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On the direct detection of extragalactic weakly interacting massive particles

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We consider the direct detection of weakly interacting massive particles (WIMPs) reaching the Earth from outside the Milky Way. If these WIMPs form a distinct population they will, although of much lower flux than typical galactic halo WIMPs, have a number of features which might aid in their ultimate detectability: a high and essentially unique velocity ($\sim$600 km/s in the galactic rest frame) due to their acceleration in entering the Milky Way, and most likely one or two unique flight directions at the Earth. This high velocity may be experimentally advantageous in direct detection experiments, since it gives a recoil signal at relatively high energy where background is generally much reduced. For a density of extragalactic WIMPs comparable to the critical density of the universe the count rate expected is very roughly the same as that of fast galactic WIMPs. If there is an increased density relative to critical density associated with the Local Group of galaxies, say 10–30 times the critical density, there is a corresponding increase in rate and the extragalactic WIMPs would show up as a high energy shoulder in the recoil energy distribution. Evidence of such WIMPs as a separate population with these distinct properties would offer interesting information on the formation and prehistory of the galaxy.

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I. INTRODUCTION

The nature of the dark matter in the universe is one of the outstanding questions in astrophysics and particle physics. One of the favored candidates is the WIMP (weakly interacting massive particle). These particles are presumed to have masses in the range 1 GeV/c^2 to a few TeV/c^2.

Much work has been done studying the possibilities for detecting these particles. Possibilities include direct detection [1], where the particle interacts with a nucleus in a low temperature detector, and is identified by the keV of energy it deposits in the nucleon; and indirect detection, whereby the particles are captured in the Sun or Earth, sink to their centers, annihilate with one another in the core, giving rise to particles including neutrinos which can be detected by experiments on the surface of the Earth [2,3]. Alternatively, the particles annihilate in the galactic halo [4] or the galactic center [5] and produce anomalous components in cosmic rays.

Most of this work has focused on detection of WIMPs that belong to the virialized halo of the Milky Way galaxy. However, some authors [6–10] have started to consider the detection of WIMPs that are not yet virialized with the Milky Way halo, specifically because they have fallen into the Milky Way galaxy only relatively recently. The general result is that late-falling WIMPs reach the solar neighborhood as streams of cold dark matter particles with small velocity dispersion and high bulk velocity. Only a few of these streams are expected, each one coming from a particular direction.

In this paper we discuss the possibility of detecting "extragalactic WIMPs." These would appear as streams of WIMPs passing the solar neighborhood. The existence of a separate dark matter population with properties distinct from that of galactic WIMPs is plausible although not certain. Its presence depends on the prehistory and the evolution of the galaxy. So verification of such an extragalactic component would provide very interesting information on the history and dynamics of the galaxy.

We address the question of the detectability of extragalactic WIMPs within a simple model in which the galaxy is moving through a background field of ambient WIMPs. In this way, there is a simple distinction between bound and extragalactic WIMPs. Although this is not the commonly used picture at the present time—where the galaxy is still in a state of formation and there is no clear distinction between bound and extragalactic WIMPs—our infalling ambient WIMPs may resemble the streams found in detailed simulations. Indeed, the investigation of our simplified model provides the opportunities to look at possibilities for detection and to begin to investigate this complex of questions.

Any such extragalactic WIMPs reaching the Earth from outside the halo will have been accelerated considerably by falling in the potential well of the Milky Way. They may thus be expected to have the local escape velocity from the Galaxy, roughly 600 km/s, to be compared with the 200–300 km/s typical of halo WIMPs. Furthermore, in a picture where the galaxy is moving through a background of WIMPs es-

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sentially at rest, they will appear in the rest frame of the galaxy as a "wind" with definite incoming direction and velocity at infinity. In the case of a spherically symmetric halo, this leads to their appearance at the Earth from one or two essentially unique directions, one direction corresponding to WIMPs on their way into the galaxy, the other to WIMPs on their way out of the galaxy. Hence in this picture one has extragalactic WIMPs at the Earth with the distinct features of essentially unique energy and direction of motion.

In direct detection based on the observation of nuclear recoil, the unique energy has two interesting implications. First, the recoil energy for extragalactic WIMPs is higher than that for typical halo WIMPs. Since, in general, the background decreases sharply with energy [11], this may be a very favorable point experimentally. Secondly, a monoenergetic incoming flux produces a characteristic recoil spectrum quite different from the recoil spectrum from galactic WIMPs. For example, if the WIMP is not too heavy with respect to the target nucleus, the recoil energy spectrum has a "box-like" shape, quite different from the tail of a Maxwellian distribution.

Concerning the definite arrival direction at the Earth, exploitation of this should eventually be possible with direction-sensitive detectors [12]. The directional sensitivity is of particular interest in connection with extragalactic WIMPs, and indeed could be used as a filter to increase the extragalactic WIMP signal relative to halo WIMPs and background.

When directional sensitivity becomes available [12], an improvement of the signal can be hoped for, in addition to interesting additional information on the extragalactic WIMPs population. Unfortunately with present knowledge it is not possible to calculate this direction at the Earth, since it depends on the unknown shape of the Galactic gravitational potential (see below). On the other hand were a definite direction ever to be verified, this direction, like the feature of a distinct energy, would provide very interesting information on the structure and evolution of the galaxy.

Naturally the absolute rate to be expected from extragalactic WIMPs is very small and this is undoubtedly the major obstacle to a practical realization of these ideas. However with progress in background reduction and increasing mass of cryogenic detectors it is perhaps of interest to consider detection of extragalactic WIMPs for the future in a preliminary way. We shall find that under the assumption of an extragalactic WIMP population density of the order of the universal critical density, the expected count rate is very roughly the same as that of fast halo WIMPs. If there is an enhanced density associated with the Local Group of galaxies, say by a factor of 10–30 (see below), then the rate is correspondingly increased. Thus a distinct monoenergetic WIMP population—if it exists—could show up as a shoulder above the halo WIMP spectrum. Verification of such a component would provide very interesting information on the history and dynamics of the galaxy.

We proceed as follows. First, in Sec. II, we discuss the nature of the incoming flux of extragalactic WIMPs for an observer in the Solar System, i.e., the velocity, the direction, and the flux enhancement due to gravitational focusing. Then, in Sec. III, we discuss interactions in a detector: cross sections and recoil energy spectrum. In Sec. IV, we discuss the expected density of fast galactic WIMPs, and how we will use it as a basis for comparison. In Sec. V, we discuss the expected count rates due to our galaxy moving through a critical density of extragalactic WIMPs. Finally, in Sec. VI, we discuss the expected count rate due to Local Group WIMPs overdense by a factor of 10–30 relative to critical. We present our numbers relative to the expected count rates for fast halo WIMPs; the precise count rates depend on the details of the detectors, details which are the same for Halo and extragalactic WIMPs.

A general note for this paper: given the level of uncertainty tolerated, we neglect the small velocity of the Earth in its motion around the Sun, and also neglect the peculiar velocity of the Sun with respect to a frame rotating with the galaxy at the Sun location (the so-called local standard of rest). In other words, we take the Earth and the Sun as moving at the rotation velocity of the galaxy at the Sun's position, $v_{\rm rot} = 220$ km/s. With these considerations in mind, we will refer to Earth, Sun, and Solar System interchangeably. This simplifying assumption precludes us from considering the annual modulation of the signal due to the Earth motion around the Sun [13,14]. We only comment that the amplitude of this modulation is approximately equal to the ratio of the Earth's speed, 30 km/s, to the WIMP speed, and so decreases as the speed of the WIMPs increases. Hence for extragalactic WIMPs arriving on Earth at the escape velocity, the amplitude of the modulation is expected to be approximately 5%.

II. LOCAL FLUX OF EXTRAGALACTIC WIMPS

In this section, we discuss the flux of extragalactic WIMPs as seen by an observer moving with the Solar System. The WIMP flux is an essential quantity for estimating the total interaction rate, which is proportional to the flux F and the interaction cross section σ,

$$R_{tot} = F\sigma, \tag{1}$$

where $F = nw$ is the product of WIMP number density times velocity. However, the WIMP speed itself is also significant in that it governs the recoil energy in the detector, and so determines the differential interaction rate (see Sec. III). Furthermore, given the prospects for a directional WIMP detector [12], also the direction of the WIMP velocity is of interest.

A. Velocity

When an extragalactic WIMP enters the Milky Way, it is accelerated to at least the local escape velocity $v_{\rm esc}$ by the potential well of our galaxy. The local escape velocity from the Milky Way at the position of the Sun is not well determined. Following the discussion in [14], we take a value of about $v_{\rm esc} = 600$ km/s. The local speed v of an extragalactic WIMP with velocity v_∞ far away from the Milky Way follows from energy conservation

$$v = \sqrt{v_\infty^2 + v_{\rm esc}^2}. \tag{2}$$

Since we anticipate that WIMPs outside the galaxy have rather low velocities compared to the escape speed, to a first approximation WIMPs of extragalactic origin at the Earth will have a unique velocity of $v \sim v_{esc}$ in the rest frame of the galaxy. In a simple model we consider below, the increase in v over v_{esc} due to an initial velocity of the WIMPs is on the order of a few percent.

The transformation from the WIMP velocity in the galactic rest frame to the WIMP velocity relative to the Earth is obtained by a simple vectorial addition of its velocity relative to the galaxy and the galactic rotation velocity. As such, it depends on the arrival direction of the WIMP when it approaches the Solar System. We define ψ to be the angle between the following two vectors: the vector starting at the Earth and pointing in the direction the WIMPs are moving near the Earth, and the vector starting at the Earth and pointing in the direction of the galactic rotation velocity. Then the WIMP speed relative to the Sun is

$$w = \sqrt{v^2 + v_{rot}^2 - 2 v v_{rot} \cos \psi}. \tag{3}$$

B. Arrival direction

Statements on the arrival direction of extragalactic WIMPs are model dependent. We will focus on a plausible model in which the Milky Way is in motion with respect to the extragalactic WIMPs in its vicinity. Viewed in the rest frame of the Milky Way, there is then a parallel flux of extragalactic WIMPs entering the galactic halo with a definite velocity at infinity v_∞. (Section VI discusses a particular model of this type in more detail.)

The further progress of the particles, and in particular their direction upon reaching the Sun, depends very much on the details of the potential in the galactic halo. In a flattened halo, WIMPs may follow complicated non-planar trajectories. We will consider instead a spherical potential. In this case, conservation of energy and angular momentum impose that the WIMP trajectories lie on the plane defined by the Sun, the direction of galactic motion, and the galactic center. Spherical symmetry would also impose that only a discrete number of trajectories on the orbital plane intersect the location of the Earth. Thus the detector should see WIMPs coming from a few unique directions, which would be somewhat blurred by the velocity dispersion of the extragalactic WIMPs. On the one hand, it is not easy to predict what these directions are, because they strongly depend on the radial behavior of the galactic gravitational potential. (Some examples will be given later in Sec. VI and in the Appendix.) On the other hand, detecting and determining the direction of possible extragalactic WIMPs would provide very interesting information on the gravitational potential of the galactic halo.

C. Focusing

A further interesting effect of the galactic potential is gravitational focusing, which will tend to increase the flux at the Sun relative to its value at infinity. Again, specific results even in the simple model depend on the details of the galactic potential.

However it is interesting that one may find a simple result for a kind of average enhancement by the following argument. We view the galaxy, in its rest frame, as being subject to a parallel flux of particles, which at great distances is $F_\infty = \rho_\infty v_\infty / m$, where $\rho_\infty = \rho_{extragal}$ is the density of extragalactic WIMPs far away from the Milky Way and m is the WIMP mass. The flux is the appropriate quantity to study since it is divergenceless in a stationary situation. Consider a particle incident at very great impact parameter. In its trajectory through the galaxy, or rather the galactic halo, it will remain far from the center and suffer a very small deflection. As we reduce the impact parameter, the trajectory will pass closer and closer to the center. Consider the sphere around the center of the galaxy on which the Sun is located, of radius R_{Sun}. As the incident impact parameter is reduced, we will reach a critical value of the impact parameter where the trajectory of the particle just touches the sphere of radius R_{Sun}.

Now every incident particle with less than this impact parameter, which we call b_{max}, will pass through the sphere. Hence the total number of particles per second passing through the entire surface of the sphere is

$$N_{sphere} = 4 \pi b_{max}^2 F_\infty . \tag{4}$$

If we divide this by the surface area of the sphere we will obtain the average flux over the sphere:

$$\bar{F}_{sphere} = \frac{N_{sphere}}{4 \pi R_{Sun}^2} = \frac{b_{max}^2}{R_{Sun}^2} F_\infty . \tag{5}$$

We now use angular momentum conservation to find b_{max}. The angular momentum per unit mass as calculated at infinity for the particle on the critical trajectory is $v_\infty b_{max}$. On the other hand when it just touches the sphere it is $v R_{Sun}$. We thus obtain

$$b_{max} = \frac{v}{v_\infty} R_{Sun} \tag{6}$$

and the average flux

$$\bar{F}_{sphere} = F_\infty \frac{v^2}{v_\infty^2}. \tag{7}$$

This equation is invalid when $v_\infty = 0$, in which case there is no WIMP wind and the simplified model described here should be amended by including the WIMP velocity dispersion.

We can also express this result in terms of WIMP densities instead of fluxes. From $\bar{F}_{sphere} = \bar{\rho}_{sphere} v / m$ and $F_\infty = \rho_\infty v_\infty / m$, it follows that the WIMP density averaged over the sphere of radius R_{Sun} is given by

$$\bar{\rho}_{sphere} = \rho_\infty \frac{v}{v_\infty}. \tag{8}$$

With $v \approx v_{esc} \approx 600$ km/s and $v_\infty \approx 75$ km/s (see Sec. VI), we obtain a magnification factor $\bar{\rho}_{sphere} / \rho_\infty \approx 8$.

This is an essentially exact result within the model but unfortunately it is only the average over the entire sphere in question, while instead we would like to know the density of extragalactic WIMPs at the location of the Solar System $\rho_{\text{extragal,local}}$. This density depends on the details of the gravitational potential, and is in general correlated with the arrival direction of the extragalactic WIMPs at the Sun. Obtaining the exact value of the enhancement factor at the position of the Earth is quite complicated. Given the greater uncertainties of other numbers here, this calculation is not currently justified. We therefore parametrize the uncertainty in the focusing enhancement by a factor b_{foc}, and write

$$\rho_{\text{extragal,local}} = b_{\text{foc}} \bar{\rho}_{\text{sphere}}. \tag{9}$$

Since the position of the Sun on this sphere is not exceptional,[1] the average should roughly characterize the enhancement factor to be expected. In more detailed calculations using various potential shapes, we find there can be variations of two or three in the enhancement factor but, barring very special conditions, not of an order of magnitude; i.e., we find $0.3 \lesssim b_{\text{foc}} \lesssim 3$.

III. INTERACTION RATE IN A DETECTOR

The total interaction rate of WIMPs with a target detector nucleus is given by

$$R_{\text{tot}} = F\sigma, \tag{10}$$

where $F = \rho w/m$ is the WIMP flux, m is the WIMP mass, and σ is the cross section for the interaction. Since the flux of Milky Way WIMPs largely dominates over any imaginable flux of extragalactic WIMPs, the total interaction rate in a detector is expected to be largely dominated by galactic, instead of extragalactic, WIMPs.

However, as seen in Sec. II, the local speed of an extragalactic WIMP, and so its kinetic energy, is higher than that of any galactic WIMP. In scattering with a detector nucleus, the more energetic extragalactic WIMP can deposit more energy, and so impart a higher recoil energy to the detector nucleus than any galactic WIMP. Hence selecting events with high recoil energy helps in searching for a signal due to extragalactic WIMPs. It is therefore important to consider the interaction rate differential in the nucleus recoil energy, and compare this differential rate for extragalactic WIMPs with that expected from fast galactic WIMPs. In the rest of the section, we review the generalities of the differential interaction rate, that is the recoil spectrum, and in the following sections we discuss and compare the recoil spectra expected from extragalactic and fast galactic WIMPs.

In general, the differential cross section for elastic scattering of a WIMP of mass m with a nucleus of mass M can be written

$$\frac{d\sigma}{dq^2} = \frac{\sigma_0}{4\mu^2 w^2}\mathcal{F}(q) \tag{11}$$

where w is the speed of the WIMP relative to the detector, q is the momentum transfer in the elastic collision, $\mu = mM/(m+M)$ is the WIMP-nucleus reduced mass, and $\mathcal{F}(q)$ is a nuclear form factor that takes into account the loss of coherence in WIMP-nucleus interactions for momentum transfers comparable to or larger than the inverse nuclear radius [we normalized $\mathcal{F}(0)=1$].

For purely scalar interactions,

$$\sigma_{0,\text{scalar}} = \frac{4\mu^2}{\pi}[Zf_p + (A-Z)f_n]^2. \tag{12}$$

Here Z is the number of protons, $A-Z$ is the number of neutrons, and f_p and f_n are the neutralino couplings to nucleons. For purely spin-dependent interactions,

$$\sigma_{0,\text{spin}} = (32/\pi) G_F^2 \mu^2 \Lambda^2 J(J+1). \tag{13}$$

Here J is the total angular momentum of the nucleus and Λ is determined by the expectation value of the spin content of the nucleus (see [1,13,15–17]).

The nucleus recoil energy equals the energy lost by the WIMP and is simply

$$E = \frac{q^2}{2M}. \tag{14}$$

The maximum momentum transfer is $q_{\text{max}} = 2\mu v$, and the maximum recoil energy imparted by a WIMP of speed w follows as

$$E_{\text{max}} = \frac{2\mu^2 w^2}{M}. \tag{15}$$

The recoil spectrum for WIMPs of a given speed w then follows from a change of variables as

$$\frac{dR}{dE} = 2F\frac{d\sigma}{dq^2} = \begin{cases} \dfrac{\rho\sigma_0}{2\mu^2 m w}\mathcal{F}(q) & \text{for} \quad E \leq E_{\text{max}}, \\ 0 \quad \text{for} \quad E > E_{\text{max}}. \end{cases} \tag{16}$$

We have used $F = \rho w/m$, and we have divided by the nucleus mass so as to obtain a rate per unit detector mass, as customary.

As argued in Sec. II, extragalactic WIMPs in the solar neighborhood have essentially a single speed w. Inserting the focusing enhancement factor as in Eq. (9), we obtain that extragalactic WIMPs give rise to the following recoil spectrum

$$\left.\frac{dR}{dE}\right|_{\text{extragal}} = \begin{cases} \dfrac{b_{\text{foc}}}{w}\dfrac{v}{v_\infty}\dfrac{\rho_\infty \sigma_0}{2\mu^2 m}\mathcal{F}(q), & \text{for} \quad E \leq E_{\text{max}}, \\ 0 \quad \text{for} \quad E > E_{\text{max}}. \end{cases} \tag{17}$$

[1]As discussed in Sec. VI, our best guess is that WIMPs enter the Galaxy from the direction of Andromeda, which lies ~120° from the Galactic center as seen from the Solar System.

Here w is given by the expression in Eq. (3). Notice that both the focusing enhancement factor $b_{\rm foc}$ and the WIMP speed relative to the Sun w depend on the details of the galactic gravitational potential.

The nuclear form factor $\mathcal{F}(q)$ depends on the type of WIMP-nucleus interaction and on the mass and spin distributions within the nucleus. Some of these distributions can be found in [15–17]. For the present simple considerations, we consider only the case where form factor effects are negligible, $q \ll \hbar/R$, where R is the radius of the nucleus. (For a velocity of 600 km/s this translates into either $m \ll M/[(Mc^2/20\ \text{GeV})^{4/3}-1]$ or $M \lesssim 20\ \text{GeV}/c^2$ independently of m.) The cross section is then simply σ_0.

With extragalactic WIMPs all moving at a single velocity and form factor effects neglected, the differential cross section is constant up to the kinematic maximum. Thus from Eqs. (11) and (14), for low mass extragalactic WIMPs, the recoil spectrum in a detector is "box-like," i.e., $dR/dE = (\rho\sigma_0)/(2\mu^2 m w) = \text{const}$ is flat as a function of recoil energy E up to the maximum recoil energy $E_{\rm max}$ given in Eq. (15). This signature is completely different from that obtained from a Maxwellian velocity distribution; for halo WIMPs, one must average these boxes over different velocities. We recall the procedure in the next section.

IV. COMPARISON WITH FAST GALACTIC WIMPS

It is useful to have a simple comparison of the expected detection rates for extragalactic and galactic WIMPs. For galactic WIMPs, we use the detection rates previously calculated by many authors as a basis for the comparison. In more detail, we compare the extragalactic rates with the galactic rates obtained under the simple assumption that the galactic WIMP speeds obey a Maxwellian distribution in the rest frame of the galaxy, without truncating it at the escape velocity. This is the WIMP velocity distribution used in the earliest analyses of direct detection by Drukier, Freese, and Spergel [13] and Freese, Frieman, and Gould [14]. We write it as

$$f_{\rm MW}^{\rm gal}(v)dv = 4\pi\rho_{\rm MW}\left[\frac{3}{2\pi\bar{v}^2}\right]^{3/2} v^2 \exp\left[-\frac{3v^2}{2\bar{v}^2}\right]dv, \tag{18}$$

where we take the dispersion velocity $\bar{v} = 270$ km/s. Assuming an isothermal sphere, the local galactic WIMPs have a density of $\sim 0.3\ \text{GeV/cm}^3/c^2$, which amounts to

$$\rho_{\rm MW} \sim 60\,000 \rho_{\rm crit} \tag{19}$$

(for a Hubble constant of 70 km s^{-1} Mpc^{-1}). Here $\rho_{\rm crit} = 5 \times 10^{-6} h_{70}^2\ \text{GeV/cm}^3/c^2$ where h_{70} is the Hubble constant in units of 70 km/s/Mpc.

The distribution in Eq. (18) that we use as a basis for comparison, although useful for this purpose, may not be the real distribution of WIMP velocities, first, because it allows galactic WIMPs with speeds in excess of the escape speed, and second, because the real nature of the WIMP velocity distribution in the galaxy is unknown. The effects of other halo distributions on direct detection have been studied for example in Refs. [7–10,18].

We now present two ways to estimate the ratio of extragalactic to galactic event rates. One method gives an order of magnitude estimate, the other is more detailed, but they give essentially the same result.

The quick method is based on the comparison of the galactic and extragalactic WIMP fluxes at the escape speed in the galactic rest frame, $v_{\rm esc} \approx 600$ km/s. How many galactic WIMPs are moving this fast? We use the Maxwellian velocity distribution of Eq. (18) to find that the fraction of fast galactic WIMPs (out of the total galactic WIMPs) that are within $\Delta v = 1$ km/s of escape velocity (corresponding to 1% energy resolution) is given by

$$\rho_{\rm MW,esc} \simeq f_{\rm MW}^{\rm gal}(v_{\rm esc})\Delta v \sim 4 \times 10^{-5} \rho_{\rm MW} \tag{20}$$

so that

$$\rho_{\rm MW,\,esc} \sim 3\rho_{\rm crit}. \tag{21}$$

With a reasonable guess of 3 for the focusing enhancement factor, the local density of extragalactic WIMPs is

$$\rho_{\rm extragal,\,local} \sim 3\rho_\infty. \tag{22}$$

Hence

$$\rho_{\rm extragal,\,local} \sim \rho_{\rm MW,\,esc}\frac{\rho_\infty}{\rho_{\rm crit}}. \tag{23}$$

The ratio of interaction rates for extragalactic and galactic WIMPs moving with escape speed then follows as

$$\frac{R_{\rm extragal}}{R_{\rm MW,\,esc}} \sim \frac{\rho_\infty}{\rho_{\rm crit}}. \tag{24}$$

We remind the reader that, in the above equation, we have used the integrated rates for extragalactic and galactic WIMPs, with the proviso that for the galactic WIMPs the rate is only integrated over velocities near local escape speed (and hence is *not* the total rate integrated over all energies).

We now discuss the more detailed method of comparing the galactic and extragalactic rates. Here we calculate the ratio of differential rates rather than of the integrated rates as given in Eq. (24). First, we convert the speed distribution in the galactic rest frame to a speed distribution as seen by an observer moving with the Solar System. We do this by means of a Galilean transformation of the velocities. Denoting with w the WIMP speed relative to the Sun, and remembering that we identify the velocity of the Sun with the velocity of galactic rotation $v_{\rm rot}$, we find

$$f_{\rm MW}(w)dw = 2\rho_{\rm MW}\left[\frac{3}{2\pi\bar{v}^2}\right]^{1/2}\frac{w}{v_{\rm rot}}\sinh\left[\frac{3wv_{\rm rot}}{\bar{v}^2}\right] \times \exp\left[-\frac{3(w^2+v_{\rm rot}^2)}{2\bar{v}^2}\right]dw. \tag{25}$$

The differential interaction rate is then obtained by integrating Eq. (16) over the WIMP speed distribution $f(w)$. In general,

$$\frac{dR}{dE}=\frac{\rho\sigma_0}{2\mu^2 m}\mathcal{F}(q)\int_{w_E}^{\infty}\frac{f(w)}{w}dw, \tag{26}$$

where $w_E=(EM/2\mu^2)^{1/2}$ is the minimum velocity of the WIMP required to deposit an amount of energy E. For the Maxwellian we use as comparison, we find

$$\left.\frac{dR}{dE}\right|_{\rm MW}=\frac{\rho_{\rm MW}\sigma_0}{2\mu^2 m}\mathcal{F}(q)\frac{1}{2v_{\rm rot}}\left\{\mathrm{erf}\left[\frac{\sqrt{3}(w_E+v_{\rm rot})}{\sqrt{2}\bar{v}}\right]-\mathrm{erf}\left[\frac{\sqrt{3}(w_E-v_{\rm rot})}{\sqrt{2}\bar{v}}\right]\right\}. \tag{27}$$

The ratio between the extragalactic and galactic recoil spectra $dR/dE|_{\rm extragal}$ and $dR/dE|_{\rm MW}$ depends on the recoil energy and on the arrival direction of the extragalactic WIMPs. For our comparison, we adopt the maximum value of this ratio of differential rates,

$$\mathcal{R}^{\rm max}=\max\frac{dR/dE|_{\rm extragal}}{dR/dE|_{\rm MW}}. \tag{28}$$

Clearly the ratio of differential rates is maximal when the recoil energy equals the maximum recoil energy $E_{\rm max}$. Furthermore, if we neglect the dependence of the focusing factor on the arrival direction, the local extragalactic flux is maximal when the WIMP velocity is opposite to the direction of galactic rotation, that is $\psi=\pi$ in the notation of Sec. II. This is therefore the value of $\mathcal{R}^{\rm max}$ that we adopt for our comparison. Explicitly, we use

$$\mathcal{R}^{\rm max}=b_{\rm foc}\frac{v}{v_\infty}\frac{\rho_\infty}{\rho_{\rm MW}}\frac{2v_{\rm rot}}{w_{\rm max}}\frac{1}{\mathrm{erf}\left[\frac{\sqrt{3}(w_{\rm max}+v_{\rm rot})}{\sqrt{2}\bar{v}}\right]-\mathrm{erf}\left[\frac{\sqrt{3}(w_{\rm max}-v_{\rm rot})}{\sqrt{2}\bar{v}}\right]} \tag{29}$$

where $w_{\rm max}=v_{\rm rot}+v$ and $v=\sqrt{v_\infty^2+v_{\rm esc}^2}$. For $v_\infty\approx 75$ km/s (see Sec. VI), and with the values adopted for escape speed, rotation velocity, and velocity dispersion, the $\mathcal{R}^{\rm max}$ ratio becomes

$$\mathcal{R}^{\rm max}\sim 40\,000 b_{\rm foc}\frac{\rho_\infty}{\rho_{\rm MW}}, \tag{30}$$

and using Eq. (19),

$$\mathcal{R}^{\rm max}\sim 0.7 b_{\rm foc}\frac{\rho_\infty}{\rho_{\rm crit}}. \tag{31}$$

This is the ratio that will appear in Fig. 1 below. This ratio of differential rates is within a small factor equal to the analogous expression for the ratio of integrated rates in Eq. (24) (integrated only over velocities near the escape speed).

We conclude that within an order of magnitude the ratio of extragalactic to galactic WIMP-detection rates is

$$\frac{\text{rate(extragalactic)}}{\text{rate(galactic near } v_{\rm esc})}\sim\frac{\rho_\infty}{\rho_{\rm crit}}. \tag{32}$$

A few comments are in order. The detection rate of galactic WIMPs used for comparison in Eqs. (24), (31) and (32) is a fictitious extrapolation of the detection rate of galactic WIMPs to WIMP velocities exceeding the escape speed. By definition, no galactic WIMPs, i.e., no WIMPs bound to the galaxy, exist with speeds above the escape speed. The comparison ratios in Eqs. (24), (31) and (32) should then be

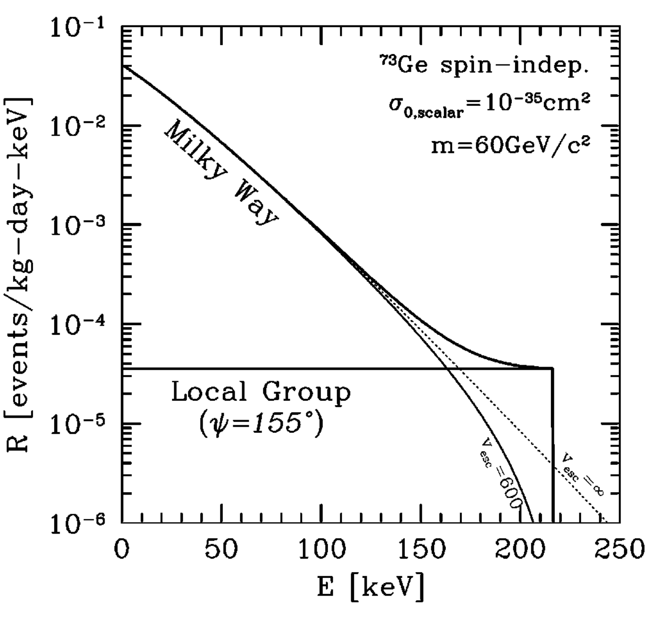

FIG. 1. Expected recoil spectra for Milky Way and Local Group WIMPs. Detector and WIMP parameters as indicated. The Local Group spectrum corresponds to an angle $\psi=155°$ between the velocity of the Local Group WIMPs and the velocity of galactic rotation at the solar neighborhood (both velocity vectors starting at the location of the Earth). The Milky Way spectrum reflects a velocity distribution truncated at the escape speed (curve marked $v_{\rm esc}=600$). The dotted line, marked $v_{\rm esc}=\infty$, does not include this truncation. Thick lines are the sum of the galactic (with $v_{\rm esc}=600$ km/s) and extragalactic spectra.

understood not as actual ratios of detection rates at specific recoil energies, but only as an informative way to gauge the rates due to extragalactic WIMPs with respect to the galactic signal. Indeed, the ratio of extragalactic to galactic WIMP-detection rates may in reality be much larger than what Eq. (32) indicates: The galactic WIMPs are limited to speeds smaller than the escape velocity, while the extragalactic WIMPs may have speeds higher than the escape velocity. Hence the upper edge of the "box-like" spectrum of extragalactic WIMPs may extend beyond the highest recoil energy from galactic WIMPs. This may make the extragalactic signal stick out above the galactic signal more than the numbers in this section (and below) would indicate. On the other hand, the ratio of extragalactic to galactic rates may be much smaller than what Eq. (32) indicates: The shape of the galactic potential or the motion of the galaxy with respect to the extragalactic WIMPs may conspire to make the extragalactic WIMPs move in a direction almost parallel to the direction of galactic rotation at the location of the Sun in the galaxy. In this case, the speed of extragalactic WIMPs relative to the Sun would be a low 380 km/s, and the recoil spectrum of extragalactic WIMPs would be much lower than the recoil spectrum of galactic WIMPs even at the highest recoil energy. This is an unfortunate possibility that should be kept in mind. Hopefully, a detector with directional sensitivity may be able to help in this situation.

V. EXTRAGALACTIC WIMPS WITH CRITICAL DENSITY

We consider smoothly distributed extragalactic WIMPs with density equal to the critical density $\rho_{\rm crit}$. We expect that the typical velocities of these WIMPs away from our Galaxy are small compared to the local escape velocity from the Milky Way. Applying the considerations in the previous sections, we find that the density of extragalactic WIMPs near the Earth is comparable to the density of galactic WIMPs moving at close to escape velocity (600 km/s),

$$\rho_{\rm crit,local} \sim \rho_{\rm MW,600}. \tag{33}$$

The subscript (crit,local) is intended to remind the reader that this is the local density of extragalactic WIMPs that at infinity have a critical density. We find a similar relation for the detection rates:

$$R_{\rm crit} \sim R_{\rm MW,600}; \tag{34}$$

that is, the expected extragalactic WIMP count rate is comparable to that for fast halo WIMPs moving at close to escape velocity.

However, these estimates are very uncertain, because, not knowing the relative motion of the galaxy with respect to critical-density extragalactic WIMPs, the considerations at the end of the Sec. IV unfortunately apply fully. A somewhat less uncertain case is discussed next.

VI. LOCAL GROUP WIMPS

It would be interesting if there would be an overdensity (above critical) of extragalactic WIMPs near the Milky Way. If so, these extragalactic WIMPs would have a count rate in excess of the count rate from halo WIMPs at escape velocity. For example, in this section, we consider the possibility that our galaxy may be sitting in an overdensity of WIMPs due to the fact that we are in the Local Group of galaxies. If the Local Group would be overdense relative to critical by a factor of 10–30, then the expected count rate in a detector would be 10–30 times that of fast halo WIMPs near the escape velocity. In addition, since the Milky Way is moving relative to the rest frame of the Local Group (likely in the direction of Andromeda, as argued below), we would be moving into a wind of Local Group WIMPs. In this case, the Local Group WIMPs would approach our detectors from one or at most two unique directions. In this section we discuss the resulting rates and directions.

The Local Group of galaxies is a group of galaxies dominated by the Milky Way and M31 (a.k.a. the Andromeda galaxy). Currently the number of galaxies stands at 35. For a discussion of Local Group properties, see [19,20]. Each of these big galaxies has a few small satellites (these include the large and small Magellanic clouds for the Milky Way). In addition, there are some isolated small galaxies. Most of the mass of the Local Group is concentrated in the Milky Way and Andromeda. The Milky Way is roughly 400 kpc from the center of gravity of the Local Group. The zero-velocity surface, which separates the Local Group from the field that is expanding with the Hubble flow, has a radius 1.2–1.8 Mpc from the center of gravity of the Local Group.

The question is, are there any Local Group WIMPs that are not virialized with the Milky Way? We will stipulate the existence of a $10^{12} M_{\odot}$ Local Group WIMP halo (not virialized), with the caveat that in the worst case there may be no such halo at all. Recent studies of galaxies [21–23] indicate that galaxies are larger than previously believed, with radii at least 250–300 kpc, and mass-to-light ratios of 145–200. Hence it is possible that much of the dark matter in the Universe is concentrated in the galaxies themselves. However, even if this is the case, it is plausible to believe that when galaxies come into contact with one another some of the dark matter is stripped off. Hence it is possible that there are Local Group WIMPs not associated with either the Milky Way or M31; these form a Local Group halo but are not virialized. In particular, explaining the rapid motions of galaxies near the outskirts of the Local Group may require the existence of a Local Group halo not associated with either the Milky Way or Andromeda galaxy. Hence in the remainder of this section we will stipulate the existence of a $10^{12} M_{\odot}$ Local Group halo of WIMPs.

We now specialize the considerations of the previous sections to Local Group WIMPs.

A. WIMP velocity and arrival direction

The velocity dispersion within the Local Group is $\sigma_r = 61 \pm 8$ km s^{-1} [19]. The Milky Way is moving towards the center of gravity of the Local Group with a similar velocity (about 75 km/s). The tangential velocity of the Milky Way towards the Local Group has not been measured. However, it is likely to be small: the Local Group is an overdense region that was expanding and now collapsing, so that the Milky Way is likely to be falling into M31. Hence we can assume the Milky Way is moving in the direction of M31 with a speed of 75 km/s. This gives rise to a WIMP wind from the direction of M31 with a velocity at infinity $v_\infty \approx 75$ km/s.

The Local Group dispersion velocity of 60 km/s is only 10 percent of the Milky Way escape velocity of 600 km/s, and the associated energy is only 1%. Hence energy conservation tells us we need to modify the velocity near Earth by only 1%. Thus as above, we find that, with respect to the galactic rest frame, the speed of Local Group WIMPs in the solar neighborhood is to a good approximation ~ 600 km/s, that is the escape speed from the galaxy.

As discussed in Sec. II, the specific arrival direction of extragalactic WIMPs at the location of the Sun and Earth depends on the details of the galactic gravitational potential. For the sake of illustration, we have assumed a spherical Milky Way halo and we have calculated WIMP trajectories for two radial dependences of the gravitational potential: a Navarro-Frenk-White potential and a logarithmic potential. (See the Appendix for details.) We have found that at most two arrival directions at the Sun are possible, one for WIMPs on their way into the Milky Way and one for WIMPs on their way out. Thus there are one or at most two values of the WIMP speed at Earth.

Without any constraint on the direction and speed of the relative motion of the Milky Way and the Local Group WIMPs, these directions could cover the whole possible range from opposite to parallel to the Sun motion in the galaxy (which is roughly the direction of galactic rotation). In this case, the WIMP speed relative to the Earth, obtained subtracting the Sun velocity from the WIMP velocity vectorially, could have a value (or at most two values) in the range 380–820 km/s, which is 600 ± 220 km/s, according to the arrival direction at the Sun.

However, if we assume that the WIMPs are approaching the galaxy at infinity from the direction of M31, we find that the WIMP speed relative to the Earth must be larger than ~ 640 km/s. This gives a higher ratio of Local Group and galactic WIMPs counting rates. The reason for the selection of the higher range in speeds is the conservation of the sign of angular momentum. To keep track of the angular momentum, it is helpful to introduce an observer at the Earth that faces the galactic center with the north galactic pole above his/her head. Then M31 lies in the southern hemisphere on his/her left-hand side, at $\sim 120°$ from the galactic center (M31 has galactic longitude $l = 121.2°$ and galactic latitude $b = -21.6°$). Now consider the WIMPs that come from M31 and that reach the Earth on their way *into* the galaxy, that is *before* they reach the point of closest approach to the galactic center. Their initial angular momentum points toward the north galactic pole. When these WIMPs arrive at the solar neighborhood, their angular momentum still points toward the north galactic pole (conservation of the sign of angular momentum). So for our hypothetical observer they possess a velocity component (tangential velocity) directed toward the *right*, together with another velocity component (radial velocity) directed toward the galactic center. The key point is that the direction of galactic rotation, and also the direction of the solar motion, is to the *left* of the observer. Hence the angle between the WIMP velocity (sum of radial and tangential components) at the solar neighborhood and the direction of galactic rotation cannot be smaller than $90°$. It follows that the WIMP speed at the Earth cannot be smaller than $(600^2 + 220^2)^{1/2} = 640$ km/s. A more detailed analysis of the geometry also shows that the angle between the WIMP velocity at the Earth and the direction of galactic rotation cannot be larger than $155°$, which gives a maximum WIMP speed relative to the Earth of 805 km/s.

B. Density at the Earth

Using the numbers given above for the total mass $10^{12} M_\odot$ in WIMPs and a Local Group radius of 1.2–1.8 Mpc, we find an average Local Group WIMP mass density of

$$\rho_{\rm LGW} = (1-3) \times 10^{-28} \ \text{gm/cm}^3. \tag{35}$$

For a Hubble constant of 70 km s^{-1} Mpc^{-1}, this amounts to $(10–30)\rho_{\rm crit}$. Our galaxy is roughly 400 kpc from the center of gravity of the Local Group, about 1/3 of the way out to the zero-velocity surface; for a $1/r^2$ density profile, we should thus be at roughly $10\rho_{\rm crit}$. (Since the system is not virialized we cannot be sure of the density profile, but this estimate still gives us an indication that it is not unreasonable to use an average WIMP density in the halo as its value at the outskirts of the Milky Way.)

Thus the density ratio of nearby Local Group WIMPs to galactic WIMPs moving with 600 km/s is given by

$$\rho_{\rm LGW,local} \sim (10-30)\rho_{\rm MW,600}, \tag{36}$$

where $\rho_{\rm LGW,local}$ is the density of Local Group WIMPs near the Earth and $\rho_{\rm MW,600}$ is the density of Milky Way WIMPs moving with 600 km/s.

The count rate of Local Group WIMPs in detectors would also be 10–30 times that of galactic WIMPs

$$R_{\rm LGW} \sim (10-30) R_{\rm MW,600}. \tag{37}$$

These Local Group WIMPs would have a unique signature in that (i) they come only from one or at most two specific directions and (ii) the lighter ones would have a flat recoil spectrum.

Unfortunately the numbers of events in current detectors would still be tiny. The expected count rate is 0.1 below that for typical halo WIMPs moving ~ 220 km/s, and 10^{-3} below the signal from the sum total of halo WIMPs.

However, as discussed in the Introduction and at the beginning of Sec. II, selecting events with high recoil energy should help in the detection of Local Group WIMPs, because the recoil spectrum of extragalactic WIMPs would show up as a high-energy shoulder which may stick out above the spectrum of galactic WIMPs at high recoil energies. We calculate the recoil spectra for extragalactic WIMPs using Eq. (17) and compare it with the galactic spectrum from Eq. (27). As discussed in Sec. III, the spectrum of extragalactic WIMPs is boxlike.

Figures 1 and 2 show examples of the expected recoil spectra for Milky Way and Local Group WIMPs. For the

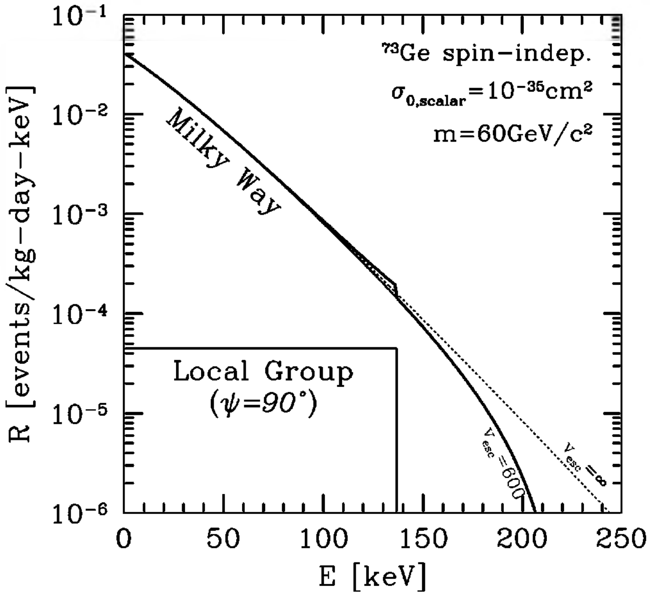

FIG. 2. Same as Fig. 1, but for an angle $\psi = 90°$ between the velocity of the Local Group WIMPs and the velocity of galactic rotation at the solar neighborhood.

sake of illustration, we have fixed the mass of the target nucleus $M = 73$ GeV/c^2, the WIMP mass $m = 60$ GeV/c^2, the WIMP-nucleus scattering cross section $\sigma_0 = 10^{-35}$ cm^2, and we have set the nuclear form factor $\mathcal{F}(q) = 1$. For the galactic WIMPs, we have obtained the recoil spectrum from Eq. (26) for a Maxwellian velocity distribution as in Eq. (18) but truncated at the escape velocity (we do not display the lengthy formulas). For comparison, the dotted line shows the spectrum due to Milky Way WIMPs for the non-truncated velocity distribution we use as a comparison in Sec. IV, that is Eq. (27). For the Local Group WIMPs we have obtained the rate using Eq. (17), taking $b_{\rm foc} = 1$, $\rho_{\rm LGW} = 20\rho_{\rm crit}$. Since we do not know the arrival direction of the Local Group WIMPs at the Solar System, we show two possible spectra for these WIMPs, Fig. 1 and Fig. 2. We believe that these two spectra bracket the plausible range. They correspond to the two values 90° and 155° for the angle ψ between the velocity of the Solar System in the galaxy and the velocity of the Local Group WIMPs when they reach the Solar System (both velocity vectors starting at the earth).

In the most favorable case of $\psi = 155°$, the high energy recoils due to Local Group WIMPs can be clearly distinguished from the recoils due to galactic WIMPs. In the less favorable case of $\psi = 90°$ the distinction is more difficult.

VII. CONCLUSIONS

Extragalactic WIMPs have several unique signatures which may aid in their ultimate detectability. As these WIMPs enter the Milky Way, they are sped by the gravitational potential of the Galaxy. Near the solar neighborhood, they are moving with the local escape velocity of 600 km/s relative to the galactic rest frame. The relatively high velocity of the extragalactic WIMPs can have the advantage of moving the recoil signal in direct detection to higher energies, where background is much lower. In addition, the extragalactic WIMPs come from only one or two directions. Although it is currently not easy to predict what these directions are, because they strongly depend on the radial behavior of the Galactic gravitational potential, still the uni- or bi-directionality will someday be a strong signature for an extragalactic WIMP population. Additionally, the essentially monochromatic spectrum of extragalactic WIMPs implies (if they are not too heavy) a nuclear recoil spectrum that is flat up to the maximum recoil energy.

For a critical density of extragalactic WIMPs, the count rate in a detector is roughly the same as that of fast galactic WIMPs, but their signature is quite different. An extragalactic population of particular interest would be WIMPs associated with the Local Group; these may have a 10–30 times higher count rate than galactic WIMPs of the same speed, for an energy resolution of 1% in the detector. Detection of extragalactic WIMPs would bring valuable new insight into the processes of galaxy formation.

We also wish to point out that the annual modulation of the signal due to the Earth moving around the Sun decreases as one goes to higher velocity WIMPs. The Earth's annual motion of 30 km/s is 5% of the escape velocity and hence 5% of the velocity of extragalactic WIMPs.

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APPENDIX: TRAJECTORIES OF EXTRAGALACTIC WIMPS

In this appendix, we mention some details of the calculation of the trajectories of extragalactic WIMPs entering the galaxy. In particular, if a WIMP at infinite distance from the galaxy is approaching with one angle α_∞, we calculate the angle α at which it arrives at the Sun. The deviation of the angle from its initial value can be huge. One might assume that radial infall through the location of the Sun to the center of the galaxy is a good approximation to the motion; however, this assumption is not always true, depending on the galactic potential. In an extreme case, the incoming particle can instead have a purely tangential velocity at the location of the Sun, with no radial component (towards the Galactic Center) at all. We considered only galactic potentials which are spherically symmetric. Specifically, we present our results for (i) a Keplerian (point mass) potential, (ii) a Navarro-Frenk-White (NFW) potential, and (iii) a logarithmic potential. For comparison, we also consider (iv) the free particle case.

1. Gravitational potentials

(i) As a simple check, we consider a Keplerian potential with $\phi_{\rm Kep}(r)=-GM/r$, a point mass at the center of the galaxy. This is *not* a realistic potential.

(ii) An NFW potential corresponds to an NFW density profile $\rho=\rho_s(r/r_s)^{-1}(1+r/r_s)^{-2}$, where r_s is a length scale and ρ_s is four times the density at $r=r_s$. Explicitly,

$$\phi_{\rm NFW}(r)=-4\pi G\rho_s r_s^2\frac{\ln(1+r/r_s)}{r/r_s}. \qquad \text{(A1)}$$

We fix the scale r_s by imposing that the escape velocity from the galaxy at the Sun position r_0 is $v_{\rm esc}$, a given quantity. This gives the following equation for $u_s=r_0/r_s$,

$$\frac{u_s}{1+u_s}\ln(1+u_s)=1-\frac{2v_c^2}{v_{\rm esc}^2}. \qquad \text{(A2)}$$

Here v_c is the galactic circular velocity at the Sun position. Taking $v_c=220$ km/s and $v_{\rm esc}=600$ km/s gives $u_s=2$, in other words $r_s=r_0/u_s=r_0/2.00$ or $r_s=4$ kpc if $r_0=8.5$ kpc. [We are aware that this value of r_s is an approximation to what may arrive in a realistic scenario.]

(iii) A logarithmic potential is interesting in this context because it is a simple function with constant circular velocity. However, it increases without limit as $r\to\infty$, and so it needs to be cut off at a certain radius R to allow for unbound orbits. Hence we use the truncated logarithmic potential

$$\phi_{\rm Log}(r)=\begin{cases} v_c^2\left[\ln\left(\frac{r}{R}\right)-1\right], & \text{for} \quad r\leqslant R,\\ -v_c^2\frac{R}{r}, & \text{for} \quad r\geqslant R,\end{cases} \qquad \text{(A3)}$$

fixing the cutoff radius R by again imposing that the escape speed at the Sun position is $v_{\rm esc}$. This gives the condition

$$u_R=\frac{r_0}{R}=\exp[1-v_{\rm esc}^2/(2v_c^2)]. \qquad \text{(A4)}$$

With $v_{\rm esc}$ and v_c as above, $u_R=0.0659$.

2. Arrival directions

Since we assume spherical symmetry for the galactic potential, angular momentum is conserved and the orbit is planar. The particle trajectories that pass by the Sun lie on a single plane, which is defined by the particle velocity vector at infinity (the stream velocity) $\mathbf{v}_\infty$ and the vector from the galactic center to the Sun $\mathbf{r}_0$. [The only exception is if the WIMPs come towards the Sun from directly behind the galactic center, as discussed below.]

In the plane of the orbit, we define the angles α_∞ and α. To characterize the direction from which the particle entered the Galaxy, we define the angle α_∞ to be the angle between the following two vectors that originate at the Galactic Center: the particle velocity vector at infinity towards the Galactic Center, and a vector pointing from the Sun towards the Galactic Center (but translated to the location of the Galactic Center). To characterize the direction the particle is moving once it reaches the Sun, we define α as the angle between the following two vectors that originate at the Sun: the tangent vector of the particle's motion as it passes the Sun, and the vector from the Sun towards the Galactic Center. As an example, see Fig. 4. In order to obtain α_∞ as a function of α, we need to solve the orbit equations.

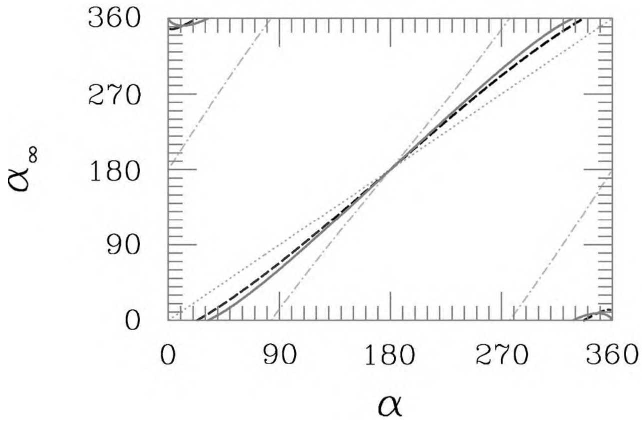

FIG. 3. Graph relating the particle direction at infinity α_∞ to the particle direction at the Sun position α. Clearly for any given incoming velocity from infinity, at most two directions in a detector are possible. Solid lines are for the NFW potential, dashed lines for the logarithmic potential, dash-dotted lines for the Keplerian potential, and dotted lines for the free particle case.

Introduce polar coordinates (r,θ) in the plane of the orbit with the origin at the Galactic Center. Let r_0 be the distance of the Sun from the Galactic Center, and v_∞ be the particle velocity at infinity. Then conservation of energy gives

$$\frac{1}{2}\dot{r}^2+\frac{1}{2}r^2\dot{\theta}^2+\phi(r)=\frac{1}{2}v_\infty^2, \qquad \text{(A5)}$$

where we have chosen the zero of the potential $\phi(r)$ at infinity. Conservation of angular momentum gives

$$r^2\dot{\theta}=v_0 r_0\sin\alpha \qquad \text{(A6)}$$

where $v_0=\sqrt{v_\infty^2+v_{\rm esc}^2}$ is the particle speed at the Sun location r_0 and $v_{\rm esc}=\sqrt{-2\phi(r_0)}$ is the escape speed at the Sun location. Substituting Eq. (A6) into Eq. (A5), and defining $u=r_0/r$, we have

$$\frac{du}{d\theta}=\pm\frac{1}{\sin\alpha}\sqrt{K(u)-u^2\sin^2\alpha}, \qquad \text{(A7)}$$

where the upper (positive) sign in Eq. (A7) applies to particles going in, and the lower (negative) sign to particles going out. Here

$$K(u)=\frac{v_\infty^2-2\phi(r_0/u)}{v_0^2} \qquad \text{(A8)}$$

is the ratio of the particle kinetic energies at galactocentric distances $r=r_0/u$ and r_0. We evaluate $K(u)$ for an NFW, a

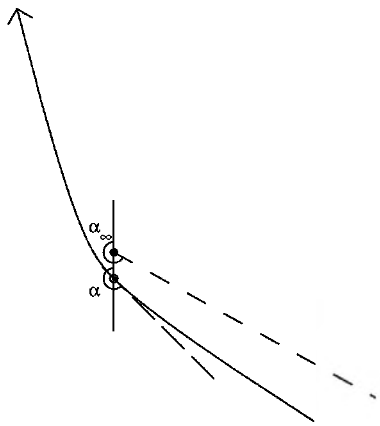

FIG. 4. An example of an NFW trajectory with incoming angle at infinity $\alpha_\infty = 225°$. The lower dot indicates the position of the Sun, the upper dot denotes the galactic center. The arrival direction and the incoming direction at infinity are shown by dashed lines. Also shown are the angles α_∞ and α.

logarithmic, and a Keplerian gravitational potential, and for the free particle case. Then we integrate Eq. (A7) from $\theta = 0$ to $\theta = \pi - \alpha_\infty$ with initial condition $u(0) = 0$ to find

$$\alpha_\infty = \begin{cases} \pi + \Theta(\alpha) - \Psi(\alpha), & \text{for} \quad \cos\alpha \geqslant 0, \\ \pi - \Theta(\alpha), & \text{for} \quad \cos\alpha \leqslant 0, \end{cases} \tag{A9}$$

where

$$\Theta(\alpha) = \sin\alpha \int_0^1 \frac{du}{\sqrt{K(u) - u^2 \sin^2\alpha}}, \tag{A10}$$

and

$$\Psi(\alpha) = 2\sin\alpha \int_0^{u_\alpha} \frac{du}{\sqrt{K(u) - u^2 \sin^2\alpha}}. \tag{A11}$$

Here u_α is the solution of

$$K(u_\alpha) - u_\alpha^2 \sin^2\alpha = 0; \tag{A12}$$

$\Psi(\alpha)$ is the angle between asymptotes for particle trajectories passing at the Sun position with speed v_0.

We now present the results of calculating α_∞ as a function of α. Figure 3 plots the four functions obtained for the NFW potential, the logarithmic potential, the Keplerian potential, and the free particle case. The axes have been labeled in degrees.

The graph in Fig. 3 can be inverted to obtain the particle direction α at the Sun position once its direction at infinity α_∞ is given. This inverse function is in general multi-valued: for instance, $\alpha_\infty = 20°$ in Fig. 3 corresponds to two values of α for each gravitational potential plotted. The physical interpretation is that for these incoming directions of the particle stream, particles reach the Sun position from two separate directions: one beam has passed to the "left" of the galactic center, the other to the "right." Notice that for some stream directions there is only one possible direction for the particle flow at the Sun. It must be remembered that except for $\alpha_\infty = 0$ the particle directions at the Sun position lie in the plane defined by the directions of the galactic center and of the stream velocity at infinity. The case $\alpha_\infty = 0$ is special, in that it corresponds to the Sun being on the downstream axis. The values of α read from the graph in Fig. 3 then give the half-angle of the cone of directions from which the particles approach the Sun.

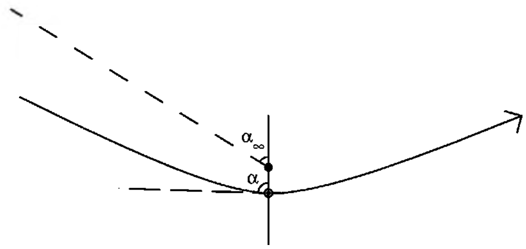

FIG. 5. The unique NFW trajectory passing by the Sun for a particle wind coming from an angle of 58.5° from the galactic center (i.e., from the direction of the Andromeda galaxy). Notice the almost tangential character of the trajectory near the Sun.

Particles approaching the galaxy from the direction of M31 with a speed $v_\infty = 70$ km/s have $v_0 = 604.1$ km/s and

$$\alpha_\infty = 58.5°, \quad \alpha_{\text{Kep}} = \{113.4°, 305.0°\},$$
$$\alpha_{\text{NFW}} = 88.3°, \quad \alpha_{\text{Log}} = 81.4°. \tag{A13}$$

There is only one particle flow for the NFW and the logarithmic potential. In the NFW case the particle velocity at the Sun position is almost tangential. This is because in those trajectories that pass by the Sun, the Sun is almost at the point of closest approach to the galactic center. At this point, the particle radial velocity vanishes, and the particle velocity is purely tangential.

In Figs. 4 and 5, we present examples of two orbits for an NFW potential. Figure 4 is a typical case with a relatively small deviation of the angle from its value at infinity, while Fig. 5 is close to the extreme case in which the orbit has a purely tangential component at the location of the Sun.

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