneling from the tip of the tunneling electron microscope (TEM) is confined to the outermost layer only, the presence of internal layers is important as they participate in the screening of the Coulomb interaction between electrons in the top layer.

We also consider a similar, yet slightly different, tunneling geometry that is realized in multi-wall carbon nanotubes (MWNT). A typical MWNT consists of a few graphine monolayer sheets rolled concentrically into cylinders with radius $R \sim 10$ nm. At zero doping they can be either metals or semiconductors, depending on the helical arrangement of the carbon hexagons. In the measurements of the tunneling DOS the tunneling current propagates through the outermost shell[8,9] while the intershell tunneling is suppressed. Depending on the degree of disorder the transport around the elastic mean-free path l can be shorter or longer than the radius of the nanotube corresponding to the diffusive ($l < R$) or ballistic ($R > l$) motion along the circumference of the nanotube. For energies ϵ exceeding the inverse time of propagation around the circumference the zero-bias anomaly is described by 2D formulas. At lower energies a crossover to the regime of a quasi-one-dimensional (quasi-1D) conductor is realized.

For diffusive quasi-1D conductors the interaction correction to DOS in the lowest order of perturbation theory was found to be more singular than in the 2D case, $\delta\nu/\nu_0 \sim -1/\sqrt{|\epsilon|\tau}$.[2] Working beyond the perturbation approximation, Nazarov[10] found that close to the Fermi surface DOS has an exponential behavior, $\ln(\nu/\nu_0) \sim |\epsilon|^{-1}$ (this result was later obtained by Levitov and Shytov[11] in a different way). Ballistic 1D conductors were studied by several authors[12,13] and were shown to have a power-law behavior $\nu/\nu_0 \sim |\epsilon|^{\alpha}$ of the tunneling DOS. The crossover between diffusive and ballistic regimes as well as the temperature behavior of DOS in multiwall carbon nanotubes were recently studied in Ref. 14 under the assumption that electrons reside on the outermost shell only.

Here we study the zero-bias anomaly due to dynamically

screened intershell and intrashell Coulomb interaction in two- and one-dimensional layered systems and its effects on the tunneling DOS. In Sec. II the system of two-dimensional layers is analyzed, with both regimes of diffusive and ballistic in-plane electron motion considered. In Sec. III we discuss the zero-bias anomaly in multiwall nanotubes assuming that the doping electrons are distributed uniformly across the shells.

II. SYSTEM OF TWO-DIMENSIONAL ELECTRON LAYERS

We consider a semi-infinite system of identical conducting two-dimensional layers separated by the distance d, as in Fig. 3 in Appendix A. The interlayer tunneling is neglected, and the tunneling electron from the TEM propagates within the upper layer only. The properties of an isotropic two-dimensional electron system are described by the in-plane Fermi velocity v and the electron-impurity scattering rate $1/\tau$. The presence of internal layers is important as they contribute to the screening of the Coulomb interaction between electrons in the top layer.

The first-order perturbation correction to the tunneling DOS (see Refs. 1,5,6) of the 2D conductor at zero temperature has the form

$$\frac{\delta\nu(\epsilon)}{\nu_0}=\int_{|\epsilon|}^{\epsilon_F} d\omega\mathcal{V}(\omega), \tag{1}$$

$$\mathcal{V}(\omega)=\mathrm{Im}\int_0^\infty \frac{qdq}{2\pi^2}\frac{(\omega+i/\tau)U(\omega,q)\Gamma^2(\omega,q)}{[(\omega+i/\tau)^2-q^2v^2]^{3/2}}, \tag{2}$$

where ν_0 is the thermodynamic two-dimensional density of states $\nu_0=m/\pi$, counting both spin directions. The (inverse) impurity-dressed vertex function is given by

$$\Gamma^{-1}(\omega,q)=1-\frac{i/\tau}{[(\omega+i/\tau)^2-q^2v^2]^{1/2}}. \tag{3}$$

The function $U(\omega,q)$ denotes the Coulomb interaction of two electrons residing in the top plane and dynamically screened by the infinite number of conducting layers below it. To find this function, we consider the Coulomb potential $\phi(\omega,\mathbf{r})$ created by the tunneling electron located in the outermost plane $z=0$. It satisfies the Poisson equation, that has the following form in the Fourier representation with respect to the in-plane coordinates:

$$\left(\frac{d^2}{dz^2}-q^2+4\pi e^2\Pi(q,\omega)\sum_{n=0}^{\infty}\delta(z-nd)\right)\phi(\omega,q,z) =4\pi e\,\delta(z), \tag{4}$$

where the last term in the brackets describes the polarization charge induced in the system of 2D layers, and the polarization operator of a single 2D electron layer is

$$\Pi(\omega,q)=\nu_0\frac{\omega+i/\tau-[(\omega+i/\tau)^2-q^2v^2]^{1/2}}{[(\omega+i/\tau)^2-q^2v^2]^{1/2}-i/\tau}. \tag{5}$$

For the solution of Eq. (4) we refer the reader to Appendix A. We obtain,

$$U(\omega,q)\equiv -e\,\phi(\omega,q,0)=\frac{4\pi e^2\sinh qd}{q(e^{kd}-e^{-qd})}, \tag{6}$$

where k is given by the solution of the equation

$$\cosh kd=\cosh qd-\frac{2\pi e^2}{q}\Pi(\omega,q)\sinh qd, \tag{7}$$

having a non-negative real part $\mathrm{Re}\,k\geqslant 0$. When the wavelength decreases $qd\to\infty$, Eq. (6) gives

$$U(\omega,q)=\frac{2\pi e^2}{q-2\pi e^2\Pi(\omega,q)}, \tag{8}$$

and the conventional expression for the screened interaction in a single 2D layer is recovered. For static interactions Eq. (8) gives $U(\omega\to 0,q)=2\pi e^2/(q+\kappa)$, where $\kappa=2\pi e^2\nu_0$ is the inverse static screening length. In what follows we assume that $\kappa d\gg 1$. This condition ensures that different layers are (at least for low frequencies) weakly coupled. In the opposite limit $\kappa d\ll 1$ the system could be treated effectively as the $3d$ metal with the cylindric Fermi surface. This regime is beyond the scope of our paper.

Within the random-phase approximation electrons can be viewed as noninteracting but moving is a fluctuating electric field whose propagator is given by Eq. (7). In this picture Eqs. (1)–(2) describes the suppression of the tunneling density of states due to fluctuations of the electric field with various frequencies ω and various momenta q.

A. Ballistic motion

First we consider the ballistic limit when the tunneling bias exceeds the scattering rate $\epsilon\gg 1/\tau$. In this case the main contribution to the integral in Eq. (2) comes from the electric field fluctuations from two regions of the momentum space: i) low-momentum region, $qv\ll\omega$, where the fluctuating electric field has a plasmon resonance; ii) particle-hole continuum $qv\gtrsim\omega$ where the fluctuations of the electric field decay due to Landau damping.[15] We analyze the contribution of the plasmon region in Sec. II A 1 and that of the particle-hole region in Sec. II A 2.

1. Plasmon region contribution

In the plasmon region, $qv\ll\omega$, the polarization operator (5) may be approximated by

$$\Pi(\omega,q)=\frac{\nu_0q^2v^2}{2\Omega^2}, \tag{9}$$

where we introduced the notation $\Omega^2=\omega(\omega+i/\tau)$. Solving Eq. (7) with respect to e^{kd}, we obtain

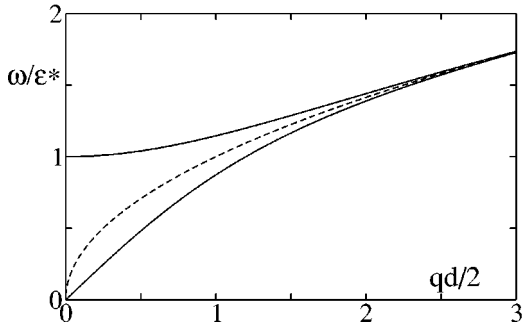

FIG. 1. The plasmon spectra of an infinite number of 2D metallic layers are shown. The frequency ω is plotted as a function of the in-plane momentum q. The dashed line shows the plasmon spectrum of a single 2D layer. Plasmons of different layers interact with each other thus creating a band (inner area between solid lines). The upper solid line $\omega=\omega_+(q)$ represents the upper boundary $\omega=\omega_+(q)$ of the plasmon continuum, with the uniform charge distribution across the layers ($q_\perp=0$) while the lower solid line marks the lower boundary corresponding to the alternating charge in adjacent layers ($q_\perp=\pi/d$).

$$\mathcal{V}_{\rm pl}(\omega)=\frac{1}{4\pi\epsilon_F\omega^2}\,{\rm Im}\int_0^{q_\omega}\frac{dq}{q}(e^{2qd}-1)\times[\Omega^2-\omega_+\omega_--\sqrt{(\Omega^2-\omega_+^2)(\Omega^2-\omega_-^2)}]. \tag{10}$$

Here $\omega_\pm(q)$ represents the upper/lower boundaries of the continuum of plasmons with a fixed in-plane momentum q in an infinite system of layers $z=0,\pm1,\pm2,\ldots$. Each member of the continuum can be parametrized by the wave vector $q_\perp$ along the z direction,

$$\omega_\pm^2(q)=\kappa\frac{qv^2}{2}\left\{\begin{matrix}\coth\dfrac{qd}{2}\\ \tanh\dfrac{qd}{2}\end{matrix}\right\}. \tag{11}$$

The mode $\omega_+(q)$ corresponds to the uniform charge distribution across all layers, $q_\perp=0$, while the other mode, $\omega_-(q)$, describes the alternating charge in adjacent planes ($q_\perp=\pi/d$). The former mode has a frequency gap $\epsilon^*=v(\kappa/d)^{1/2}$, and at $qd\to0$ gives the usual three-dimensional plasmon (in an anisotropic metal). At $qd\to\infty$, both branches tend to the usual plasmon spectrum of a two-dimensional electron gas. This is illustrated by Fig. 1.

We chose the momentum cutoff q_ω in Eq. (10) to be larger than the characteristic momenta of the plasmon modes but still less than the momentum of the particle-hole excitations, i.e., $(\omega/\kappa v^2)\max[\omega,\epsilon^*]\ll q_\omega\ll\omega/v$.

When the frequency is higher than the plasmon gap, $\omega>\epsilon^*$, the integral over the momentum is dominated by large values of $q>1/d$ where ω_+ and ω_- converge exponentially, $\omega_\pm^2=\kappa qv^2[\frac{1}{2}\pm\exp(-qd)]$. Simple calculations give

$$\mathcal{V}_{\rm pl}(\omega)=-\frac{1}{2\epsilon_F}-\frac{1}{2\pi\epsilon_F\omega\tau}\ln\frac{\kappa q_\omega v^2}{\omega^2}. \tag{12}$$

2. Particle-hole region contribution

The region of high transferred momenta $q\gtrsim\omega/v\gg q_\omega$ represents the contribution of particle-hole pairs. In this region propagating modes of the electromagnetic fluctuations are absent because of Landau damping.[15] Under the condition $\kappa d\gg1$, the second term on the right-hand side of Eq. (7) always exceeds the first term (screened interaction) and the interaction takes the form

$$U(\omega,q)=\frac{1}{\nu_0}\frac{\sqrt{(\omega+i/\tau)^2-q^2v^2}-i/\tau}{\sqrt{(\omega+i/\tau)^2-q^2v^2}-\omega-i/\tau}. \tag{13}$$

Substituting Eqs. (13) and (3) into Eq. (2) and integrating from q_ω to infinity we find

$$\mathcal{V}_{\rm p\text{-}h}(\omega)=\frac{1}{4\epsilon_F}-\frac{1}{2\pi\epsilon_F\omega\tau}\ln\frac{\omega}{q_\omega v}. \tag{14}$$

We note that although this calculation for a finite scattering rate is rather simple and straightforward, taking the limit $1/\tau\to0$ in the calculation should be done with care. Assuming $1/\tau=0$ in Eq. (13), and hence restricting oneself to the principal part of the integral, leads to the incorrect answer $\mathcal{V}_{\rm p\text{-}h}(\omega)=1/8\epsilon_F$ that is twice as small as the correct result (14). One has to keep $1/\tau$ a positive infinitesimal to bypass the square-root singularity at $\omega=qv$. We discuss this point in greater detail in Appendix B.

3. Total density of states

Adding the contributions of the plasmon and the particle hole regions (12) and (14) we obtain the following expression for the total spectral weight function:

$$\mathcal{V}(\omega)=\mathcal{V}_{\rm pl}(\omega)+\mathcal{V}_{\rm p\text{-}h}(\omega)=-\frac{1}{4\epsilon_F}-\frac{1}{2\pi\epsilon_F\omega\tau}\ln\frac{\kappa v}{\omega},\quad \omega>\epsilon^*. \tag{15}$$

The results (12) and (15) hold as long as the frequency exceeds the plasmon gap $\epsilon^*=v(\kappa/d)^{1/2}$. In this large frequency and large wave number $qd>1$ region the plasmons have virtually no dispersion along the z direction due to weak interaction of electron densities on different layers. At lower frequencies and wave numbers, $\omega<\epsilon^*$ and $qd<1$, electron densities on different layers interact strongly, and the plasmon spectrum acquires a significant dispersion along the z direction. In this region we may approximate $\omega_-^2=\kappa dq^2v^2/4$, $\omega_+^2=\kappa v^2/d$, $\exp(2qd)=1+2qd$, and take the inverse interlayer distance as the momentum cutoff $q_\omega\sim2/d$. Taking the integral in Eq. (2) explicitly and extracting the imaginary part, we find instead of Eq. (12),

$$\mathcal{V}_{\rm pl}(\omega)=-\frac{1}{4\epsilon_F}-\frac{1}{2\pi\epsilon_F\omega\tau}\ln\frac{\epsilon^*}{\omega}. \tag{16}$$

The high-momentum (particle-hole) contribution does not depend (as long as $\kappa d \gg 1$) on the distance between layers. This can be readily seen from Eq. (13) that does not contain d. This is quite natural as the particle-hole pairs do not induce long-range oscillations of electric field. Therefore, we can take the old expression (14) but with the new cutoff $q_\omega \sim 2/d$, and obtain

$$\mathcal{V}(\omega) = -\frac{1}{2\pi\epsilon_F \omega\tau} \ln\frac{\kappa v}{\epsilon^*}, \quad \omega < \epsilon^*. \tag{17}$$

Integrating the expressions (15) and (17) over the frequency according to Eq. (1), we found the perturbation correction to the tunneling DOS in the ballistic regime

$$\frac{\delta\nu_b(\epsilon)}{\nu_0} = \frac{\max(|\epsilon|,\epsilon^*) - \epsilon_F}{4\epsilon_F} - \frac{1}{4\pi\epsilon_F\tau} \ln\left[\frac{\kappa v}{\max(|\epsilon|,\epsilon^*)}\right] \ln\left[\frac{\kappa v}{\epsilon^2}\max(|\epsilon|,\epsilon^*)\right]. \tag{18}$$

We observe that the correction (18) is less singular near the Fermi surface than in the case of a single 2D layer.

The suppression of the tunneling DOS for a single isolated plane is given by the $d \to \infty$ ($\epsilon^* \to 0$) limit of Eq. (18). This differs from the result of Khveshchenko and Reizer,[6] as the leading contribution [the first term of Eq. (18)] is twice as small as their result. The difference arises from the contribution of the particle-hole excitations, $\mathcal{V}_{\text{p-h}}(\omega)$ in Eq. (15) whose first term partially cancels the first term in the plasmon contribution (12). The overall sign of the DOS correction remains unchanged (suppression).

The contribution of the particle-hole region (15) arises from the low-frequency region $\omega \lesssim vq$ where the interaction is screened. It is, therefore, independent of the bare interaction between the particles. The contribution of the high-frequency region $\omega \gg vq$ arises from the collisionless collective modes that depend essentially on the bare interaction. The overall correction to the tunneling DOS must remain negative for an arbitrary interaction. Indeed, $\mathcal{V}(\omega)$ is related to the retarded correlator of fluctuating electric field.[16] Therefore, its imaginary part is strictly negative for $\omega > 0$, which leads to the suppression of the tunneling density of states (1). We show in Sec. II A 4 in the case of the short-range interactions the contribution of the particle-hole region (15) cancels the contribution of the collective modes in the high-frequency region $\omega \gg vq$. Thus, in order to reproduce the results of the Fermi-liquid theory it is essential to take into account the particle-hole contribution (15).

4. Finite-range interaction

Let us consider a single conducting plane in the case when the Coulomb interaction between electrons is screened at some finite length scale $1/q_0$. We consider a relatively long-range interaction, $q_0 \ll \kappa$, when it can be treated within the random-phase approximation. The screened interaction can then be written in the form

$$U(\omega,q) = \frac{2\pi e^2}{q_0 - 2\pi e^2 \Pi(\omega,q)}. \tag{19}$$

For $q_0 \ll \kappa$, like in the case of the Coulomb interaction considered above, the main contribution to Eq. (2) comes from two well separated regions of the momentum space: (i) the collective excitation (zero sound) region, $q \sim (q_0/\kappa)\omega/v$, and (ii) the particle-hole region, $q \gtrsim \omega/v$.

Following the procedure outlined above we introduce a momentum cutoff q_ω, such that $(q_0/\kappa)\omega/v \ll q_\omega \ll \omega/v$. In the particle-hole region $q \gtrsim \omega/v$ the first term in the denominator in Eq. (19) may be neglected (the interaction is completely screened). Therefore, the contribution of this region is identical to that for the case of the Coulomb interaction, Eq. (15). To evaluate the contribution $\mathcal{V}_{\text{zs}}(\omega)$ of the zero sound region, $q \ll q_\omega$, we substitute the high-frequency asymptotics of the polarization operator (9) into Eq. (19) and use Eq. (2) to obtain

$$\mathcal{V}_{\text{zs}}(\omega) = -\frac{1}{4\epsilon_F} - \frac{1}{4\pi\epsilon_F\omega\tau} \ln\frac{\kappa v^2 q_\omega^2}{2q_0\omega^2}. \tag{20}$$

Adding the particle-hole contribution (15) and using Eq. (1) we obtain for the DOS correction

$$\frac{\delta\nu(\epsilon)}{\nu_0} = -\frac{1}{4\pi\epsilon_F\tau} \ln\frac{\kappa}{2q_0} \ln\frac{\epsilon_F}{\epsilon}. \tag{21}$$

Note that the linear in energy τ-independent correction, which appears for the Coulomb interaction [first term in Eq. (18)], is now absent in agreement with the Fermi-liquid theory. This comes about because the first term in the zero sound contribution (20) cancels with the first term in the particle-hole contribution (15). Thus, when considering the interaction correction to the tunneling DOS for a two-dimensional gas with a finite-range interaction, it is essential to take into account the contribution of the particle-hole excitations in order to obtain results consistent with the Fermi-liquid theory.

B. Diffusive motion

Next we address the regime of strong impurity scattering, $\epsilon \ll 1/\tau$, when the motion of the tunneling electron is diffusive and the polarization operator has the form, $\Pi(\omega,q) = -\nu_0 D q^2/(Dq^2 - i\omega)$, where $D = v^2\tau/2$ is the diffusion coefficient.

The main contribution to the integral (2) comes from the region of transferred momenta $\max\{\omega/D\kappa, \sqrt{\omega/D\kappa d}\} < q < \sqrt{\omega/D}$. In this region the interaction takes the form

$$U(\omega,q) = -\frac{i\omega}{\nu_0 D q^2},$$

and we obtain

$$\mathcal{V}(\omega) = -\frac{1}{4\pi^2\nu_0 D\omega} \ln\frac{D\kappa^2}{\max(\omega, D\kappa/d)}. \tag{22}$$

For a strongly disordered system, $\tau D\kappa/d<1$, and for a relatively high bias, $|\epsilon|>D\kappa/d$ integration over the frequency gives the following DOS correction:

$$\delta\nu_d(\epsilon)=\frac{1}{8\pi^2 D}\ln\left(\frac{D^2\kappa^4\tau}{|\epsilon|}\right)\ln(|\epsilon|\tau)+\delta\nu_b\left(\frac{1}{\tau}\right), \quad (23)$$

where the first term is the well-known double logarithmic correction of Altshuler, Aronov, and Lee[3] and the second term is the constant contribution of ballistic frequencies $\omega>1/\tau$ given by Eq. (18).

At lower bias, $|\epsilon|<D\kappa/d$, we have instead of Eq. (23),

$$\delta\nu_d(\epsilon)=\delta\nu_b\left(\frac{1}{\tau}\right)+\frac{1}{4\pi^2 D}\left(\ln(\kappa d)\ln(|\epsilon|\tau)+\frac{\ln^2\frac{\tau D\kappa}{d}}{2}\right). \quad (24)$$

For a relatively clean system with $\tau D\kappa/d>1$ (i.e., such that $l^2>d/\kappa$) the DOS correction is given by Eq. (24) with the second term in the brackets absent.

Comparing Eq. (24) with the result by Zuzin[4] for the DOS correction for a 2D electron system screened by a bulk metal, we observe that in the diffusive regime at low energies the role of an infinite set of 2D electron layers $z=d,2d,3d,\ldots$ is equivalent to a bulk metal screen located at a distance $d/2$ from the outermost plane.

C. Interlayer tunneling

Throughout the analysis performed above we disregarded completely the possibility of tunneling electrons between different layers. We now consider the corrections to the density of states originating from interlayer tunneling and establish conditions for neglecting such processes. We treat them perturbatively, in the lowest- (second) order in the tunneling Hamiltonian

$$\hat{H}_t=\frac{t}{2}\sum_i\int\frac{d^2p}{(2\pi)^2}[\hat{\psi}^\dagger_{i+1}(\mathbf{p})+\hat{\psi}^\dagger_{i-1}(\mathbf{p})]\hat{\psi}_i(\mathbf{p}), \quad (25)$$

which conserves the in-plane momentum during tunneling. Having in mind mainly applications to the system with strong in-plane scattering (like high-T_c materials) we focus here on the diffusive transport regime $\epsilon\ll 1/\tau$. The corrections to the density of states to the lowest order in the interlayer tunneling amplitude t are shown in Fig. 2. Of the six diagrams drawn here the more important ones are Figs. 2(e) and 2(f) that contain four diffusions each. They correspond to the tunneling corrections to the screened Coulomb interaction rather than to corrections to Green's function of the tunneling electron itself, which are given by Figs. 2(a)–2(d).

To evaluate the contributions of these diagrams one has to know the interaction $U_{nm}(\omega,q)$ between electrons residing on arbitrary layers n and m. As finding such a general expression seems to be quite a cumbersome task for a semi-infinite system of layers, we utilize the corresponding expressions for the infinite system to obtain a qualitative estimate of the effect. It has the form (see Appendix A)

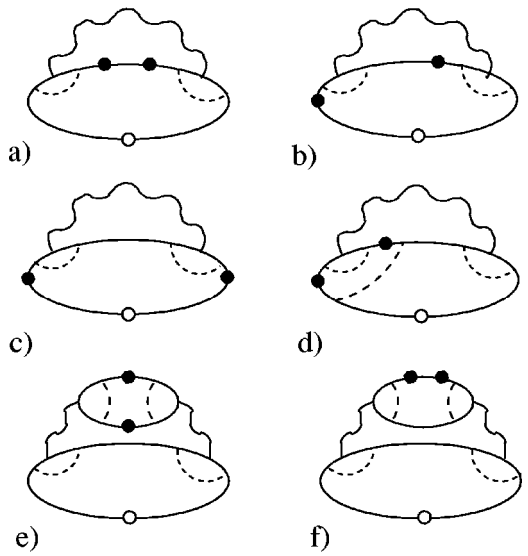

FIG. 2. The diagrammatic representation of the correction to the single-particle density of states from the interlayer tunneling processes to the lowest order of perturbation theory in the tunneling amplitude t. The electron from the microscope probe tunnels into white circles. Solid lines stand for the Green's functions, wavy lines denote the dynamically screened Coulomb interaction, black dots represent the tunneling matrix elements, and the dashed lines are the impurity ladders (diffusons).

$$U_{nm}(\omega,q)=U_{00}(\omega,q)e^{-|n-m|kd},$$

$$U_{00}(\omega,q)=\frac{2\pi e^2\sinh qd}{q\sinh kd}. \quad (26)$$

Figures 2(e) and 2(f) are computed to yield for the tunneling correction,

$$\mathcal{V}_t(\omega)=t^2\nu_0\omega\tau\,\mathrm{Re}\int\frac{q\,dq}{2\pi^2}\frac{U_{00}^2(\omega,q)\tanh kd/2}{(Dq^2-i\omega)^4}. \quad (27)$$

If the frequency is large, $\omega>D\kappa/d$, the main contribution to the integral in Eq. (27) arises from the interval $1/d<q<\omega/\kappa D$, where we can approximate $U_{00}\tanh kd/2\simeq\kappa^2/\nu_0^2q^2$, to obtain

$$\frac{\delta\nu_t(\epsilon)}{\nu_0}=\frac{e^4t^2\nu_0\tau}{\epsilon^2}\ln\frac{|\epsilon|d}{D\kappa}. \quad (28)$$

For smaller frequencies $\omega<D\kappa/d$, the leading contribution comes from the integral $0<q<1/d$ and reads

$$\frac{\delta\nu_t(\epsilon)}{\nu_0}=-\frac{e^4t^2\nu_0\tau d}{2|\epsilon|D\kappa}. \quad (29)$$

It is worth noting that the interlayer correction changes its sign around the point $|\epsilon|\sim D\kappa/d$ and in fact leads to the *increase* in the density of states for $|\epsilon|>D\kappa/d$. Comparing the expressions (28) and (29) with the formulas (23) and (24), we observe that interlayer tunneling correction is small provided that $t\ll|\epsilon|/(\epsilon_F\tau)$. This much stronger condition than the one that might be naively expected ($t\ll|\epsilon|$) is due to

the enhancement of the electron-electron interaction by the interlayer tunneling. When this condition is violated the summation of a wider class with all possible transitions of diagrams is necessary.

III. MULTIWALL NANOTUBES

In this section we consider the interaction suppression of the tunneling DOS in multiwall carbon nanotubes. A multiwall nanotube is built up of M concentric carbon tubes (or shells) each of which can be obtained by rolling a graphine sheet into a cylinder.

In tunneling experiments[9] the tunneling current is believed to propagate along the outermost shell only while the tunneling between the shells is largely suppressed. Thus, the role of electrons in the inner shells is reduced to merely screening the interaction between the outer shell electrons.

The electron band structure of a single metallic carbon nanotube consists of N (this number is usually of the order of a few tens and depends on the doping level and gate voltage) conducting subbands $\epsilon_n(k)$ characterized by the Fermi velocities along the tube axis v_n and around the circumference $v_{\perp n}$. Each of the M shells has, in general, its own band structure $\epsilon_n(k)$. The electrons are scattered between different bands (but mostly within the same tube) by impurities, lattice imperfections and incommensurate potential of ajacent tubes. Not all the tubes are necessarily metallic, some of them could well be insulating. Experimental evidence of the internal structure of MWNT is usually not easy to obtain and leaves some room for speculation. The scenario when the dopants are situated outside the nanotube was considered in Ref. 14. In this case the doping electrons reside in the outermost shell only to minimize their electrostatic energy, and the screening of the interaction due to inner shells may be neglected.

Here we study the case when the dopant distribution results in a finite number M of conducting shells. To simplify the problem we consider an approximation in which all M shells have the same band structure and the same doping level. In Sec. III A we obtain the first-order pertubative correction to the tunneling DOS (TDOS). This correction diverges at low energies. Therefore, the study TDOS at low energies and temperatures requires a nonperturbative approach. This regime is addressed in Sec. III B.

A. Perturbation theory

Unlike the case of the 2D conductors considered above, in the one-dimensional wires the contribution of plasmon frequencies $\omega \gg qv$, logarithmically exceeds that of the electron-hole interval $\omega < qv$. Therefore, the weight function can be written in a simple form [compare this with Eqs. (2) and (3)],

$$\mathcal{V}(\omega) = \frac{1}{\omega^2} \mathrm{Im} \sum_{q_m} \int_{-\infty}^{\infty} \frac{dq}{2\pi^2} U_{00}(\omega, q, q_m), \tag{30}$$

where the sum is taken over the quantized transverse momentum $q_m = m/R_0$, $m = 0, \pm 1, \pm 2, \ldots$ The function $U_{00}(\omega, q, q_m)$ should be understood as the 00 element of the matrix $U_{ij}(\omega, \mathbf{q})$ that represents the dynamically screened Coulomb interaction of an electron in shell i with an electron in shell j and satisfies the matrix equation

$$\hat{U}(\omega, \mathbf{q}) = \hat{V}(\mathbf{q}) + \hat{V}(\mathbf{q}) \hat{\Pi}(\omega, \mathbf{q}) \hat{U}(\omega, \mathbf{q}), \tag{31a}$$

where $\Pi_{ij}(\omega, q, q_m)$ is the polarization operator that, according to our assumptions, is proportional to the unit matrix and in the plasmon region $\omega \gg qv$ has the form

$$\Pi_{ij}(\omega, q, q_m) = \delta_{ij} \Pi = \delta_{ij} \nu_1 \frac{q^2 v_\parallel^2 + q_m^2 v_\perp^2}{\omega(\omega + i/\tau)}. \tag{31b}$$

Here we introduced the one-dimensional density of states $\nu_1 = \Sigma_n (\pi v_n)^{-1}$ and the average squares of the longitudinal, $v_\parallel^2$ and the transverse, $v_\perp^2$ electron velocities,

$$v_\parallel^2 = \frac{\sum_n v_n}{\sum_n v_n^{-1}}, \quad v_\perp^2 = \frac{\sum_n v_n^{-1} v_{\perp n}^2}{\sum_n v_n^{-1}}.$$

In Eq. (31a) $V_{ij}(q, m)$ denotes the bare Coulomb potential,

$$V_{ij}(q, q_m) = \frac{2e^2}{\pi} \int_0^\pi d\phi K_0(q R_{ij}) \cos m\phi. \tag{31c}$$

In this equation we introduced the notations $R_{ik}^2(\phi) = R_i^2 + R_k^2 - 2R_i R_k \cos\phi$, with R_i being the radii of concentric shells forming the MWNT ($i = 0$ is the external shell, $i = M - 1$ the innermost shell).

To proceed further with Eqs. (31) we assume that the radius of the jth shell is a linear function of its number $R_j = R_0(1 - j\xi)$. In the long-range limit $qR_0 \ll 1$, we obtain from Eq. (31c) for the bare interaction,

$$\frac{V_{ij}(q, q_m)}{e^2} = \begin{cases} \dfrac{1}{|m|} \left(\dfrac{1 - \xi \max(i,j)}{1 - \xi \min(i,j)} \right)^{|m|}, & m \neq 0 \\ \ln \dfrac{\beta}{(qR_0)^2} + 2\xi \min(i,j), & m = 0. \end{cases} \tag{32}$$

Here $\beta = 4e^{-2\mathcal{C}} \simeq 1.26$, with $\mathcal{C}$ being the Euler constant. While the second line in Eq. (32) holds provided $\xi \ll 1$, the first line utilizes only the approximation $qR_0 \ll 1$.

We use the set of eigenvectors $w_i^{(k)}(q)$ and eigenvalues $V_k(q, m)$ of the bare interaction matrix (31c) to write the screened Coulomb interaction in the form

$$U_{ij}(\omega, q, q_m) = \sum_k \frac{w_i^{(k)}(m) w_j^{(k)}(m)}{V_k^{-1}(q, q_m) - \Pi(\omega, q, q_m)}. \tag{33}$$

The eigenvalues $V_k(q, q_m)$ determine the spectrum of collective plasmon excitations $\omega_m^{(k)}(q)$ through the poles of the interaction $U_{ij}(\omega, q, q_m)$ in Eq. (33), and the eigenvectors $w_i^{(k)}(m)$ determine the distribution of charge between different shells in these plasmon oscillations.

Typically, MWNT's exhibit ballistic transport around the circumference $l \equiv v\tau \sim 10$–100 nm$> R$. The plasmon modes with $m \neq 0$ have finite gaps, ranging between $v\sqrt{\kappa\xi/R}$ and $v\sqrt{M\kappa/R}$, where κ is the inverse 2D Debye screening length for one layer. These modes do not contribute to the spectral density (30) at frequencies ω below the corresponding gaps since their contribution to the imaginary part of the screened interaction (33) vanishes. Therefore, the contribution of the $m \neq 0$ modes to the TDOS correction (1) is energy-independent at low energies. The $m=0$ plasmons, on the other hand, are gapless. Their contribution to Eq. (30) continues to depend on frequency and leads to a singular correction to TDOS at $\epsilon \to 0$. Therefore, to study the energy dependence of TDOS at $\epsilon \to 0$ we can neglect the nonsingular contribution of the $m \neq 0$ plasmons and retain only the $m=0$ term in Eq. (30).

The index k labels the plasmon modes and is equal to the number of nodes in the charge distribution in the kth plasmon across the section of a multishell nanotube. The mode $k=0$ is characterized by the uniform distribution of the oscillating charge and corresponds to the logarithmic eigenvalue, $V_0(q) \simeq e^2 M \ln(\beta/(qR_0)^2)$, with $w_i^{(0)} \simeq 1/\sqrt{M}$. All other $M-1$ modes correspond to q-independent eigenvalues of the bare interaction, $V_k = 2\xi e^2 \alpha_k$ and, therefore, have soundlike spectrum. The coefficients α_k are to be computed numerically. They range roughly from a few tenths to a few units, e.g., for $M=10$ we obtain $\alpha_k = 10.0; 2.6; 1.2; 0.72; 0.50; 0.38; 0.31; 0.28; 0.26$. The components $w_0^{(k)}$ of the eigenvectors are also computed numerically. As the eigenvalues of the matrix (33) depend (at most) only logarithmically on the momentum q, we can perform the integral in Eq. (30) taking these eigenvalues at characteristic plasmon momenta, $q^2 \sim \omega(\omega + i/\tau)/NMv^2$,

$$\mathcal{V}(\omega) = -\sqrt{\frac{g}{N}} f_M(\omega) \mathrm{Re} \frac{\sqrt{\omega + i/\tau}}{\omega^{3/2}}, \tag{34}$$

where we defined the average Fermi velocity $\bar{v} = \Sigma_n v_n / N$ and the dimensionless coupling strength $g = e^2/(2\pi\bar{v})$ in a single channel. The function $f_M(\epsilon)$ is given by the 00-element of the matrix $\hat{V}^{1/2}$,

$$f_M(\omega) = \sum_k (w_0^{(k)})^2 \sqrt{V_k/2e^2} = \frac{1}{\sqrt{M}} \ln^{1/2} \frac{\sqrt{NM}\bar{v}}{R\sqrt{|\omega(\omega + i/\tau^{-1})|}} + \gamma_M \sqrt{\xi}, \tag{35}$$

the last term representing the contribution of the $M-1$ soundlike plasmons, with values of γ_M given in the table.

M	1	2	3	5	10
γ_M	0	0.35	0.60	0.84	1.38

The contribution of the $k=0$ plasmon decreases with the number of tubes M due to the screening by internal shells, while the contribution of soundlike plasmons increases roughly linearly with M. Since the number of shells may not exceed $1/\xi$ the second term in Eq. (35) is never greater than the first one, and at most becomes comparable to it at $M \approx 1/\xi$. The approximation $\omega > qv$ used above assumes that the case of strong interaction is realized, $\xi\alpha_k > 1/4e^2\nu_1 \sim 1/4N$. This is equivalent to the condition that the two-dimensional Debye screening length for one layer be shorter than the spacing between the shells, $\kappa R\xi > 1$.

Substituting the expression (34) into Eq. (30) and evaluating the frequency integral we obtain the following perturbation correction to the tunneling DOS of an M-shell multiwall nanotube,

$$\frac{\delta\nu_M(\epsilon)}{\nu_1} = -\sqrt{\frac{g}{N}} \begin{cases} \left(\frac{\sqrt{2}}{\sqrt{|\epsilon|\tau}} f_M(|\epsilon|) + f_M(\tau^{-1}) \ln \frac{\sqrt{NM}\bar{v}\tau}{R} \right), & |\epsilon| < 1/\tau \\ f_M(\epsilon) \ln \frac{\sqrt{NM}\bar{v}}{|\epsilon| R}, & |\epsilon| > 1/\tau. \end{cases} \tag{36}$$

If a metallic gate is present at a distance $\mathcal{D} \gg R$ from the nanotube the long-range Coulomb interaction is screened at long wavelengths,[4] and the first term in the function $f_M(|\epsilon|)$, Eq. (35), should be replaced by $1/\sqrt{M} \ln^{1/2} \mathcal{D}/R$. Then Eq. (36) reproduces the lowest-order interaction correction (i) for a diffusive wire with a short-range interaction[1] at $\epsilon > 1/\tau$, and (ii) for a Luttinger liquid in the ballistic regime at $\epsilon > 1/\tau$. In the absence of the screening gate the correction (36) exhibits an additional $\ln^{1/2}|\epsilon|$ dependence on energy in comparison to the case of a finite-range interaction.

B. Tunneling DOS at low energies

We now discuss how the results obtained for multiwall nanotubes can be extended to the very vicinity of the Fermi surface where the perturbation correction (36) diverges and the nonperturbative approach is required. This question was addressed for a single-wall metallic nanotube in Ref. 14 that utilized the phase approximation first proposed by Nazarov[10] and further developed in Ref. 16. Here we concentrate on the dependence of the DOS on the number of shells M, skipping the derivation that could be found in Ref. 14.

At zero temperature the tunneling DOS can be expressed through the spectral density $\mathcal{V}(\omega)$ in the form[16]

$$\frac{\nu(\epsilon)}{\nu_1} = \mathrm{Re} \int_{-\infty}^{\infty} \frac{dt \sin|\epsilon| t}{\pi t} \exp\left\{ \int_0^{\infty} d\omega \mathcal{V}(\omega)(1 - e^{-i\omega t}) \right\}. \tag{37}$$

At low energies the t integration in Eq. (37) can be carried out in the saddle-point approximation. The saddle point lies on the imaginary axis at $-it^*$ and corresponds to the imaginary time that the particle spends under the Coulomb barrier in the approach of Ref. 11.

Using the spectral density (34) we obtain that in the ballistic regime the saddle point is given by $t^* \sim \sqrt{g/(NM)}/\epsilon < \tau$, and thus corresponds to energies $\epsilon > (\sqrt{NM}\tau)^{-1}$, and not to $\epsilon > 1/\tau$ as may be naively expected. This is related to the fact that the characteristic plasmon velocity is propor-

tional to the square root of the number of channels $\sqrt{NM}$. In the ballistic regime the tunneling density of states is given by the exponential of the perturbative correction (36),

$$\nu(\epsilon) \propto \exp\left\{ -\sqrt{\frac{g}{N}} \ln\frac{\bar{v}}{R|\epsilon|}\left(\sqrt{\frac{1}{M}\ln\frac{\bar{v}}{R|\epsilon|}} + \gamma_M\sqrt{\xi}\right)\right\}. \tag{38}$$

In the regime of strong diffusive suppression of DOS, $|\epsilon| < g/(NM\tau)$, the integral in Eq. (37) can again be evaluated in the saddle-point approximation. The saddle point is given by $t^* \sim g/(NM\epsilon^2\tau)$, and we obtain

$$\nu(\epsilon) \propto \exp\left\{ -\frac{\epsilon_0}{|\epsilon|}\left(\sqrt{\frac{1}{M}\ln\frac{\bar{v}}{R|\epsilon|}} + \gamma_M\sqrt{\xi}\right)^2\right\}, \tag{39}$$

where $\epsilon_0 = \pi g/2N\tau$. This result holds provided that the argument of the exponential is large.

At finite temperature the nonperturbative expression for the tunneling DOS can be obtained using Eq. (14) of Ref. 14. At low energies, $\epsilon^2/\epsilon_0 < T < \epsilon$, this equation gives

$$\nu(T) \propto \exp\left\{ -1.07\sqrt{\frac{\epsilon_0}{T}}\left(\sqrt{\frac{1}{M}\ln\frac{\bar{v}}{R\sqrt{T\epsilon_0}}} + \gamma_M\sqrt{\xi}\right)^2\right\}. \tag{40}$$

At higher energies, $\epsilon^2/\epsilon_0 > T$, the tunneling DOS in the diffusive regime, $\epsilon < g/(NM\tau)$, is given by the zero temperature result (39).

In the case when the Coulomb interaction is screened by a metal gate it is straighforward to show following Ref. 14 that in the crossover region between the high-energy, Eq. (39), and the low-energy, Eq. (40), regimes the tunneling DOS depends on energy only through the dimensionless variable $|\epsilon|/\sqrt{\epsilon_0 T}$, see Eqs. (16) and (17) in Ref. 14.

IV. DISCUSSION

We considered the zero-bias anomaly in the tunneling density of states in layered 2D and quasi-1D materials with dynamically screened Coulomb interaction. The theory presented above is applicable to high-T_c materials and semiconductor heterostructures as well as to the multiwall carbon nanotubes. We showed that the presence of many conducting shells in 2D systems weakens the singularity in the DOS correction at the Fermi level: for energies below $\kappa D/d$ in diffusive systems or below the plasmon gap $v(\kappa/d)^{1/2}$ in ballistic systems the DOS correction becomes logarithmic $\propto \ln|\epsilon|$, rather than double logarithmic $\propto \ln^2|\epsilon|$ as in the case of a single layer.

In addition to the logarithmic terms mentioned above that originate from the broad region of the transferred frequency and momentum there are also terms coming from the regions around the plasmon and particle-hole singularities. These contribute correction to DOS $\max(|\epsilon|,\epsilon^*)/4\epsilon_F$ that is not singular but dominant in the wide range of energies $|\epsilon| > \ln(\epsilon_F\tau)/\tau$. The contribution of the particle-hole continuum is especially interesting in 2D ballistic systems. It turns out that the contribution of the particle-hole continuum to the DOS correction, which corresponds to the region $\omega \leqslant qv$ in the interaction frequency/momentum transfer, of the same order as the contribution originating rom the plasmon pole but has a *different* sign. Namely, the particle-hole continuum *increases* the density of states. For the long-range Coulomb interaction the total effect is still a suppression of DOS for low energies as the plasmon contribution larger than that of the particle-hole continuum (twice as large for a single 2D layer). For a finite-range interaction and for a single 2D layer the contribution of the particle-hole continuum exactly cancels that of the collective (zero-sound mode) nullifying the linear in $|\epsilon|$ correction to TDOS in agreement with the Fermi liquid theory.

The validity condition for neglecting the intershell tunneling was found for the diffusive regime. It was shown that instead of the naively expected requirement of the tunneling time t^{-1} being greater than the characteristic measurement time ϵ^{-1}, the much more stringent condition $t^{-1} \gg \epsilon_F\tau/\epsilon \gg \epsilon^{-1}$ must be satisfied.

With respect to multiwall carbon nanotubes our analysis was concentrated on the dependence of DOS on the number M of coaxial shells for both the regimes of perturbative and strong suppression. We found that this dependence can be described by a single function γ_M found numerically. The energy and temperature dependence of DOS found in Ref. 14 for a nanotube with a single conducting shell is preserved for an arbitrary number of shells. When comparing our results with experiments it is important to bear in mind the the crossover between the ballistic (38) and the diffusive (39) behavior of the tunneling DOS in multichannel wires occurs not at $\epsilon \sim 1/\tau$ but at much lower energies $\epsilon \sim \sqrt{(g/NM)/\tau}$. This is related to the fact that the plasmon velocity in a multichannel wire is proportional to the square root of the total number of channels $\sim v\sqrt{NM}$.

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APPENDIX A: SOLUTION OF THE POISSON EQUATION

To solve the Poisson equation (5) we first derive the Green function for the infinite stack of layers,

$$\left(\frac{d^2}{dz^2} - q^2 + 4\pi e^2\Pi \sum_{n=-\infty}^{\infty} \delta(z-nd)\right) G(z,z') = \delta(z-z'), \tag{A1}$$

with the boundary condition $G(z,z') \to 0$ when $|z-z'| \to \infty$. The homogeneous solutions of Eq. (A1) are easily written in terms of quasiperiodic Bloch functions,

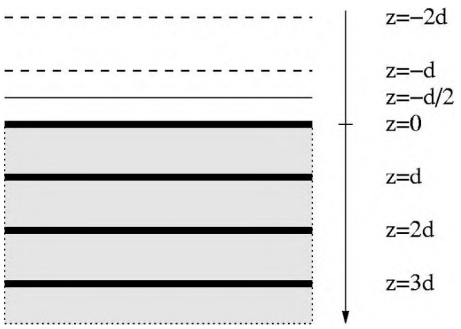

FIG. 3. The crystal consisting of a semi-infinite system of 2D metallic layers ($z=0,d,2d,\ldots$) is shown. The thin solid line at $z=-d/2$ hosts a fictitious (image) charge Q that accounts for the absent $z=-d,-2d,\ldots$ layers.

$$\psi_{\pm}(z)=e^{\pm knd}(\sinh[q(z-(n+1)d]-e^{\pm kd}\times\sinh[q(z-nd)]),\quad nd<z<(n+1)d \tag{A2}$$

with k given by the solution of the equation

$$\cosh kd=\cosh qd-\frac{2\pi e^2\Pi}{q}\sinh qd, \tag{A3}$$

having a positive real part, $\mathrm{Re}\,k\geqslant 0$. This choice means that the function $\psi_-(z)$ decreases and the function $\psi_+(z)$ increases with z increasing. The Green function can then be written in the conventional form

$$G(z,z')=-\frac{1}{2q\sinh qd\sinh kd}\begin{cases}\psi_-(z)\psi_+(z'), & z>z'\\ \psi_+(z)\psi_-(z'), & z<z'.\end{cases} \tag{A4}$$

To find the solution for the semi-infinite problem, Eq. (4), we impose a fictitious charge Q located at $z=-d/2$ (see Fig. 3). This charge has to be found from the condition that the total potential of the electronic charge $-e$ and the fictitious charge Q,

$$\phi(z)=4\pi[eG(z,0)-QG(z,-d/2)], \tag{A5}$$

contain only the exponent e^{qz} for the negative values of the coordinate, $-d/2<z<0$.

This condition ensures that the electric field in the outside region ($z<0$) decays exponentially with the distance from the outermost plane, with the fictitious charge, therefore, taking care of the absent ($z=-dn$, $n=1,2,\ldots$) planes. We obtain the following value of the fictitious charge:

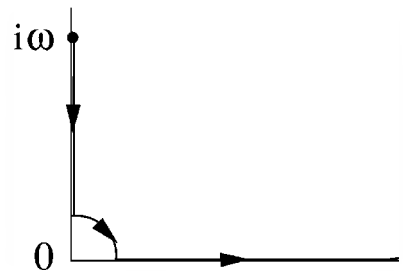

FIG. 4. The contour of integration in the complex plane z.

$$Q=e\frac{\psi_-(0)}{\psi_+(-d/2)}\frac{e^{kd}-e^{qd}}{e^{-kd}-e^{qd}}. \tag{A6}$$

Substituting Eq. (A6) into Eq. (A5) and making use of Eqs. (A2) and (A4), we obtain the final expression (6).

In Sec. II C the formula for interaction U_{nm} of two electrons residing at different layers $z=nd$ and $z=md$ in an infinite system is used. To obtain it one can write $U_{nm}=-4\pi e^2G(nd,md)$, which leads to the expression (26).

APPENDIX B: PARTICLE-HOLE CONTRIBUTION IN THE CLEAN LIMIT $1/\tau\to 0$

The particle-hole contribution to the spectral density (2) in the clean limit has the form [see Eq. (13)]

$$\mathcal{V}_2(\omega)=\frac{\omega}{2\pi^2\nu_0}\mathrm{Im}\int_0^{\infty}\frac{qdq}{(\omega+i\eta)^2-q^2v^2}\times\frac{1}{\sqrt{(\omega+i\eta)^2-q^2v^2}-\omega-i\eta}, \tag{B1}$$

where the lower limit could be extended to zero. Making the substitution $z=\sqrt{q^2v^2-(\omega+i\eta)^2}$ we write it in the form

$$\mathcal{V}_2(\omega)=-\frac{\omega}{2\pi^2\nu_0}\mathrm{Im}\int_{i\omega}^{\infty}\frac{dz}{z(iz-\omega)}. \tag{B2}$$

The integrand in this expression does not have singularities in the upper half plane. We can, therefore, deform the integration contour as shown in Fig. 4. The integral along the imaginary axis is real and does not contribute to $\mathcal{V}_2$. The integral around $z=0$ point and the integral along the real axis give $1/4\epsilon_F$ each. We, therefore, establish the first term in the expression (14).

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