Raman scattering in a two-dimensional electron gas: Boltzmann equation approach

E. G. Mishchenko*

L. D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, Kosygin 2, Moscow 117334, Russia
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The inelastic light scattering in a two-dimensional electron gas is studied theoretically using the Boltzmann equation techniques. Electron-hole excitations produce the Raman spectrum essentially different from the one predicted for the 3D case. In the clean limit it has the form of a strong nonsymmetric resonance due to the square-root singularity at the electron-hole frequency \( \omega = \nu k \), while in the opposite dirty limit the usual Lorentzian shape of the cross section is reestablished. The effects of electromagnetic field are considered self-consistently, and the contribution from collective plasmon modes is found. It is shown that unlike 3D metals where plasmon excitations are unobservable (because of very large required transferred frequencies), the two-dimensional electron system gives rise to a low-frequency \((\omega \simeq k^{1/2})\) plasmon peak. A measurement of the width of this peak can provide data on the magnitude of the electron-scattering rate.

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where the fluctuation $\delta n_{\gamma}$ expressed via the nonequilibrium partition function $\delta f_{p}(s,t)$,

$$\delta n_{\gamma}(s,t) = \int \frac{2d^{2}p}{(2\pi)^{2}} \gamma_{p} \delta f_{p}(s,t), \quad (2)$$
differs from the electronic density only by the anisotropic dimensionless factor $\gamma_{p}$ (electron-light vertex). This factor depends on the light polarization and accounts for the virtual interband transitions. Its exact form is not essential for the following (see Ref. 14).

Varying the expression (1) over the vector potential $A$, we obtain the electron current induced by the incident light with the frequency $\omega^{(i)}$ and the in-plane wave vector $k_{x}^{(i)}$,

$$j^{(i)}(s,t) = -\frac{e^{2}}{mc} \delta n_{\gamma}(s,t) A^{(i)} \exp(-i\omega^{(i)} t + ik_{x}^{(i)} s). \quad (3)$$

The 2D current [Eq. (3)] produces a scattered electromagnetic wave with different frequency $\omega^{(j)}$ and wave vector $k_{z}^{(j)}$. The solution of the corresponding nonuniform Maxwell equation is straightforward. After some simple calculations, it gives for the amplitude of light scattered into the half-space $z>0$ the expression

$$A(\omega^{(s)} k_{z}^{(s)}) = \frac{2\pi e^{2} A^{(s)}}{mc^{2} k_{z}^{(s)}}, \quad (4)$$

where $k_{z}^{(i)} = \omega^{(i)}/c^{2} - k_{z}^{(i)}$; the Fourier component of the density fluctuations depends on the transferred energy $\omega = \omega^{(j)} - \omega^{(i)}$ and momentum $k_{z} = k_{z}^{(j)} - k_{z}^{(i)}$.

The Raman-scattering cross section defined as the normalized energy $\langle \omega^{(s)} A^{(s)} \omega^{(j)} A^{(j)} \rangle$ related to the interval $d\omega^{(s)} d^{2}k_{z}^{(s)}/(2\pi)^{3}$ has the form

$$\frac{d^{2}\sigma}{d\omega^{(s)} d\omega^{(j)}} = \frac{e^{1/2} e^{4}}{2\pi m^{2} c^{5}} \omega^{(s)3} K(\omega, k_{z}), \quad (5)$$

where $K(\omega, k_{z})$ is the Fourier component of the correlator of density fluctuations

$$K(s-s', t-t') = \langle \delta n_{\gamma}(s, t) \delta n_{\gamma}(s', t') \rangle.$$

One can argue that the expression (5) diverges as the direction of scattered light approaches the electron plane: $k_{z}^{(j)} \to 0$. In fact, this means that as soon as the “width” of a two-dimensional system is assumed to be the smallest one of all of the characteristic lengths of the problem, we are restricted to the limit $k_{z}^{(j)} \gg l^{-1}$.

To evaluate this correlator we apply the fluctuation-dissipation theorem that expresses it via the imaginary part of the generalized response $\delta n_{\gamma}(\omega, k)$ to an arbitrary external potential $U(\omega, k)$ (in what follows we omit the subscript $s$),

$$K(\omega, k) = -\frac{2}{1 - \exp(-\omega/T)} \text{Im} \frac{\delta n_{\gamma}(\omega, k)}{U(\omega, k)} \quad (6)$$

The most simple way to derive the generalized response (6) is to make use of the linearized Boltzmann equation for the nonequilibrium part of the distribution function $\delta f_{p} = \chi_{\rho} \delta f_{0}/\partial \varepsilon$; $^{12,13}$

FIG. 2. Diagrammatic representation of the density-density correlation function (10). The solid lines represent electron propagators, the dashed line stands for the electromagnetic Green function, and the empty vertex for the effective electron-light interaction.

$$-i(\omega - kv + iv\tau^{-1}) \chi_{\rho}(k, \omega) = i\omega \gamma_{\rho} U(k, \omega) - e\varepsilon \varepsilon E(k, \omega), \quad (7)$$

where $f_{0}(x)$ is the local-equilibrium Fermi-Dirac partition function. The second term on the right-hand side of Eq. (7) accounts for the fluctuating electromagnetic field $E$. It satisfies the Maxwell equation with the nonequilibrium electric current determined from Eq. (7),

$$\begin{align*}
\text{rot} \text{ rot } E(z, s, \omega) = \frac{e\omega^{2}}{c^{2}} E(z, s, \omega) = & -\frac{4\pi e^{2}}{c^{2}} \delta(z) \\
& \times \langle \chi_{\rho}(s, \omega) \rangle.
\end{align*} \quad (8)$$

Here the angular brackets denote the integral over the Fermi line

$$\langle \cdots \rangle = \int \frac{2dp_{F}}{v(2\pi)^{2}} \langle \cdots \rangle.$$

We are interested in the solution of Eq. (8) at $z=0$. The straightforward derivation gives for the Fourier component of the electric field

$$E(z=0, k, \omega) = \frac{2\pi e^{2}}{\epsilon \omega \sqrt{k^{2} - \epsilon^{2} c^{2}}} \langle \chi_{\rho}(k, \omega) \rangle. \quad (9)$$

Substituting the solution (9) into the Boltzmann equation (7) one gets the integral equation for the electronic density fluctuation $\chi_{\rho}(k, \omega)$. Such an equation has a simple solution, which after the substitution into Eq. (2) and then into the fluctuation-dissipation theorem (6), gives the Raman cross section. Finally, we obtain (see Fig. 2)

$$K(k, \omega) = -\text{Im} \left( \frac{\omega \gamma_{\rho}}{\omega - kv + i\tau^{-1}} \right)$$

$$+ \text{Im} F_{a}(k, \omega) D_{a\beta}(k, \omega) F_{\beta}(k, \omega) \quad \text{or} \quad (10)$$

where the proportionality coefficient (Bose factor) is omitted, see Eq. (6); $D(k, \omega)$ is the two-dimensional electromagnetic Green function,

$$D_{a\beta}(k, \omega) = \frac{1}{\omega} \left( \frac{v_{a} v_{\beta}}{\omega - kv + i\tau^{-1}} \right) - \frac{\epsilon \delta_{a\beta}}{2\pi e^{2} \sqrt{k^{2} - \epsilon^{2} c^{2}}}, \quad (11)$$

and $F_{a}(k, \omega)$ is the oscillator strength,

$$F_{a}(k, \omega) = \left( \frac{v_{a} \gamma_{\rho}}{\omega - kv + i\tau^{-1}} \right). \quad (12)$$

We devote the rest of this paper to the discussion of different terms in the Raman cross section [Eq. (10)].
where \( v_z \) means the component of electron velocity in the \( k \) direction. The typical momenta transfer is \( k \) of the order of light momenta. Hence from the formula (14) one can see that \( \omega \gg \nu k \) proving the initial assumption was correct. We can also omit the term \( \epsilon \omega k^2 \) (this term cares for the finite plasmon velocity) in Eq. (14) in comparison with the term \( k^2 \) due to the fact that \( c^2 k \gg v^2_p \eta \) for typical values of \( k \). Indeed, this means that it is enough to use the Poisson equation for electromagnetic fluctuations instead of the Maxwell equation (8). The only difference occurs at very small transferred momenta where the Poisson equation gives the infinite plasmon velocity in the limit \( k \rightarrow 0 \).

The formula (14) is valid only if the plasmon wavelength becomes large compared to the layer thickness, \( k \gg l^{-1} \). If this condition is violated, the three-dimensional problem has to be solved with boundary conditions satisfied on both sides of the layer. Its solution gives the expression

\[
\omega_{pl}(k) = \frac{4 \pi \epsilon^2}{\epsilon - \nu(k)^2} \frac{m \gamma^2}{\nu(k)} \cosh^{-2} \left( \frac{\sqrt{\nu(k)^2}}{2} \right),
\]

where the electron velocity along the perpendicular direction \( v_z \) appears. Note that the angular brackets now denote the integral over the three-dimensional Fermi surface. When \( l \rightarrow 0 \), this expression reduces to Eq. (14) and for \( l \rightarrow \infty \), it gives the frequency of ordinary 3D plasmon.

Near the plasmon resonance the Raman cross section has the symmetric Lorentzian line shape

\[
K_p(k, \omega) \propto \frac{\omega^2}{8 \pi \omega [\omega_{pl}(k) - \omega]^2 + \tau^{-2}}.
\]

The relative height of two resonances (13) and (15) is \( K_{c,h}/K_{p} \propto k^{3/2} \nu^{3/2} \tau^{-1/2}/\omega_{pl} \) and can be either more or less than unity depending on the momentum transfer \( k \) and scattering rate \( \tau^{-1} \).

In conclusion, we have calculated the Raman-scattering intensity from two-dimensional electronic fluctuations. The main distinctive features from a usual three-dimensional metal are the more singular electron-hole contribution (13) and low-frequency plasmon resonance (15). The electronic Raman scattering in a 2D system in a transverse magnetic field can be considered studied the same Boltzmann equation technique as it has been done for the 3D electron system.\(^{16}\)

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\(^{*}\)Present address: Lorentz-Instituut, Leiden University, P.O. Box 9506, 2300 RA Leiden, The Netherlands.


