Faster than Fourier: Ultra-Efficient Time-to-Frequency-Domain Conversions for FDTD Simulations

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Abstract

This tutorial compares several methods of converting from the time to the frequency domain for FDTD simulations. Applications include calculations of field or power distributions, antenna impedance, and radiation patterns. The traditional Fourier-transform methods are compared to two methods based on the solution of linear equations. This tutorial describes how to program and use these techniques, and evaluates their effectiveness for several applications, including analysis of a millimeter-resolution human model underneath a 60-Hz power line, antenna-radiation pattern and impedance calculations, the calculation of the coupling of a cellular telephone to the human head, and geophysical-prospecting simulations.

Keywords: FDTD methods; computation time; frequency domain analysis; time domain analysis; discrete Fourier transform; antenna radiation patterns

1. Introduction

Many applications of the Finite-Difference Time-Domain (FDTD) method require conversion of time-domain field data to frequency-domain data (magnitude and phase) over large regions of the model. Applications include bioelectromagnetic dosimetry calculations of the human body for analysis of cellular telephones [1-4], power lines [5, 6], and other EM-safety studies [7]; antenna impedance and radiation patterns [8]; radar cross section calculations [9]; and calculations of S parameters of microwave circuits [10]. Time-to-frequency-domain conversions have traditionally been done with either the fast Fourier transform (FFT) [11-12] or the discrete Fourier transform (DFT) [13-14]. When multiple frequencies are of interest, the FDTD method is commonly used with a pulsed excitation, and the Fourier-transform methods are used to obtain the desired results at these frequencies. More recently, methods based on the solution of linear equations have been found to be more efficient than the Fourier-transform methods [15].

This tutorial compares the traditional Fourier-transform methods and methods based on the solution of linear equations for time-to-frequency-domain conversions for FDTD simulations. Equations are given for computing memory and computational requirements for individual applications. In addition, the use of these methods for specific electromagnetic applications is discussed.

A brief outline of the computational aspects of the FDTD method is given in Section 2. Section 3 briefly describes the Fourier-transform methods and their computational requirements. It also describes methods that have been used to optimize the Fourier methods. Section 4 describes two methods based on linear equations that can be used in place of Fourier-transform methods. The first of these methods is the Two-Equations Two-Unknowns (2E2U) method that can be used for single-frequency FDTD simulations. The second of these methods is its extension to multiple frequencies, called the N-Equations and N-Unknowns (NENU) method. Section 5 gives examples of the types of calculations that are needed in realistic applications, and compares the computational requirements and relative advantages of each of the methods, with the hope of providing the user with guidance for choosing the optimal method for a specific application.

2. Computational Aspects of the FDTD Algorithm

The FDTD algorithm has been described in detail in the available literature [16, 17], so it will not be described here. The general forms of the FDTD equations for the three electric-field components (E_x, E_y, E_z) and three magnetic-field components (H_x, H_y, H_z) are shown below:

$$E_x(i,j,k) = E_x(i,j,k) + C_1\left[H_z(i,j,k) - H_z(i,j-1,k) + H_y(i,j,k-1) - H_y(i,j,k)\right],$$

$$H_x(i,j,k) = C_2 H_x(i,j,k) + C_3\left[E_z(i,j,k) - E_z(i,j+1,k) + E_y(i,j,k+1) - E_y(i,j,k)\right], \quad (1)$$

where the constants C_1, C_2, and C_3 depend on the electrical properties of the material at each point (i,j,k) in the model. The number of real multiplications required for computing this algorithm per time step is

Number of multiplications = $9N_{xyz}$,

where N_{xyz} is the number of FDTD cells = $N_x N_y N_z$.

For the purposes of this tutorial, additional computational overhead for initialization of the simulation, boundary conditions, and incidental calculations will be neglected. In practice, it is common for boundary conditions to take 10% to 30% of the total computational time, depending on the simulation size and boundary conditions used, so these computational costs are not necessarily negligible, in practice. They are neglected here because they have no effect on the time-to-frequency-domain conversions, which is the topic of this paper.

The storage requirements for the FDTD simulation (again, neglecting boundary conditions and incidental storage) are one real value for each of the six vector field components, and one for an integer indicating what dielectric material exists at each FDTD cell location:

RAM required for FDTD = $7N_{xyz}$
Disk required for FDTD = 0.

These values are shown in Table 1, which is used throughout this tutorial to summarize computational requirements.

Table 1. The computational requirements for time-to-frequency conversion methods.

	Multiplications or Divisions	Number of FDTD Simulations	Storage Locations
FDTD only	$9N_{FDTD}N_{xyz}$	1	$7N_{xyz}$
DFT**	$2N_{FDTD}N_{Pxyz}N_PN_F$	N_F (CW FDTD)	$2N_P$ N_F N_{Pxyz}
FFT** (Radix 2: N_{FDTD} must be 2^n)	$2N_{FDTD} \times \log_2\left(N_{FDTD}N_{Pxyz}N_P\right)$	1 (pulsed FDTD)	$2N_P$ N_{Pxyz} N_{FDTD}
2E2U (storing t_1)	$4N_PN_{Pxyz}$	N_F (CW FDTD)	N_PN_{Pxyz}
2E2U (no storage: use last two time steps)	$4N_PN_{Pxyz}$	N_F (CW FDTD)	0
NENU	$9N_{\Delta FDTD}N_{xyz}$ $+12N_{Pxyz}N_P\left(2N_F\right)^3$	1 (pulsed FDTD)	$\left(2N_F\right)^2$ $N_{Pxyz}N_P$

	Values used in Figure 1
N_{FDTD} = # of FDTD time steps	2000
N_{xyz} = # of FDTD cells	100 × 100 × 100
N_F = # of frequencies of interest	10
N_P = # of parameters of interest	6 (all **E** and all **H**)
N_{Pxyz} = # of FDTD cells of interest	100 × 100 × 100

$N_{\Delta FDTD}$ = additional FDTD time steps required for time-to-frequency-domain conversion (depends on simulation)

**Complex multiplications in the DFT and FFT are given the weight of approximately two real multiplications

3. Fourier-Transform Methods (DFT or FFT)

Fourier-transform methods are the traditional methods of converting from the time to the frequency domain for most discrete applications, including FDTD. They are very accurate, can be used with either single- or multiple-frequency simulations, and there are numerous commercial software packages available for making these calculations.

The discrete Fourier transform (DFT) [18] is based on a running summation of the time-domain field, as given below:

$$G(m\Delta f) = \Delta t \sum_{n=0}^{N-1} g(n\Delta t)\exp\left[\frac{-j2\pi mn}{N}\right] \quad (2)$$

for $m = 0,1,2,\ldots,N-1$,

where

$G(m\Delta f)$ = the complex value of the magnitude and phase of the equivalent steady-state sine wave at frequency $m\Delta f$;

$g(n\Delta t)$ = the time-domain value of the pulse at time $n\Delta t$;

N = length of the Fourier transform = $1/(\Delta f \Delta t)$;

Δf = the frequency resolution of the frequency-domain calculations;

m = the frequency index, $m = 0,1,2,\ldots,N-1$;

Δt = the sampling period of the DFT.

Normalization is normally required in order to obtain frequency-domain data equivalent to the magnitude and phase of a 1 V/m incident sine wave at each frequency of interest. Since commonly-

used pulse shapes do not have constant frequency responses, the final values must also be normalized by (divided by) the complex DFT of the incident pulse. This normalization step can be eliminated by using a step-function, which has a constant (unity) frequency spectrum [15, 33]. Numerical dispersion in the FDTD grid eliminates the high frequencies, so they do not cause aliasing errors or otherwise interfere with the solution. Since the normalization requires only a single Fourier transform that is reused for all points in the grid, the choice of pulse shape and the issue of normalization have negligible effects on the computational requirements of the simulation.

The computational requirements of the DFT are as follows:

Number of (complex) multiplications = $N_{FDTD}N_{Pxyz}N_PN_F$,

where

N_{FDTD} = Number of time steps in the FDTD simulation;

N_F = Number of frequencies of interest (number of DFT summations);

N_P = Number of parameters of interest (six vector field components, for instance);

N_{Pxyz} = Number of points of interest (such as all locations within the grid, for complete field distributions, a set of surfaces for a radiation pattern, or a few isolated points for impedance calculations).

The storage requirements of the DFT are as follows:

RAM required for DFT = $2N_{Pxyz}N_PN_F$;

Disk required for DFT = 0.

This assumes that the DFT summation is computed as a running sum inside the FDTD code, rather than storing all of the time-domain values for later processing with commercial software. In practice, disk storage and later computation with commercial software is only possible when a very limited number of time-to-frequency conversions are of interest (such as for impedance calculations). For problems requiring Fourier transforms for surfaces or volumes of points, the storage requirements (either RAM or disk) become prohibitively large.

The fast Fourier transform (FFT) was developed as an efficient method of computing the Fourier transform [18]. The complete time history of the values at all points of interest is stored, and the exponential components are computed iteratively. In the radix-2 algorithm–a commonly-used algorithm because of its high efficiency–the length (N) of the Fourier transform is 2^n. Most FDTD simulations are run for some arbitrary number of time steps, not necessarily 2^n. This is not a problem, however, as the FDTD pulsed data can be padded with zeros to create an array with length suitable for the radix-2 algorithm. However, even with the use of the radix-2 algorithm, the FFT has been shown to be computationally more expensive than the DFT for all FDTD simulations [19].

One of the limitations of the traditional FFT has been that it requires evenly-spaced data (which is not suitable for exponentially increasing time steps), and that it produces evenly-spaced frequency data (which makes it difficult to obtain the frequencies of interest exactly). This limitation has been partially solved by the use of the unequally-spaced FFT. [20] The storage and computational requirements are similar, however, so this method is not useful for FDTD simulations requiring a large number of time-to-frequency-domain calculations.

"Desampling" has been used to minimize the computational requirements of Fourier transform computations [14]. This minimizes the number of FDTD samples used to obtain the Fourier transform. Since FDTD calculations are over-sampled according to the Nyquist criterion [14, 19], not all of them are required for Fourier-transform calculations. If, for instance, only every tenth FDTD sample is chosen for computation of the Fourier transform, the computational requirements of the Fourier transform are divided by ten. This method still relies on the samples being evenly spaced, and it requires storage of the running sum, so the memory requirements are the same as for the DFT. Desampling rarely improves the efficiency of the Fourier transform by more than a factor of 10.

Fourier-transform methods are limited for use in low-frequency high-resolution simulations (such as analysis of a millimeter-resolution model of the human body under a 60 Hz power line), where the sampling resolution of the waveform is ultra-high. For single-frequency (CW) simulations, the Fourier-transform calculations must be made over a full half-cycle of the converged sine wave. This requires at least an additional half-cycle of FDTD calculations, which can be difficult or impossible for low-frequency calculations, and which increases the cumulative error inherent in the finite-difference calculations.

The computational cost of the radix-2 FFT scheme is typically

Number of (complex) multiplications = $(N_{FDTD})/2\log_2\left(N_{FDTD}N_{Pxyz}N_P\right)$

The storage requirements of the FFT are

RAM required for FFT = $2N_{Pxyz}N_PN_F$

Disk required for FFT = $N_{FDTD}N_{Pxyz}N_P$.

4. Linear-Equation Methods

As an alternative to Fourier-transform methods, this paper presents two methods that overcome many of their limitations, and have the added advantages of flexibility and programming simplicity. Both methods are based on the solution of linear equations. The first method, called the Two-Equation Two-Unknowns (2E2U) method–for use with single-frequency analysis–is significantly more efficient than Fourier-transform methods, and can be applied effectively for an extremely broad frequency range, from low-kHz to high-GHz and beyond. This method has the added advantage that for many applications, it can be applied with virtually no memory or computational requirements (beyond the FDTD requirements, themselves) [15].

The second method, called the N-Equation N-Unknown (NENU) method, is an extension of the first method, and can be

used for multi-frequency analysis. In theory, this method minimizes computational and memory requirements for any simulation. However, computer round-off errors somewhat limit its application. Trade-off curves are presented that show that this multi-frequency extension is the most efficient method for up to about 40 frequencies, and that the single-frequency method is preferable for larger numbers of frequencies [21].

These two methods are presented below.

4.1 Two Equations-Two Unknowns Method

The Two-Equations Two-Unknowns (2E2U) method is a simple, direct method to obtain the magnitude and phase of a sine wave in the time domain. It is based on writing two equations in two unknowns (the magnitude and phase) for the time-domain fields, and then solving them directly for the magnitude and phase. At a given location in space, we can write

$$\begin{aligned} A\sin(\omega t_1+\theta) &= q_1, \\ A\sin(\omega t_2+\theta) &= q_2, \end{aligned} \tag{3}$$

where A is the magnitude, θ is the phase angle, and ω ($= 2\pi F$) is the angular frequency. At two time steps, t_1 and t_2, the values q_1 and q_2 are obtained from the FDTD simulation. These equations can be solved for the unknowns, A and θ, to give direct relationships for these values:

$$\begin{aligned} \theta &= \tan^{-1}\left[\frac{q_2\sin(\omega t_1)-q_1\sin(\omega t_2)}{q_1\cos(\omega t_2)-q_2\cos(\omega t_1)}\right], \\ A &= \left|\frac{q_1}{\sin(\omega t_1+\theta)}\right|. \end{aligned} \tag{4}$$

The choice of t_1 and t_2 depends on the simulation. For most FDTD simulations, the spatial resolution, Δx, is on the order of $\lambda/10$ to $\lambda/100$. For these simulations, t_1 and t_2 can be the last two time steps of the simulation. For higher-resolution simulations, the time resolution is also high ($\Delta t = \Delta x/2c$), so q_1 and q_2 are nearly equal if t_1 and t_2 are very close. This results in errors due to numerical round-off when calculating A and θ. For these simulations, it is better to choose t_1 to be a few time steps (say, 50) before the end of the simulation, and t_2 to be the final time step, such as was done in [6].

The equations in Equation (4) can be programmed one of two ways, depending on t_1 and t_2. The first is to store (or output to disk) the value of q_1 at time step t_1; then, when the final time step, t_2, is reached, the values of A and θ can be calculated. This is necessary if t_1 and t_2 are not subsequent time steps. An alternate method of eliminating the memory requirement can be used when t_1 and t_2 are taken to be the last two time steps. For the final time step, q_1 is stored in a single location (not an array). Then, q_2 is calculated from the last time step of the FDTD algorithm. This gives A and θ, which can be output to disk or stored in the same locations as the fields used to compute them (remember, the FDTD is now finished). This is then repeated for each location.

The 2E2U method provides accurate magnitude and phase calculations for simulations with clean, sine-wave output. Noise and dc offsets will cause errors. Ramped-sine excitations known not to cause a dc offset should be used [22], or a pulse with a very smooth turn-on [23] should be used. Ramped-sine excitations have also been observed to reduce or eliminate numerical noise in FDTD simulations [24].

The computational requirements for this method are

Number of multiplications = $4N_P N_{P_{xyz}}$.

The memory requirements for this method are

RAM required for 2E2U (storing t_1 time steps) = $N_P N_{P_{xyz}}$

RAM required for 2E2U (using last time steps) = 0

Disk required for 2E2U = 0.

4.2 *N*-Equations *N*-Unknowns

The Two-Equations Two-Unknowns method can be extended to multiple frequencies. In this case, N equations are solved for N unknowns (NENU), which are the amplitude and phase at each frequency of interest. This requires samples at $2N$ time steps, and results in the following equations, for two frequencies:

$$\begin{aligned} A_1\sin(\omega_1 t_1+\theta_1)+A_2\sin(\omega_2 t_1+\theta_2) &= q_1, \\ A_1\sin(\omega_1 t_2+\theta_1)+A_2\sin(\omega_2 t_2+\theta_2) &= q_2, \\ A_1\sin(\omega_1 t_3+\theta_1)+A_2\sin(\omega_2 t_3+\theta_2) &= q_3, \\ A_1\sin(\omega_1 t4+\theta_1)+A_2\sin(\omega_2 t_4+\theta_2) &= q_4. \end{aligned} \tag{5}$$

This can be extended to multiple frequencies where the source is a sum of sine waves:

$$q_i = \sum_{n=1}^{N} A_n \sin(\omega_n t_i+\theta_n),\ i=1,2,3,\ldots,2N, \tag{6}$$

$$\begin{bmatrix} \sin(\omega_1 t_1) & \cos(\omega_1 t_1) & \sin(\omega_2 t_1) & \cos(\omega_2 t_1) & \cdots & \sin(\omega_N t_1) & \cos(\omega_N t_1) \\ \sin(\omega_1 t_2) & \cos(\omega_1 t_2) & \sin(\omega_2 t_2) & \cos(\omega_2 t_2) & \cdots & \sin(\omega_N t_2) & \cos(\omega_N t_2) \\ \vdots & \vdots & \vdots & \vdots & & \cdots & \cdots \\ \sin(\omega_1 t_N) & \cos(\omega_1 t_N) & \sin(\omega_2 t_N) & \cos(\omega_2 t_N) & \cdots & \sin(\omega_N t_N) & \cos(\omega_N t_N) \end{bmatrix} \times \begin{bmatrix} A_1\cos(\theta_1) \\ A_1\sin(\theta_1) \\ A_2\cos(\theta_2) \\ A_2\sin(\theta_2) \\ \vdots \\ A_N\cos(\theta_N) \\ A_N\sin(\theta_N) \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{bmatrix}$$

Using trigonometric identities on the sine function, this can be broken into a matrix equation that can be solved for functions $\left[A_n \cos(\theta_n)\right]$ of A_n and θ_n. A standard matrix-solution method, such as Gaussian elimination, is used to obtain the vector of functions $A_n \sin$ or $\cos(\theta_n)$. The unknowns A_n and θ_n are then found from these functions.

This form of the NENU method requires a multi-frequency source in the form of Equation (6), which does not utilize methods such as sine-wave ramping to prevent high-frequency transients or dc offsets. These specialized ramps could be included in the source type, and the same solution method could be followed by changing the specifics of the matrix above.

In theory, the NENU method provides an exact conversion from the time to the frequency domain. In practice, however, the matrix can be ill-conditioned, because of computer round-off error. This happens when the cosine and sine samples become very close together, so that they are numerically indistinguishable when the time samples (t_1, t_2, t_3, etc.) are too close together; when a very large number of frequencies are involved; or when the frequencies are too close together. There is a also a problem when the relative magnitudes of the source are several orders of magnitude different (which can be prevented simply by scaling magnitudes *after* time-to-frequency-domain calculations). This paper discusses each of these problems and the efficiency tradeoffs for solving them.

As an example of the application of the NENU method, solutions were computed for twenty-five different frequencies as a function of the spacing of the time samples ($t_2 = t_1 + n\Delta t$). The frequencies were evenly spaced from 0.1 to 1 MHz, and had equal magnitudes. The time resolution was $\Delta t = \Delta x/2c$, where Δx was the spatial resolution of the FDTD grid, and $\Delta x = \lambda_{\min}/20$. Figure 1 shows the inverse of the condition number. A large condition number (a small inverse) indicates a poorly-conditioned matrix. The spikes seen in the inverse condition number indicate sample spacings providing most-accurate matrix solutions. These are clearly sporadic. In particular, note that simply taking time samples that are far apart does not ensure accurate matrix solution. Figures 2a and 2b show the maximum error in the computation of the amplitude, when using the simple $Ax = b$ form and Gaussian elimination, as a function of sample spacing. Errors of less than 1% were obtained when samples were spaced at least seven samples apart (Figure 2a), and low errors were obtained for all of the sample spacings that provide high inverse condition numbers in Figure 1. A difficulty with using sample spacings that are far apart is that additional FDTD time steps must be run, beyond convergence of the solution. Improvements can be made. The Singular Value Decomposition (SVD) is commonly used to solve ill-conditioned matrices by removing or reducing the near-zero eigenvalues. Figure 2c shows the maximum error in the computation of the magnitude using the Singular Value Decomposition and the related pseudo-inverse to solve the matrix equation. Using SVD provided accurate calculations (less than 1% error) for all time-sample spacings. This enabled calculations of magnitude and phase of twenty-five frequencies using the last fifty converged time steps of the FDTD simulation.

The accuracy advantage of using the SVD becomes more pronounced as the number of frequencies increases. For instance, it was found that for 100 frequencies, evenly spaced from 0.1 to 1 MHz, the SVD could provide calculations with less than 1% error for sample spacings greater than four, whereas the direct

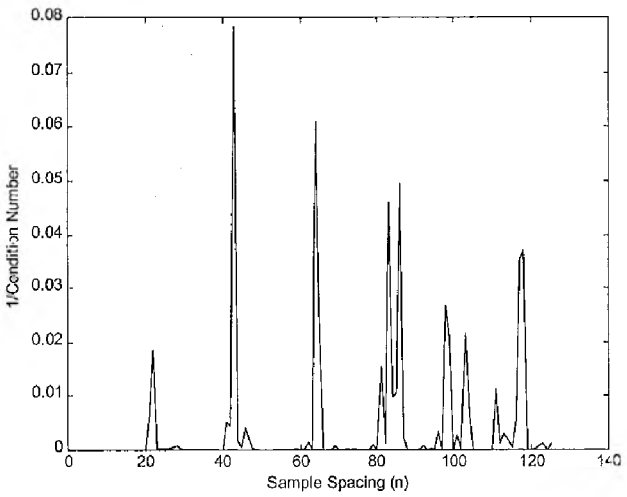

Figure 1. The inverse of the condition number of the *N*-Equation *N*-Unknown matrices as a function of the sample spacing (n) for 25 frequencies evenly spaced from 0.1 to 1 MHz. Large values indicate small condition numbers and ill-conditioned matrices.

method, without SVD, required a spacing of at least twenty-one samples.

The computational requirements for the *N*-Equation *N*-Unknown method, using Gaussian elimination, are [25]

Number of multiplications =

$$9N_{\Delta FDTD}N_{xyz} + N_{Pxyz}N_P(2N_F)^3/3.$$

The computational requirements for the N-Equation N-Unknown method, using SVD, are [26]

$$\text{Number of multiplications} = 9N_{\Delta FDTD}N_{xyz} + 12N_{Pxyz}N_P N_F^3.$$

The memory requirements for NENU are

$$\text{RAM required for NENU} = (2N_F)^2 N_{Pxyz}N_P$$

Disk required for NENU = 0.

It is clear that there is a substantial tradeoff between accuracy and efficiency in the NENU method using Gaussian elimination or SVD. This will be examined below for specific applications.

5. Applications

This section outlines several applications of time-to-frequency-domain calculations and the computational aspects surrounding them. Four specific applications will be considered. These are the computation of antenna impedance; radiation pattern; absorbed power or field distributions, such as for cellular-telephone analysis; and low-frequency high-resolution simulations, such as analysis of a millimeter-resolution model of the human body in the presence of a 60 Hz field.

The computational requirements for the different methods of converting from time to frequency domains are compared in

Table 1. Their relative efficiency depends on the number of frequencies of interest, and the number of parameters and locations where the conversions must be made. This is application dependent. Figure 3 shows the effect of frequency for an application where a large number of time-to-frequency-domain conversions are required (such as analysis of cellular telephones). For a small number of frequencies (one or two), the 2E2U method is the most efficient. For a larger number of frequencies (up to about 37), the NENU method, using Gaussian elimination, is most efficient. For more frequencies than that, the DFT is more efficient, because it does not require additional FDTD time steps. This figure also emphasizes the importance of choosing the most efficient method for the application because of the extremely large computational burden of computing magnitude and phase as compared to simply running the FDTD simulation.

The relative efficiency of the methods depends strongly on the application. Table 2 summarizes the computation and memory requirements of the various methods for the applications discussed below.

5.1 Impedance Calculations

Impedance calculations require relatively few time-to-frequency-domain calculations. Impedance is generally computed as $Z = V/I$, where Z, V, and I are complex values. The voltage is found from the line integral of the electric field (requiring one electric-field component), and the current is found from the closed contour integral of the magnetic field (requiring four magnetic-field components) around the electric field that is used to find the voltage [1,8]. Thus, only five time-to-frequency-domain conversions may be required to compute impedance. Table 2 shows the relative computation and memory requirements for an example that has the following parameters:

Number of frequencies $N_F = 1$ and 25

Number of parameters $N_P = 5$

Number of FDTD cells $N_{xyz} = 100 \times 100 \times 100$

Number of FDTD time steps $N_{FDTD} = 2000$
(typical for low-Q simulations)

From Table 2, it is clear that the 2E2U and NENU methods are more efficient than the DFT for single-frequency calculations. For

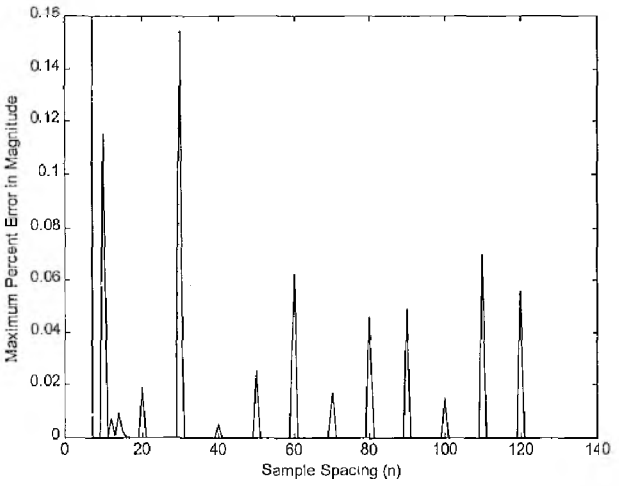

Figure 2a. The error in the calculation of the magnitude using Gaussian elimination, for twenty-five frequencies evenly spaced from 0.1 to 1 MHz, where the inverse of the condition number is shown in Figure 1. Errors of less than 1% are obtained for sample spacings greater than seven. Errors for sample spacings less than seven (shown in Figure 2b) are 5% - 105%, and are generally unusable. This means that to compute the magnitudes for twenty-five frequencies, the last (7)(2)(25) time steps of the simulation would be required, or an additional 350 time steps after convergence.

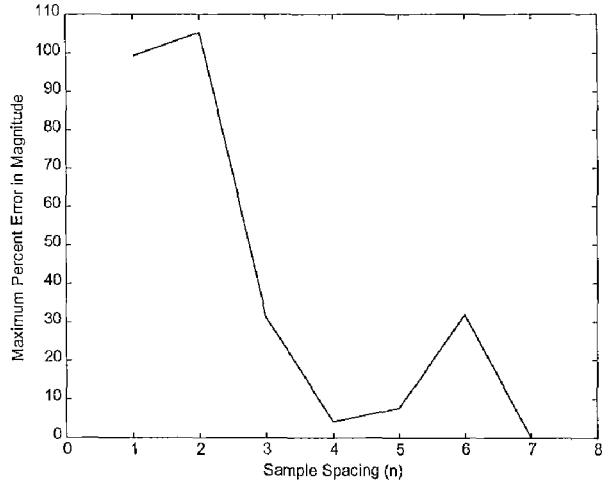

Figure 2b. The error in the calculation of the magnitude using Gaussian elimination for sample spacings less than seven (see Figure 2a).

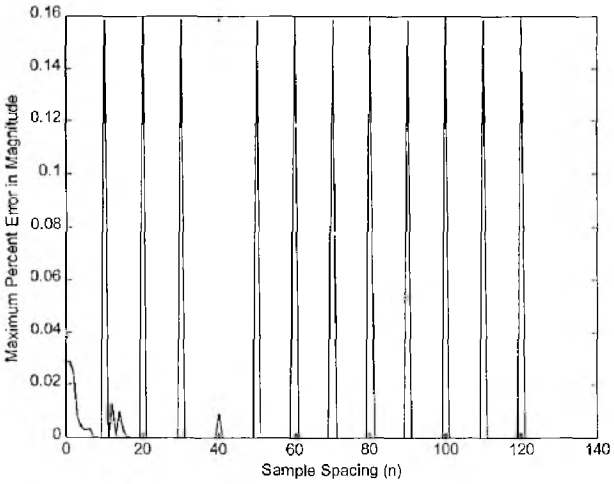

Figure 2c. The error in the calculation of the magnitude using Singular Value Decomposition (SVD) for twenty-five frequencies evenly spaced from 0.1 to 1 MHz. Errors of less than 1% are obtained for all sample spacings. The magnitude and phase can be computed, for all twenty-five frequencies, from the last 50 time steps of the simulation.

Table 2. The computational requirements for several different classes of simulations. For all cases, the FDTD space is 100 × 100 × 100, and the simulation is run for 2000 time steps. Comparisons are made between simulations with one frequency ($N_F = 1$) and twenty-five frequencies ($N_F = 25$). Values shown for time-to-frequency-domain methods do *not* include FDTD time steps.

Multiplications Required	Impedance $N_P = 5$ $N_{Pxyz} = 1$		Radiation Pattern $N_P = 4 \times 6$ $N_{Pxyz} = 90 \times 90$		Field Distribution $N_P = 3$ $N_{Pxyz} = 100 \times 100 \times 100$	
	$N_F = 1$	$N_F = 25$	$N_F = 1$	$N_F = 25$	$N_F = 1$	$N_F = 25$
FDTD only	1.8×10^{10}	1.8×10^{10}	1.8×10^{10}	1.8×10^{10}	1.8×10^{10}	1.8×10^{10}
DFT*	2.0×10^{4}	5.0×10^{5}	7.8×10^{8}	1.9×10^{10}	1.2×10^{10}	3.0×10^{11}
2E2U**	20	500	7.8×10^{5}	1.9×10^{7}	1.2×10^{5}	3.0×10^{8}
NENU (Gaussian Elimination)***	20	2.0×10^{5}	1.8×10^{7}	8.1×10^{9}	8.0×10^{6}	1.2×10^{11}
NENU (SVD)***	800	1.4×10^{7}	5.2×10^{7}	5.3×10^{11}	5.3×10^{8}	8.2×10^{12}

*If a pulsed FDTD is used, no additional FDTD time steps are required after convergence.

**Requires a separate FDTD simulation for each frequency. This will negate the efficiency for higher numbers of frequencies.

***Requires $2N_F$ FDTD time steps past convergence.

multiple frequencies, the DFT or NENU should be used, as the expense of additional FDTD simulations would negate the efficiency of the 2E2U method. For an application such as impedance calculations, where few field components require time-to-frequency-domain conversions, all of the methods are relatively inexpensive compared to the FDTD simulation itself. Storage is minimal for all of these methods for this application. Even the FFT with commercial software could be used for this application.

5.2 Radiation-Pattern Calculations

The calculation of antenna radiation patterns requires a moderate number of time-to-frequency-domain conversions. Equivalent surface currents are found by integrating the electric and magnetic fields tangential to a cubical surface surrounding the radiator, and transformed to the far field in either the time or frequency domains [16]. If they are converted to the frequency domain and then transformed to the far field (following the method of Van Bladel [8, 27]), four tangential electric- and magnetic-field components are required on each of six surfaces surrounding the antenna. For an example where the total FDTD space is 100 × 100 × 100 cells, the radiation pattern may be taken five cells inside the boundary, so each of the six surfaces is 90 × 90 cells. This gives the number of parameters NP = (4 field components) × (6 surfaces) × (90 × 90 cells) = 194400. Table 2 shows the relative computational requirements for this example:

Number of frequencies $N_F = 1$ and 25

Number of Parameters $N_P = 194\,400$

Number of FDTD cells $N_{xyz} = 100 \times 100 \times 100$

Number of time steps $N_{FDTD} = 2000$ (typical low-Q example)

For this example, which has a moderate number of time-to-frequency-domain conversions (two-dimensional surfaces rather than three-dimensional volumes), the 2E2U method would again be most efficient for single-frequency calculations. The NENU and DFT methods would be most efficient for multiple-frequency calculations. The 2E2U and NENU methods would require less storage than the DFT method, although this storage is still small compared with the overall FDTD simulation requirements.

5. 3 Field or Power Distribution

Field, power, or current-density distributions are commonly calculated with the FDTD method for analysis of safety guidelines, or for showing color plots of these distributions. Commonly, they are done for every point in a three-dimensional grid and, therefore, require a very large number of time-to-frequency-domain conversions. Examples include bioelectromagnetics and medical imaging and inversion problems [1-4, 28], and geophysical applications [29]. In order to find the complete electric-field distribution, for instance, this would require time-to-frequency-domain conversions for three field components for every cell in the simulation. The relative computational requirements of the various methods are given in Table 5, using the values given below:

Number of frequencies $N_F = 1$ and 25

Number of Parameters $N_P = (3 \text{ fields})\,(N_{xyz} \text{ cells}) = 3000000$

Number of FDTD cells $N_{xyz} = 100 \times 100 \times 100$

Number of time steps $N_{FDTD} = 2000$ (typical low-Q example)

From Table 2, the significance of the savings that can be obtained using the 2E2U or NENU methods becomes more apparent for this application. The DFT requires about as much computational time and as much storage as the FDTD simulation itself. This means that half of the system resources are being dedicated to the time-to-frequency-domain transformation. The 2E2U method provides a significant improvement for single-frequency simulations: five orders of magnitude less computation, and virtually no storage

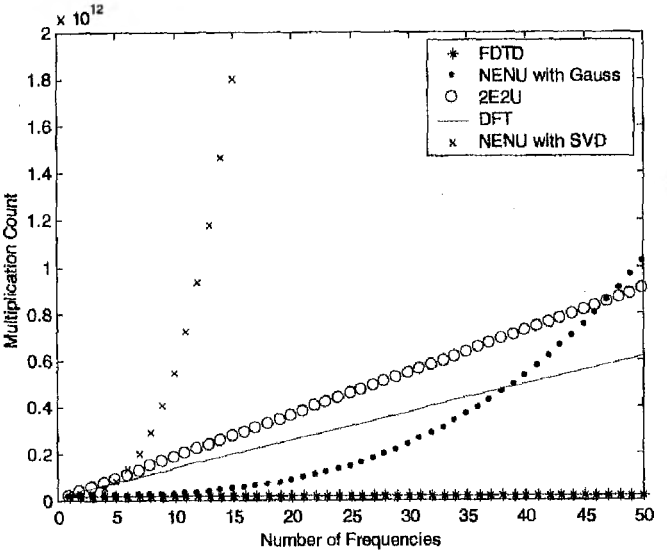

Figure 3. The computational requirements of the FDTD algorithm and associated time-to-frequency-domain conversions for the parameter values indicated in association with Table 1. This figure includes all FDTD simulation time steps necessary for each method.

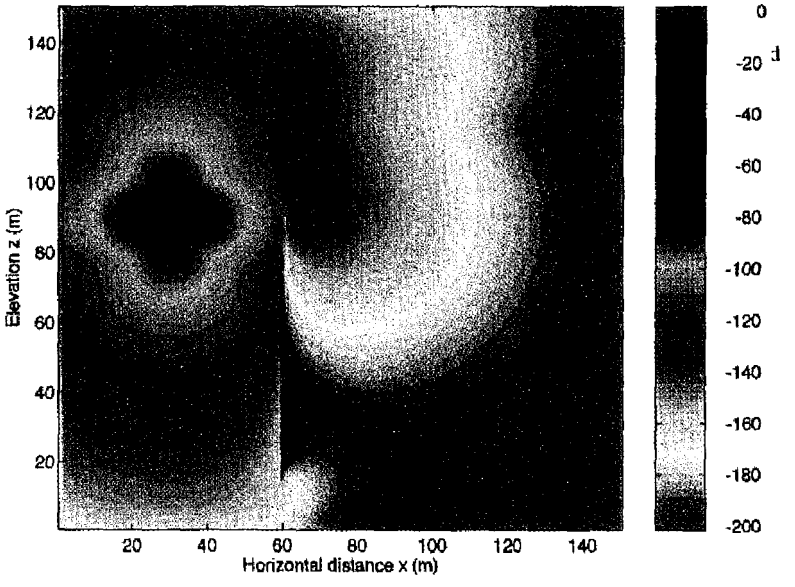

Figure 4. The horizontal magnetic-field amplitudes for a perfectly conducting slab (shown as the vertical line), illuminated by a small loop (the star-shaped element in the upper left) at 2 MHz. This application seeks the optimal receiver location to the right of the slab to delineate the slab's location and size from a source on the left. The values are expressed in decibels relative to the maximum value. The minimum has been clipped at –500 dB (from [32]).

requirement. This frees up system resources for larger problem sizes, rather than larger post-processing applications. The NENU method is slightly more efficient than the DFT for multiple frequency simulations, up to about 35 frequencies. This is an approximation, and would need to be analyzed for the specific array of frequencies of interest.

In a problem of this size, the 2E2U method would not necessarily be the most efficient method; however, it has potential application because of its ability to compute the frequency-domain fields with virtually no memory. Many realistic simulations are constrained more by memory than computer time.

5.4 Low-Frequency High-Resolution Simulations

Low-frequency high-resolution simulations present a peculiar problem for time-to-frequency-domain conversions that are solved using the 2E2U method. Until recently, the FDTD method was limited to applications where the simulation could be completed for at least one-half cycle of the lowest frequency in the wave. This precluded low-frequency analysis of high-resolution models. For instance, analyzing a 6 mm-resolution model of the human body at 60 Hz would require 1.7×10^9 time steps per cycle of the wave [6]. The use of pulsed FDTD with the Fourier transform and frequency scaling [5, 30] and the 2E2U method [6, 31, 32] have extended the range of this method to very low frequencies (down to 60 Hz). Both of these methods take advantage of the fact that the FDTD simulation actually converges in a moderate number of time steps (say 2000), and that the only difficulty is obtaining the magnitude and phase data from far less than a single cycle of the fields. This is done using the 2E2U method, and has been shown to be highly accurate when compared to analytical and measured data [6, 31, 32]. This has allowed direct calculation of the magnitude of 60 Hz fields within the human body, with a modeled resolution of 2 mm. Other methods simply cannot solve this problem, so the 2E2U method has been critical to the understanding of these low-frequency fields.

Figure 4 shows the application of the 2E2U method to geophysical simulations [32]. This simulation analyzes a typical nickel-sulfide deposit in the Kambalda formation in Australia. A perfectly conducting nickel-oxide slab (shown as the vertical red line) is illuminated by a small loop at 2 MHz (the star-shaped element in the upper left). The resolution of the simulation was 0.5 meter, and results were analyzed at 300 and 500 kHz, as well as at 2 MHz. The simulation converged in approximately 1000 time steps, and a half cycle of the wave was 2000 time steps at 300 kHz. Thus, the 2E2U method was used to obtain converged frequency-domain values without running a half cycle of the wave in the simulation. In the figure, values are expressed in decibels relative to the maximum value; the minimum has been clipped at –500 dB.

6. Conclusions and Summary

Figure 3 shows the relative cost of the Fourier-transform, 2E2U, and NENU methods, for an application requiring a large number of time-to-frequency-domain conversions. For a single frequency, the 2E2U method is most efficient. For up to about 40 frequencies, the NENU method is most efficient.

Memory is also a tradeoff for these methods. As noted in Table 1, the DFT must store a complex value (equal to two real values) for every location and parameter of interest. The 2E2U method can completely eliminate this requirement, if the sampling resolution is sufficient to allow computation of magnitude and phase from the final two time steps of the simulation. The NENU method must store a matrix that is $(2NF)^2$ in size, where NF is the number of frequencies for each location and parameter of interest. These values would generally be written to disk, with final solution being done as a post-processing step, but this may be prohibitively expensive for some applications. The optimal method to use depends on the size of the problem and the number of time-to-frequency-domain conversions required.

For applications with relatively few time-to-frequency-domain conversions, such as antenna-impedance calculations, all methods could be used. The 2E2U and NENU methods would be the most efficient. For applications with a moderate number of time-to-frequency-domain conversions, such as antenna radiation-pattern calculations, the 2E2U method would be most efficient for single frequency studies, and the DFT or NENU methods would be most efficient for multiple-frequency studies. For applications with a large number of time-to-frequency-domain conversions, such as the computation of complete field or power distributions, the efficiency of the methods is critical. The 2E2U method is clearly the most effective for single-frequency calculations, and the NENU method and DFT would be used for multiple-frequency simulations, depending on the number of frequencies being considered. The 2E2U method is also critical for use in FDTD simulations that are low frequency and high resolution (i.e., ultra-high resolution with respect to wavelength).

The time-to-frequency-domain conversion methods described in this paper give a good range of options to choose from for different applications of FDTD simulations. Choosing the optimal method can lead to huge advantages of code efficiency, and an increase in the overall problem size that can be simulated with given computer resources. The DFT, 2E2U, and NENU methods are all good choices for FDTD time-to-frequency-domain conversions.

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Introducing the Feature Article Author

Cynthia Furse is an Assistant Professor in the Department of Electrical and Computer Engineering at Utah State University (USU), and co-Director of the Center of Excellence in Smart Sensors. Her experience with millimeter-resolution bioelectromagnetic simulations of the human body exposed to cellular telephones, power-distribution lines, and implantable medical devices led to this and related studies of methods to reduce the computational requirements of FDTD simulations. Her present work incorporates computational electromagnetics to design, build, and test systems of imbedded or implanted antennas as sensors and/or communication devices for applications in medicine, geophysics, precision agriculture, and aging-aircraft maintenance.

Dr. Furse received her PhD, MS, and BSEE degrees from the University of Utah in 1994, 1988, and 1985, respectively. She was an NSF Computational Science and Engineering Research Fellow in 1993, and received IEEE Microwave Theory and Techniques Fellowships in 1991 and 1992. She is the author of over 70 scientific publications and conference proceedings. She enjoys teaching electromagnetics and microwave engineering, and was the Teacher of the Year 2000 for the USU College of Engineering.

Beacon Satellite Symposium 2001

The Beacon Satellite Symposium 2001 (BSS'01) will take place at Boston College on June 3-6 2001. This symposium will be held in conjunction with a two-day workshop on Space Weather Effects on Communication and Navigation Signals, to be held on June 7-8.

The primary topics for the BSS'01 will include:

1. Ionospheric tomography algorithms and measurements
2. Ionospheric scintillation measurements and modeling, especially for L-band systems
3. Total Electron content measurements and modeling
4. Ionospheric effects on satellite-based navigation
5. Scintillation effects on satellite-based communication
6. GPS occultation studies
7. Ionospheric effects on GPS geodesy and time transfer using GPS
8. New ionospheric research opportunities using beacon satellites

The program committee will consider contributions for oral and poster presentations from all areas connected to beacon-satellite investigations. However, contributions from the primary topics are especially encouraged. For all organizational details, visit the Web site at www2.bc.edu/~dohertpd/beacon.htm. In case of difficulties with Web access, additional information can be requested by e-mail to Patricia.Doherty@bc.edu.

The deadline for the submission of abstracts is **February 28, 2001**. Abstracts may be submitted online through the Web site, or via e-mail or postal mail. If abstracts are submitted via e-mail or postal mail, copies *must* be sent to Dr. Reinhart Leitinger, Institut fuer Geophysik, Astrophysik und Meteorologie, Universitaet Graz, Universitaetsplatz 5, A-8010 Graz, Austria; Tel: +43 316 380 5257; Fax: +43 316 380 9825; E-mail: Reinhart.Leitinger@kfunigraz.ac.at; and to Ms. Patricia Doherty, Institute for Scientific Research, Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3862 USA; Tel: +1 (617) 552-8767; Fax: +1 (617) 552-2818; E-mail: Patricia.Doherty@bc.edu.

To aid in the planning for this event and to insure receipt of future mailings, all those interested are requested to answer the questionnaire in the Call for Abstracts section of the Web site, or to send an e-mail message to Patricia.Doherty@bc.edu with their name and e-mail address, and answers to the following questions: Do you plan to submit an abstract? Will you plan to stay in College housing? Will you bring a guest?

Ground-Penetrating Radar in Sediments

The conference "Ground Penetrating Radar (GPR) in Sediments: Applications and Interpretation" will be held at the Geological Society in London and University College, London, England, August 20-21, 2001. According to the organizers, ground-penetrating radar is seeing increasing application in the fields of sedimentology and geomorphology. This international conference is intended to be the first to bring together geologists, geomorphologists, geophysicists, and engineers with an interest in the application and interpretation of GPR in sediments and sedimentary rocks. Contributions, including case studies of sedimentary environments, sedimentary architecture, sandbody geometry, shallow subsurface stratigraphy, and engineering applications, are invited. Abstracts of not more than 350 words should be sent in the body of an e-mail (not as an attachment) to the conference co-Chairs by **April 30, 2001**. The full papers are due at the conference. Authors are requested to express a preference for poster or oral presentations. Poster presentations are encouraged, and there will be a half day devoted to poster presentations, to be followed by the conference dinner in an adjacent room. The conference aims to bring together interdisciplinary scientists from around the world. Refereed papers will be published. The co-Chairs are Dr. Charlie Bristow, Birbeck College; E-mail: c.bristow@ucl.ac.uk; and Dr. Harry Jol, University of Wisconsin-Eau Claire; E-mail: jolhm@uwec.edu.