Spatial Fourier transform method for evaluating SQUID gradiometers

P. Costa Ribeiro, A. C. Bruno, and C. C. Paulsen

Departamento de Fisica, Pontificia Universidade Católica, Rio de Janeiro, RJ, CP 38071, Brazil

O. G. Symko

Department of Physics, University of Utah, Salt Lake City, Utah 84112

(Received 4 March 1987; accepted for publication 27 April 1987)

A simple method of measuring the spatial transfer function of a gradiometer, consisting of a flux transformer coupled to a SQUID, is presented and it is compared with theoretical predictions. Based, on this approach, a new method of reporting a gradiometer's performance is proposed; the rejection factor is expressed in decibels obtained directly from the transfer function plot.

INTRODUCTION

A major problem in the measurement and study of weak biomagnetic fields is the reduction of magnetic noise due to the environment. The most common solution to this is a filtering technique known as spatial discrimination. It essentially consists of connecting a set of coils, which are wound in a gradiometric configuration, to the SQUID input.[1] Usually the gradiometer consists of concentric coils connected in series and separated by distances called base lines.

In the region probed by the gradiometer the spatial distribution of the biomagnetic field and the noise magnetic fields vary monotonically with position. Thus, to study the influence of the gradiometer on the detected signals and its rejection of the noise it is natural to express the corresponding fields in terms of small variations around a given point, namely the position of the first coil of the gradiometer. This is usually done by means of a Taylor series expansion.[2]

A source located far from the gradiometer has a magnetic field spatial variation or distribution that does not vary significantly within the probed region, and hence only a few terms of the Taylor expansion are needed to represent it. However, for fields originating from sources close to the gradiometer, the spatial distribution varies rapidly from point to point within the gradiometer region, and more terms of the polynomial expansion need to be used in order to represent adequately the signal.

A gradiometer of Nth order cancels the first $N-1$ terms of a Taylor expansion.[3] Choosing N properly, the signal of distant sources detected by the SQUID will be drastically attenuated. However, detected signals of near sources should not be attenuated. Unfortunately there is some attenuation of the wanted signal as the net signal detected arises from higher-order terms of the expansion. Thus spatial discrimination is achieved at the price of distorting the signals of interest.

The extent of the distortion introduced in the measurement has been studied by using numerical means but only for a specific source, a single current dipole, and a given second-order gradiometer.[4]

For a general analytical method, a Taylor expansion formalism cannot be used. This is because the near-field sources which are usually located at a distance smaller than the gradiometer's total length would be represented, in this formalism, by a divergent series.[5]

A breakthrough to that problem emerges when it is realized that a monotonic signal can be modeled as well by a Fourier expansion in space.[6] In such an approach, the spatial distribution of a distant source will have only low spatial frequency terms while a near-field source will have both low and high frequency terms. Thus, a properly designed gradiometer will attenuate the low-frequency terms originating primarily from distant sources, and the gradiometer is then basically considered as a high-pass spatial filter.

More precisely, since the magnetic flux is coupled to it at discrete points in space, corresponding to positions of the detection coils, the gradiometer can be regarded as a digital spatial filter. In such an approach, the sampling period is the base line and the samples are weighted according to the number of turns of each coil and its polarity. The advantage of considering the gradiometer as a digital spatial filter comes from the fact that its transfer function can be easily calculated and measured, thus characterizing the gradiometer in a very formal way. With a knowledge of the transfer function of the system a quantitative analysis of the effects of the filter on the signal and noise sources can be made. Therefore, the object of this paper is to demonstrate how the transfer function can be measured and to compare it to the transfer function calculated theoretically. An alternative approach based on the plot of the transfer function will be proposed for evaluating the performance of the gradiometer.

I. EXPERIMENTAL DETERMINATION OF THE TRANSFER FUNCTION

To measure the transfer function of a filter, the gradiometer in the present case, a known signal ϕ is applied to the input of the system and the corresponding output $f(r)$ is recorded as a function of the space coordinate r. The Fourier transform of the output signal $F(k)$, where k is the spatial frequency, divided by the Fourier transform of the input signal $\phi(k)$ is then by definition equal to the measured transfer function of the system

$$H_m(k) = F(k)/\phi(k) . \quad (1)$$

 0034-6748/87/081510-04$01.30

The magnitude of $H_m(k)$ may then be compared to the magnitude of the theoretical transfer function $H_t(k)$ obtained by taking the discrete Fourier transform of the filter coefficients

$$H_t(k) = \sum_{i=0}^{N} n_i e^{-jkb_i}, \tag{2}$$

where n_i is the number of turns (with proper polarity) of each gradiometer coil at position i, b_i is the common distance between the coils, and $N + 1$ is the total number of coils.

As in all systems an input and an output must be defined. The output of the gradiometer is clearly the sum of signals seen by the various coils. The input is considered to be the signal which couples to only one coil, as the aim is to measure the magnetic field at particular points in space.

When measuring the response of the more conventional temporal filter (as opposed to the present spatial filter), the input signals are usually at fixed frequencies or they can be pulses. Unfortunately it is not a straightforward task to create a spatially monochromatic signal and it is even more difficult to get a spatial magnetic pulse. However, any known signal (i.e., a signal whose Fourier transform is known and well behaved) will suffice as the input. Thus the known signal is simply the spatial distribution of the magnetic field generated by a single turn coil.

The dimensions of such a coil were chosen according to the following requirements and criteria. The coil radius should be large enough to allow it to slip over the Dewar and also to make the measurements less sensitive to its positioning, but the coil should be small enough to produce a spatial field distribution which is well behaved for a Fourier transform processing.[8] A computer simulation was done to find a suitable radius and a good compromise was reached for a radius of 15 cm.

An apparatus was constructed which held the coil fixed with respect to the x–y plane, but allowed the movement along the z axis. The Dewar containing the gradiometer was placed on a high stand concentric with respect to the signal coil axis. The coil could then be slipped over the stand and Dewar from a position approximately 1.2 m below the pickup coil (six times the gradiometer base line) to a comparable position above. A low-frequency current (17 Hz) was applied to the coil, and the resulting signal from the SQUID was detected by a lock-in amplifier. The coil was moved by increments of 5 cm when far from the gradiometer, and by 2.5 or 1.0 cm when near it. The gradiometer used in this test was a third-order design whose total length was 20.0 cm.[9]

II. EXPERIMENTAL RESULTS

The theoretical spatial distribution of the magnetic field of the test coil was used as the input. Its variation as a function of the axial distance can be seen in Fig. 1(a), where the origin is taken at the center of the gradiometer. The corresponding Fourier transform is shown in Fig. 1(b). The limiting frequency for the Fourier plot was chosen in such a way that most of the relevant spatial frequency components of a current dipole placed at 7 cm below the gradiometer are present up to this frequency.[10] The output of the third-order gradiometer due to a signal in the test coil at various points in space is presented in Fig. 2(a). A spline interpolation technique was used to generate a function appropriate for a Fourier transform process, which then leads to the Fourier transform shown in Fig. 2(b).

The gradiometer transfer function is obtained by dividing the Fourier transform of the output by the Fourier trans-

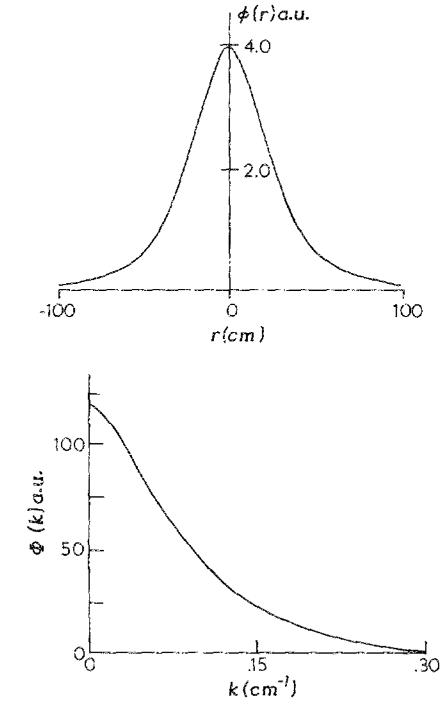

FIG. 1. (a) Magnetic field ϕ generated by the test coil as a function of the axial distance r. (b) Fourier transform of $\phi(r)$ in k space.

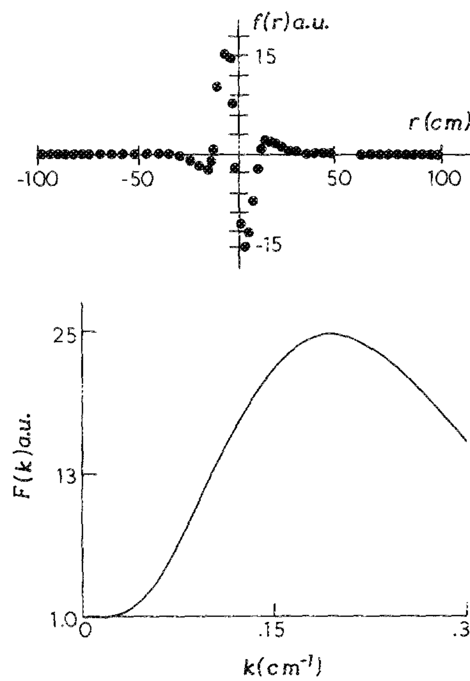

FIG. 2. (a) Measured output from the gradiometer $f(r)$ for various positions r of the test coil. (b) Fourier transform of $f(r)$ in k space.

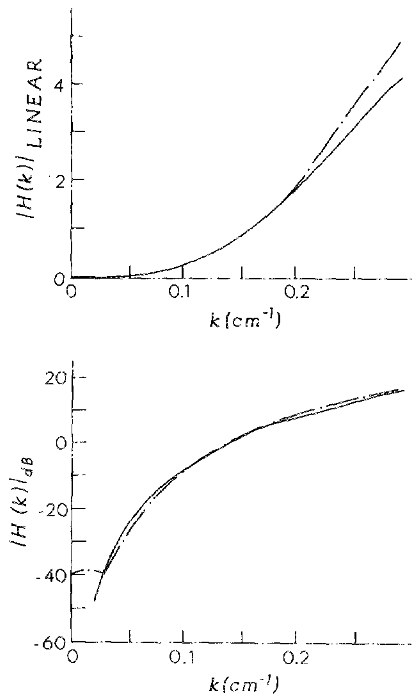

FIG. 3. (a) Theoretical (continuous line) transfer function for a third order gradiometer, and actual (dashed line) transfer function for third-order gradiometer on a linear scale. (b) Graph of (a) on a decibel scale.

form of the input as in Eq. (1). This is shown in Fig. 3(a) on a linear scale and in Fig. 3(b) on a decibel scale, where the dashed curve is the magnitude of the measured transfer function obtained by the experimental procedure described above, and the solid curve is the magnitude of the theoretical transfer function obtained by substituting in Eq. (2) the number of turns and the base line of the third-order gradiometer used. As is shown in the decibel scale plot, the real and theoretical curves are essentially the same for spatial frequencies from approximately 0.09 to 0.19 cm^{-1}. For frequencies higher than 0.19 cm^{-1} there are differences because of the integrating effect of the gradiometer loops which was not taken into account for the input calculation. It should be emphasized that, as expected, this effect appears only at high frequencies. For frequencies below 0.09 cm^{-1} the difference is due to the unbalance of the real gradiometer.

III. UNBALANCE AND TRANSFER FUNCTION

The design conditions for a Nth order gradiometer in the Taylor expansion approach,

$$\sum_{i=0}^{N} \frac{n_i b_i^\alpha}{\alpha!} = 0, \quad \alpha = 0,\ldots, N-1, \tag{3}$$

have been shown[10] to be equivalent, in the Fourier approach, to

$$\frac{d^\alpha}{dk^\alpha} H(k)\bigg|_{k=0} = 0, \quad \alpha = 0,\ldots, N-1, \tag{4}$$

where $H(k)$ is the transfer function of the gradiometer. However in practice conditions (3) or (4) cannot be really achieved. Hence we define a way to measure how perfect is the built gradiometer.[2,11] The so-called unbalance is a measure of how far from zero are the design conditions. Specifically, the uniform field rejection is according to the condition in Eq. (3)

$$\sum_{i=0}^{N} n_i = 0, \tag{5}$$

which is equivalent[10] in the Fourier approach to $H(0)$. The zeroth-order balance of the gradiometer may be then easily obtained from the magnitude of the measured transfer function curve at zero spatial frequency. Also the field gradient rejection according to Eq. (3) is

$$\sum_{i=0}^{N} n_i b_i = 0, \tag{6}$$

which is equivalent in the Fourier approach to making α equal to 1 in Eq. (4). By taking the first derivative of the measured transfer function at the same frequency, the first order unbalance can be found.

The transfer function value for the empirical plot at zero spatial frequency is 8.7×10^{-3}. As discussed earlier this value corresponds to the field balance of the gradiometer. To check this, a field balance measurement was done with a modified set of square coils,[12] based on the Rubens design with 80.0 cm for each side, where the measured uniformity was better than 5.0×10^{-5} over a cube of 20.0 cm^3. A value of 8.4×10^{-3} was obtained for the unbalance measurement, which is very near to the above value of 8.7×10^{-3}.

At this point, we would like to propose a new method of presenting the gradiometer's performance. The magnitude of the transfer function at zero spatial frequency is -41.2 on a decibel scale. Thus, instead of reporting the gradiometer's performance in terms of a percentage error in the loop areas, the performance could be described by its rejection in decibels for that spatial frequency. Concerning the gradient unbalance, the value obtained for it from a numerical calculation was 0.19. This aspect of the performance could be referred to simply as a rejection of -14.4 dB at constant gradients for a third order gradiometer. This method of specifying the performance makes it easier to compare gradiometers between various groups; it also clearly shows that a gradiometer is truly a spatial high-pass filter. This formalism makes the problem, of measuring biomagnetic signals in the presence of noise, a question of system analysis dealing with noise, filters, and bandwidth.

ACKNOWLEDGMENTS

This work was supported jointly by CNPq, FINEP, Brazil, and the National Science Foundation (International Division), USA.

[1] J. E. Zimmerman and N. V. Frederick, Appl. Phys. Lett. 18, 16 (1971).
[2] J. E. Opfer, Y. K. Yeo, J. M. Pierce, and L. H. Rorden, IEEE Trans. Magn. MAG-9, 536 (1974).
[3] P. Karp and D. Duret, J. Appl. Phys. 51, 1267 (1980).
[4] G. L. Romani, S. J. Williamson, and L. Kaufman, Rev. Sci. Instrum. 53, 1815 (1982).
[5] A. C. Bruno and P. Costa Ribeiro, Rev. Bras. Eng. 2, 21 (1984).
[6] A. C. Bruno, P. Costa Ribeiro, J. P. von der Weid, and I. R. Eghrari,

Biomagnetism: Applications and Theory, edited by H. Weinberg, G. Stroink, and T. Katila (Pergamon, New York, 1984), p. 67.
[7] M. Schwartz and L. Shaw, *Signal Processing* (McGraw–Hill, Tokyo, 1975).
[8] T. Chen, *One Dimensional Digital Signal Processing* (Dekker, New York, 1979).
[9] A. C. Bruno and P. Costa Ribeiro, Cryogenics **23**, 324 (1983).
[10] A. C. Bruno, P. Costa Ribeiro, J. P. von der Weid, and O. G. Symko, J. Appl. Phys. **59**, 2584 (1986).
[11] J. Vrba, A. A. Fife, M. B. Burbank, H. Weinberg, and P. A. Brickett, Can. J. Phys. **60**, 1060 (1982).
[12] R. Merrit, C. Purcell, and G. Stroink, Rev. Sci. Instrum. **54**, 879 (1983).