Heavy-Light Decay Constants with Dynamical Gauge Configurations and Wilson or Improved Valence Quark Actions

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We describe a calculation of heavy-light decay constants including virtual quark loop effects. We have generated dynamical gauge configurations at three β values using two flavors of Kogut-Susskind quarks with a range of masses. These are analyzed with a Wilson valence quark action. Preliminary results based on a "fat-link" clover valence quark action are also reported. Results from the two methods differ by 30 to 50 MeV, which is presumably due to significant — but as yet unobserved — lattice spacing dependence in one or both of the approaches.

Decay constants for the B and B_s mesons are crucial for the accurate determination of the CKM mixing matrix. Reference [1] describes our evaluation of these decay constants in the quenched approximation; the results are consistent with those from several other groups [2]. The effects of quenching in [1] were estimated by comparing with results including dynamical quark effects at fixed lattice spacing. We now have enough results with $N_F = 2$ dynamical quarks to start to study the continuum limit in the dynamical theory. This is the crucial step to go from quenched answers with estimates of quenching effects to true dynamical answers.

Dynamical gauge configurations have been generated with two flavors of staggered quarks at $\beta = 5.445$, 5.5, and 5.6, with a range of dynamical masses. (See Table 1.) We have analyzed each set with Wilson valence quarks (both heavy and light) as well as static heavy quarks, as in [1]. In addition, we have begun to use heavy and light "fat-link" [3,4] clover valence quarks.

In the fat-link clover case, we implement the full Fermilab program [5] through $\mathcal{O}(a)$ and through $\mathcal{O}(1/M)$, including the 3-dimensional rotations ("d_1" terms). The shift to the kinetic mass is done as in the Wilson case [1], except that tadpole improvement is not needed.

In general, we treat the dynamical quark configurations as fixed backgrounds and perform chiral extrapolations in the valence quark mass only; *i.e.*, we do "partial quenching." However, we also try extrapolating with $m_{\rm valence} = m_{\rm dynamical}$ ("full unquenching"). The difference is treated as

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Table 1
Lattice parameters. All sets use $N_F = 2$ dynamical staggered quarks and are analyzed with Wilson valence quarks. To date, 98 configurations of set R have been analyzed with fat-link clover valence quarks. Set G was generated by HEMCGC.

name	β	am_q	size	# configs.
L	5.445	0.025	$16^3 \times 48$	100
N	5.5	0.1	$24^3 \times 64$	100
O	5.5	0.05	$24^3 \times 64$	100
M	5.5	0.025	$20^3 \times 64$	199
P	5.5	0.0125	$20^3 \times 64$	199
U	5.6	0.08	$24^3 \times 64$	201
T	5.6	0.04	$24^3 \times 64$	202
S	5.6	0.02	$24^3 \times 64$	201
G	5.6	0.01	$16^3 \times 32$	200
R	5.6	0.01	$24^3 \times 64$	200

a systematic error, although in most cases it is smaller than the statistical errors.

Most other systematic errors (excited states, chiral extrapolation, fitting errors in 1/M, perturbation theory [in Wilson case], difference between m_2 amd m_3 [in Wilson case]) are estimated the same way as in the quenched approximation [1].

Finite volume errors are estimated by comparing results of sets G and R. Since set G has a smaller physical volume than all other runs, this is an overestimate.

The hardest errors to control with our data are discretization errors and the effects of omitting the dynamical strange quark. We discuss them below.

Details about the fat-link clover approach can be found in Refs. [6,7]. Throughout the current work, we use $N = 10$ smearing steps and smearing parameter $c = 0.45$ ($c/6$ is the coefficient of the staple sum). This amount of fattening completely suppresses exceptional configurations in the range of masses we are studying [3]. With the standard ("thin link") nonperturbative clover action, we found exceptional configurations to be a very serious problem on our perforce somewhat coarse dynamical lattices.

The clover coefficient has been chosen equal to the tree-level value, $C_{SW} = 1$. The fact that fattening suppresses perturbative corrections [7] leads us to expect that this value should be very close to the all-orders (in g) value for our fat links. We plan a nonperturbative evaluation to check this.

Bernard and DeGrand [7] have computed fat-link clover Z factors in perturbation theory. For light-light (ll) and static-light (sl) Z_A, they find:

$$Z_A^{\rm ll} = 1 + \frac{g^2 C_F}{16\pi^2}(-0.241)$$

$$Z_A^{\rm sl} = 1 + \frac{g^2 C_F}{16\pi^2}\left(3\log\left(aM_B\right) + 0.393\right) \ . \qquad (1)$$

For $Z_A^{\rm ll}$, $q^* = 0.71/a$. For $Z_A^{\rm sl}$, q^* has not yet been calculated; we use the light-light q^*. The mass-dependent heavy-light $Z_A^{\rm hl}$ has also not yet been computed. We expect that for moderately large masses, the difference between $Z_A^{\rm hl}$ and $Z_A^{\rm sl}$ will be small: such finite numbers are strongly suppressed by fattening. We currently use $Z_A^{\rm sl}$ for heavy-lights.

At present, we have analyzed only a subset of one lattice set (R) with fat-link clover valence quarks. With only two light quark masses currently available, we choose to focus here on f_{B_s}.

Figure 1 shows f_{B_s} as a function of a in both the Wilson and fat-link clover cases. The Wilson valence points are consistent with constant behavior in a; allowing a linear term in the fit makes almost no difference in the extrapolated value at $a = 0$. However the extrapolated values are inconsistent with the fat-link clover result. Possible explanations for this discrepancy are:

(1) The apparently constant behavior of the Wilson results is misleading. Indeed one expects the Wilson results on dynamical configurations to decrease as $a \to 0$ with roughly the same slope as in the quenched Wilson case. (The quenching effect on this slope should be roughly like the quenching effect on physical quantities, *i.e.*, ~ 5–30%.) In this scenario, the reason that the Wilson results look constant is that the effects of dynamical quarks (which should raise decay constants by deepening the potential well at $r = 0$) are turning on as the lattice spacing becomes fine enough to see the small r behavior. For smaller a, they would begin to fall. If we assume that the $a = 0$ limit of the Wilson data is equal to the

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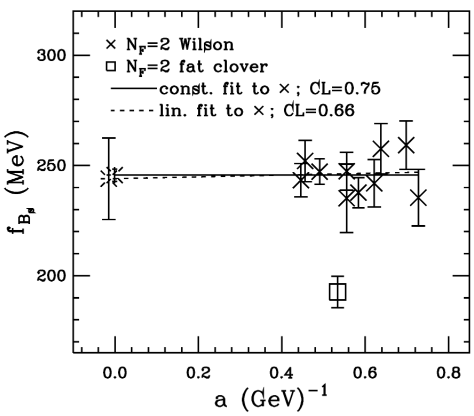

Figure 1. f_{B_s} *vs.* lattice spacing.

fat-link clover result, and that the linear slope is the same as in the quenched case, we can make a quadratic fit to the Wilson data with a confidence level of 0.23 and a reasonable quadratic term of scale $(390\,\mathrm{MeV})^2$. This does not validate the scenario, of course, but only shows that it is a consistent possibility.

(2) Too much fattening has done violence to the physics governing f_{B_s}. This could be the case if, *e.g.*, the smearing softens the Coulomb potential at the origin enough to reduce significantly the decay constants. This would not mean that fattening is "wrong," but that this much fattening introduces significant lattice spacing dependence. This dependence presumably would occur at $\mathcal{O}(a^2)$ or higher, since we have argued that $C_{SW} = 1$ is close to the nonperturbative value needed for $\mathcal{O}(a)$ improvement.

(3) Perturbation theory, used to find the renormalization constants in the fat-link case, has broken down. This may be the case because the fattening has so reduced the large q behavior of the integrands that the integrals are IR dominated, and the resulting effective coupling constant is too large. The small values obtained for q^* are indicative of this potential problem. In retrospect, less fattening would have been preferable [7].

The full explanation is probably some combination of these three scenarios. Scenario (1) makes it clear that, while extrapolating the Wilson valence results with a constant may produce significant systematic errors, it should give an *upper bound* to the correct result. If scenario (2) were the only problem with the fat-clover data, then the fat-clover result would be a *lower bound* to the correct result. Scenario (3) complicates the situation. However it is unlikely that the correct result is much below the fat-clover result, because it is unlikely that the Wilson data would have a slope much steeper than in the quenched case. Thus we average the constant-extrapolated Wilson and the fat-clover results, and use the spread to estimate the discretization error. Clearly, this analysis is preliminary; much more study is needed.

With $N_F = 2$, we are missing the effect of a dynamical strange quark. To estimate this effect, we assume that each dynamical quark, independent of its mass, has the same effect on the decay constants. This assumption is supported by the Wilson valence data. The values of f_{B_s} in Fig. 1, *e.g.*, do not depend strongly on the dynamical quark mass (which varies from $\sim m_s/2$ to $\sim 4m_s$). We thus estimate the effect of the missing strange quark by taking 50% of the difference between the $N_F = 2$ results and our older quenched results [1].

With the above caveats, our *preliminary* results are (in MeV for the decay constants):

$$f_B = 194(3)(22)(^{+20}_{-0}); \quad f_{B_s} = 219(3)(^{+32}_{-33})(^{+25}_{-0})$$
$$f_D = 211(2)(27)(^{+10}_{-0}); \quad f_{D_s} = 235(2)(^{+36}_{-37})(^{+13}_{-0})$$

$$\frac{f_{B_s}}{f_B} = 1.12(1)(5)(^{+1}_{-2}); \quad \frac{f_{D_s}}{f_D} = 1.11(0)(^{+4}_{-5})(^{+1}_{-3}) \ .$$

The errors are statistical, systematic (within the $N_F = 2$ "world"), and systematic (due to the missing dynamical strange quark), respectively.

This work was supported by the DOE and NSF.

REFERENCES

1. C. Bernard *et al.*, Phys. Rev. Lett. **81** (1998) 4812.
2. A. El-Khadra *et al.*, Phys. Rev. **D58** (1998) 014506; S. Aoki *et al.*, Phys. Rev. Lett. **80** (1998) 5711; A. Ali Khan *et al.*, Phys. Lett. **427B** (1998) 132.
3. T. DeGrand *et al.*, hep-lat/9807002; Nucl. Phys. **B** (Proc. Suppl.) **73** (1999), 903.
4. T. DeGrand, C. DeTar, A. Hasenfratz, and M. Stephenson, in preparation.

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5. A. El-Khadra, A. Kronfeld, P. Mackenzie, Phys. Rev. **D55** (1997) 3933.
6. C. Bernard *et al.*, "Semi-leptonic decays of heavy mesons with the fat clover action," these proceedings.
7. C. Bernard and T. DeGrand, these proceedings.