Determination of the Superconductor-Insulator Phase Diagram for One-Dimensional Wires

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We establish the superconductor-insulator phase diagram for quasi-one-dimensional wires by measuring a large set of MoGe nanowires. This diagram is roughly consistent with the Chakravarty-Schmid-Bulgadaev phase boundary, namely, with the critical resistance being equal to $R_Q = h/4e^2$. Deviations from this boundary for a small fraction of the samples prompt us to suggest an alternative phase diagram, which matches the data exactly. Transport properties of wires in the superconducting phase are dominated by phase slips, whereas insulating nanowires exhibit a weak Coulomb blockade behavior.

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In quasi-one-dimensional superconductors it remains to be fully understood how the superconductivity in a wire is destroyed as its diameter is reduced. Years ago it was shown that for wires with micron size diameters the mechanism that weakens superconductivity at finite temperature T is thermally activated phase slips (TAPS), which break the superconducting phase coherence along the wire and result in a measurable resistance, R [1,2]. As temperature is reduced, thermal fluctuations freeze-out and the TAPS rate decreases following the Arrhenius law, until at $T = 0$ the TAPS rate becomes zero and the wire becomes a "true" superconductor ($R = 0$). However, in the ultrathin wires being fabricated today this simple picture is complicated by an additional phase breaking process due to quantum fluctuations [3–5]. As $T \to 0$ these quantum phase slips (QPS) remain active and the resistance of a wire remains finite, even possibly at $T = 0$. The thinner a wire is made the more readily phase slips are expected to occur in it and hence the QPS resistance is also expected to be higher [6–8].

While it would seem that ultrathin superconducting wires may loose the beneficial property of dissipationless electrical transport, the remarkable possibility exists to recover the truly superconducting state if QPS are suppressed. Recent experiments on a group of six wires with similar lengths, $L \sim 100$ nm, observed that as $T \to 0$ those wires whose normal state resistance, R_N, was less than some critical resistance, R_c, were superconducting, while wires with $R_N \gtrsim R_c$ were resistive, with increasing resistance as $T \to 0$ [9]. It was found that $R_c \approx R_Q = h/4e^2$, which is suggestive of a Chakravarty-Schmid-Bulgadaev (CSB) dissipative phase transition [10], in which QPS are inhibited by the interaction with a dissipative environment. This transition was recently theoretically generalized for thin wires [11–14]. Unfortunately, these early experiments could not provide a proof of the universality of the condition $R_c = R_Q$. Furthermore, it is not clear whether a real superconductor-insulator transition (SIT) occurs in ultrathin superconducting wires or merely a crossover from wires in which the QPS rate is too small to be of consequence and so appear superconducting to wires in which the QPS rate is so large they essentially drive the wire into the normal state. Distinguishing between these two possibilities is of critical importance not only to our understanding of the physics of quasi-one-dimensional superconducting wires but also to their applicability in miniaturized superconducting circuits [15]. Insight into phase slip phenomena is also important in other fields, such as Bose-Einstein condensed gases [16] and qubit design [17].

In this Letter, we present results obtained on a large collection of about 100 wires that can be understood in terms of a superconductor-insulator transition. The wires have been characterized by linear transport measurements as well as high-bias differential resistance measurements. The results allow us to sort most of the homogeneous samples into two categories, "superconducting" and "insulating", and to construct a phase diagram with a well-defined boundary for the SIT in thin wires. For short wires (i.e., $L < 200$ nm) the phase boundary is the same as in the CSB transition, suggesting the same physical mechanism. No indication of a crossover caused by a gradual increase of the QPS rate was found for these short wires. We also suggest an alternative representation of the diagram, which indicates that localization effects become important for longer wires, while short wires may be influenced by surface magnetic moments. This diagram will provide an experimental basis for the theory of thin superconducting wires, which is still being developed [5,11,13,14,18,19].

The phase to which a nanowire belongs is easily discerned by the transport properties of the wire. In Fig. 1 we show the $R(T)$ curves for some representative samples (sample fabrication details are given in Refs. [20,21]). Note that upon cooling all samples initially show a superconducting transition at the critical temperature of the thin film electrodes, $T_{c,\mathrm{film}}$, which are measured in series with the wire. For $T < T_{c,\mathrm{film}}$ the resistance of the electrodes is zero and we probe only the wire. The sample resistance just below $T_{c,\mathrm{film}}$ is assumed to be the wire's normal state resistance, R_N.

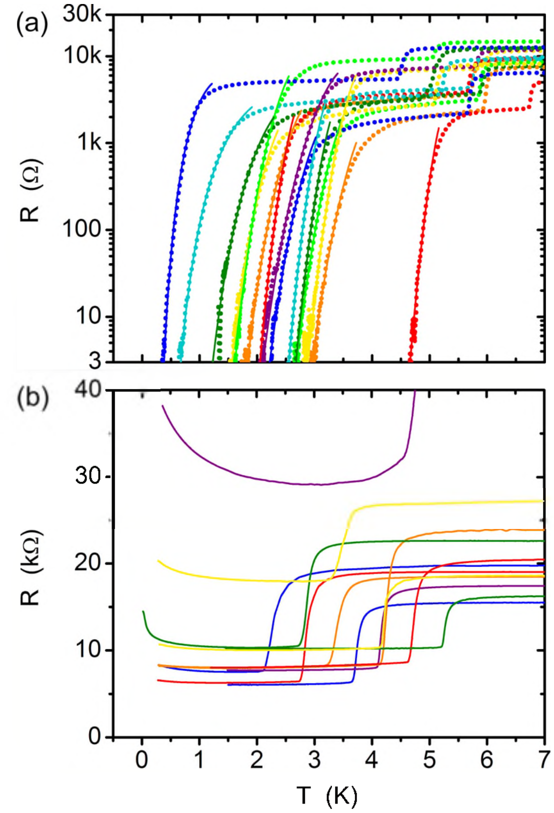

FIG. 1 (color online). (a) R vs T data (●) for wires in the superconducting phase with fits (lines) to the AL formula given in the text. (b) R vs T for wires in the insulating phase.

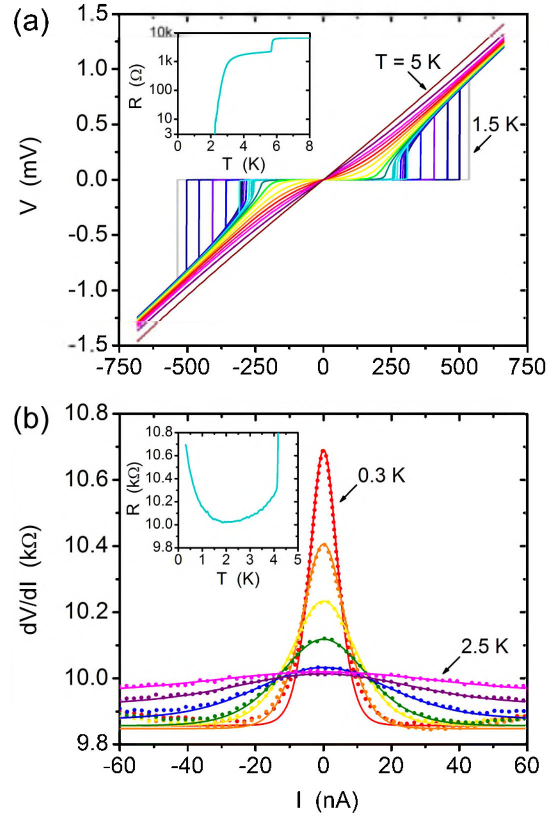

FIG. 2 (color online). (a) V vs I for a wire in the superconducting phase ($L = 61$ nm, $T_c = 3.3$ K) at $T = 1.5$, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.25, 3.5, 4.0, and 5.0 K. (Inset) R vs T for the same sample. (b) dV/dI vs I data (●) for a wire in the insulating phase ($L = 140$ nm) at $T =$ 0.3, 0.5, 0.75, 1.0, 1.5, 2.0, and 2.5 K with fits to Coulomb blockade theory (lines). (Inset) R vs T for the same sample.

Superconducting wires [Fig. 1(a)] show resistive transitions that follow an Arrhenius activation law and in this and only this sense are the wires "true superconductors". At finite temperatures TAPS are present, as evidenced by the semiphenomenological Arrhenius-Little (AL) fits [21,22], i.e., $R(T) = R_N \exp(-\Delta F(T)/k_B T)$, where $\Delta F(T)$ is the free energy barrier for phase slips. While QPS may be allowed in these wires as well for $T > 0$ [13,23], the essential point is that $R \to 0$ as $T \to 0$. We emphasize that for wires in the superconducting phase, even those near the SIT, no resistance "tails" were observed, such as those found in Refs. [4,6,7] that were attributed to a high rate of QPS. The $R(T)$ curves of wires in the insulating phase are qualitatively different, with resistance that increases upon cooling [Fig. 1(b)]. Reentrant behavior, i.e., resistance increasing with cooling and then suddenly dropping, was never observed.

Another dichotomy in transport properties is also found in the voltage vs current, $V(I)$, characteristics of the wires. In Fig. 2(a), $V(I)$ curves for a representative superconducting sample show the evolution of the $V(I)$ behavior of wires in this phase from linear for $T > T_c$, to nonlinear for $T_c > T \gtrsim T_c/2$, to hysteretic for $T \lesssim T_c/2$ with well-defined switching and retrapping currents. Insulating wires, on the other hand, display $V(I)$ characteristics that are nearly linear at all temperatures but with a zero-bias anomaly that is more pronounced in the differential resistance, $dV(I)/dI$, measurements [Fig. 2(b)]. Since a small zero-bias maximum can be observed even for those temperatures at which $R(T)$ is at its minimum, the $dV(I)/dI$ peak cannot be a result of Joule heating. Instead, as shown by the fits in Fig. 2(b), the zero-bias anomaly can be described by the theory of dynamical weak Coulomb blockade with the entire wire acting as a coherent scatterer, the same as for the $R(T)$ curves for these wires [20,24].

We now turn our attention to the main result of this Letter—the phase diagram of the SIT in quasi-one-dimensional nanowires. Based upon the transport measurements the wires fall into one of two distinct phases: superconducting or insulating [25]. In Fig. 3 we plot the phase to which a wire belongs on the coordinate plane (L, L/R_N). If the SIT in ultrathin wires is only caused by local physics then there should exist a critical cross sectional area, A_c, that separates insulating and superconducting wires. For example, as discussed in the introduction, one suggested way to understand the difference between these two regimes would be in terms of QPS. According to the microscopic theory [5], the QPS rate is

$\gamma_{QPS} \propto \exp(-aR_Q L/R_N \xi(0))$ where $\xi(0)$ is the zero-temperature coherence length and a is a numerical factor that is of order unity and depends on the actual dependence of the order parameter phase on time and space coordinates during the QPS process. If only this term is considered then a crossover model results, with the only difference between samples being that the QPS rate is lower (higher) in the superconducting (insulating) wires due to their larger (smaller) diameters. For the range of MoGe thicknesses sputtered, the resistivity does not change with thickness [27] and so the separatrix in this scenario should appear on Fig. 3 as a horizontal line with $L/R_N = \text{const}$. However, for our set of samples this is certainly not the phase boundary we observe. Rather, the CSB phase boundary, i.e., $R_N = R_Q$ provides a much better division of the data. The reason that the total wire resistance plays the role of a shunt is that the leads connected to the sample have a relatively low impedance, of the order 100 Ω, at the relevant high frequencies. Thus each phase slip in the wire is shunted by the rest of the wire and the leads, effectively connected in series with the wire itself. Since R_N is much larger than the impedance of the leads, R_N effectively acts as the only control parameter for the SIT phase boundary.

It is observed that three longer wires (~450 nm) behave as superconductors even though they have high normal resistance, i.e., $R_N > R_Q$. These deviations suggest that the SIT is only applicable to shorter wires, as predicted in the theory of Ref. [14] in which nanowire behavior is explained in terms of QPS-antiQPS dipoles. According to Ref. [14], at $T = 0$ the dipoles are bound in all wires with $R_N < R_Q$ due to dissipation; i.e., no free QPS are present, and so they will be in a truly superconducting state. For wires with $R_N > R_Q$, on the other hand, the dipoles are free and the wires characteristics are determined by the fugacity of QPS, $\zeta \propto \exp(-bL/R_N)$, where b is a constant. For shorter wires the fugacity may be large and so a proliferation of QPS will occur immediately once they are unbound when $R_N > R_Q$, driving an SIT. Longer wires, in contrast, would show a crossover with the value $R_N = R_Q$ not playing any special role. Within this framework our data for wires of various lengths is consistent with itself as well as previous work performed on long nanowires [6–8].

While the model of Ref. [14] captures most of the observations, we do point out that some short wires with R_N slightly lower than R_Q appear insulating. These deviations can be explained by assuming that our knowledge about the effective R_N is not precisely correct. The effective R_N might in fact go above the measured R_N as the temperature is reduced. A proximity effect can also be responsible for the imprecise knowledge of the effective R_N of the wire. However, these deviations may point to other phenomena occurring in these wires. In Fig. 4 we plot the state of wires on the $(R_N, R_N/L)$ coordinate plane. In this representation it is clear that a single line can be drawn that separates precisely the superconducting and insulating phases, given by $R_N/L = (2R_Q - R_N)/107$ nm. A more useful form of this boundary is obtained by using $R_N = \rho_N L/A$, where A is the cross sectional area of the wire and $\rho_N = 180$ $\mu\Omega$-cm is the typical value for the normal metal resistivity of MoGe [9]. Thus the separatrix simply is $A = A_c$ where $A_c = 14.9\ \text{nm}^2/(1 - R_N/2R_Q)$. The superconducting (insulating) phase occurs for samples with $A > A_c$ ($A < A_c$). This means that for wires that have $R_N \ll R_Q$ the superconductivity is lost if the wire diameter, d, is less than the critical diameter $d_c \approx 4.4$ nm. This critical diameter is quantitatively consistent with a recent conjecture that superconductivity in ultrathin wires is affected, in accordance with the Abrikosov and Gor'kov mechanism, by magnetic moments that spontaneously form on the wire surface [22]. Assuming that magnetic pair breaking is

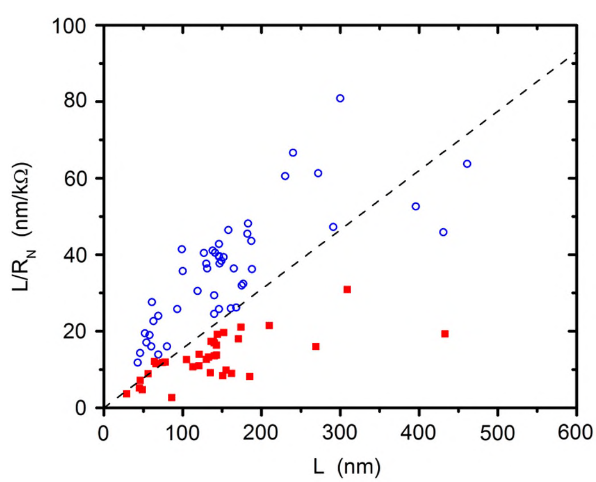

FIG. 3 (color online). Phase diagram of all superconducting (○) and insulating (■) wires in $L/R_N - L$ space. Dashed line is $R_N = R_Q$ as expected for a CSB transition.

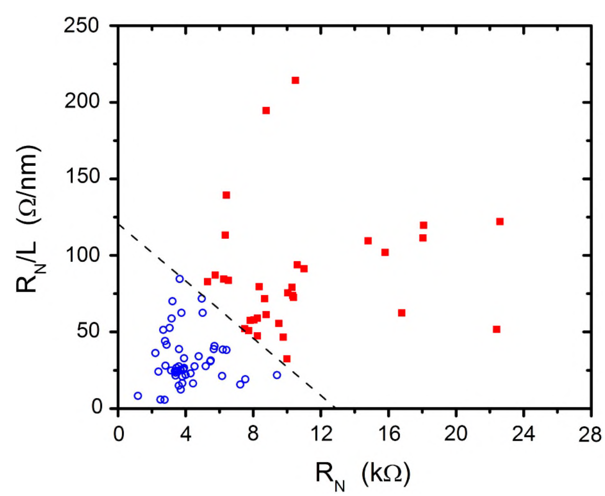

FIG. 4 (color online). Phase diagram of all superconducting (○) and insulating (■) wires in $R_N/L - R_N$ space with the separatrix (dashed line) given in the text.

responsible for the destruction of superconductivity in MoGe wires with $R_N \ll R_Q$ one can estimate the critical diameter from the empirical law relating wire diameter to the exchange scattering time, τ_B, found in Ref. [22]. The critical exchange scattering time, below which T_c is zero, is given by $\tau_{B,c} = \hbar S(S+1)/\alpha_c = 3\hbar e^\gamma/2\pi k_B T_{c0}$ where $S = 1/2$, α_c is the critical depairing factor, $\gamma = 0.577$ is Euler's constant, and T_{c0} is the critical temperature of the wire in the absence of pair breaking effects [2]. In Ref. [22], a fit to τ_B vs d data for MoGe nanowires showed that $d \approx 3$ nm/ps $\times \tau_B$ and T_{c0} was found to be in the range 4.4–5.6 K. This corresponds to $\tau_{B,c} \approx 1.2$–1.5 ps and $d_c \approx 3.5$–4.4 nm, in agreement with the value of d_c obtained from Fig. 4. Finally, we point out that the phase boundary in Fig. 4 suggests that wires with $d \gg d_c$ will become insulating if $R_N > 2R_Q$, i.e., when localization effects of single electron wave functions become strong.

In conclusion, we have studied a large set of nanowire samples with lengths and normal state resistances in the ranges of 29–490 nm and 1.17–32.46 kΩ, respectively. The phase diagram of the SIT is in good, albeit not exact, agreement with the one expected for the CSB transition, in accordance with the theory of Ref. [14]. The few deviations can be accounted for by the destruction of superconductivity due to local magnetic moments in wires that would otherwise belong to the superconducting part of the diagram. These results demonstrate that essentially dissipationless electrical transport can be retained in short superconducting nanowires with $R_N < R_Q$. Future work in this area could include the effect of the electromagnetic environment on wires near the SIT, locally modifying the wires, and high frequency transport.

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