Pseudodeconfinement and dynamical confinement in the quark plasma

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We elaborate upon phenomenological models of high-temperature hadronic matter at zero baryon density. We discuss the choice of a practical set of "fundamental" degrees of freedom, indicate their relationship to dynamically confined plasma modes, and suggest in what way they may account for the phase transition and other thermodynamic features. We offer a partial resolution to the conflict between the requirement of dynamical confinement and the apparent thermodynamic deconfinement of the plasma. When all quark masses are large, the quarks may appear to be deconfined, even though they are technically confined.

I. INTRODUCTION

The traditional notion of high-temperature deconfinement in the quark plasma, based on what is now known to be a flawed application of asymptotic freedom,[1] was bolstered by numerical simulations in pure SU(2) and SU(3) gauge theory. Early simulations measured the free energy $F(r,T)$ of a pair of test quarks in the thermal gluon ensemble as a function of their separation r at temperature T. In both gauge theories it was found that there is a single phase transition at a temperature T_c, and that the asymptotic value of the free energy,

$$F_0(T)=\lim_{r\to\infty} F(r,T)\ , \tag{1}$$

is infinite for $T<T_c$ and finite for $T>T_c$ (Ref. 2). (The free energy is calculated with lattice regularization of the ultraviolet singularity associated with the point charge.) The conventional interpretation of this result is that in the low-temperature, confining phase the Yang-Mills field is incapable of screening the quark charge, and the confining features of the theory require the unscreened flux to form a flux tube extending between the charges. In the high-temperature, deconfined phase the flux is thought to be screened by thermal fluctuations in the Yang-Mills field. Lattice simulations measured the energy density of the high-temperature plasma and found that above T_c it rapidly approached the value that is expected for a free gas of quarks and gluons.[3] Recent numerical simulations measuring the quark number susceptibility of the plasma give results suggesting the presence of "light" baryonic constituents.[4] Thus thermodynamically the plasma appears to be deconfined.

In apparent contradiction to this point of view, other recent numerical simulations and theoretical arguments have supported the interpretation that excitations of the quark plasma (at least those of low frequency) are color singlets.[5] These excitations are associated with poles in frequency-momentum-space linear response functions of the type

$$S_{AB}(\mathbf{k},\omega)=\int d\mathbf{x}\,dt\,e^{(i\mathbf{k}\cdot\mathbf{x}-i\omega t)}\langle T[A(\mathbf{x},t)B(0,0)]\rangle\ , \tag{2}$$

where $A(\mathbf{x},t)$ and $B(\mathbf{x},t)$ are local operators and the average is taken over the Gibbs ensemble at temperature T. At zero temperature these modes are the familiar hadrons. At finite temperature they correspond to peaks in certain production cross sections. Thus dynamically the plasma appears to be confined.

A corollary to confinement in the quark plasma, of course, is that the response function (2) should have no branch point singularities associated with quark-antiquark or other multiquark production thresholds.

In this report we speculate about the structure of the plasma for a wide range of quark masses and temperatures. In so doing we offer a partial resolution to these seemingly contradictory interpretations. We distinguish between the plasma excitations revealed in linear response functions (2) and "fundamental" thermodynamic degrees of freedom. When all quark species have large masses, we indicate how they can appear to be deconfined, even though they are technically confined. However, when light quarks are present, the possibility remains that at least some of the low-lying hadronic modes may be confined both from a practical as well as a technical standpoint.

II. ELEMENTARY DEGREES OF FREEDOM OF THE PLASMA AND OTHER PLASMA STRUCTURES

A traditional approach to constructing a model of high-temperature hadronic matter is to attempt an enumeration of a set of weakly interacting "fundamental" degrees of freedom, in terms of which one can discuss emission cross sections, thermodynamics, including equations of state, and hydrodynamics, including dissipation, transport and evolution. Of course, there is no guarantee that any such weakly interacting components exist, nor that any one set serves all purposes well. In this paper we try to identify the important intermediate-range excitations and plasma structures for a range of temperatures and quark masses. We shall take for granted that at *short range* quarks and gluons are the relevant degrees of freedom at all temperatures even though they never give rise to singularities in (2) and at *very long range,* hydrodynamic degrees of freedom are important.[6] We list three traditional candidates for fundamental *intermediate-range* degrees of freedom.

(1) Discrete resonances in linear response functions. The peaks in the spectral functions

$$\rho_{AB} = \mathrm{Im} S_{AB}(\mathbf{k},\omega) \tag{3}$$

derived from Eq. (2) are in principle observable in emission cross sections;[7] but, as we shall see below, they may not always be the most practical thermodynamic degrees of freedom.

(2) Quarks and gluons. Of course they are the underlying degrees of freedom, but they are weakly interacting only at short distances. At very high temperature they can be regarded as the dominant component of the plasma and they control its thermodynamic behavior over a wide temperature range, even though they never give rise to singularities in the linear response functions. At long range they are strongly interacting and confined.[8]

(3) Energy eigenstates. It has been popular to treat mesons, glueballs, and baryons in rough approximation as weakly interacting hadron gas components at low temperature. Of course, they are modified by thermal effects,[9] becoming resonances in linear response functions. At temperatures above a phase transition or a sudden "crossover" the use of unmodified energy eigenstates is unjustified.

A. The gluon plasma

1. Flux tubes at low temperature

Consider first SU(2) QCD with only gluons. At low temperature the important excitations are glueballs, of course. As the temperature is increased, increasingly highly excited glueballs appear in the thermal ensemble. The lattice flux-tube model of Patel[10] provides a particularly appealing picture of the phase transition that ensues. Excited glueball levels are represented by flux loops of increasing size and complexity. In Patel's model as well as in the model of the closed bosonic string,[11] the level density of excited states grows exponentially as

$$\rho(E) \approx \exp(E/T_0) , \tag{4}$$

where the limiting temperature[12] T_0 is identified with T_c. In Patel's SU(3) lattice flux-tube model the flux tubes form a branching network that corresponds to an even more rapidly growing level density. Nonetheless, the growth is still not greater than exponential.

The lattice flux tube model is not dynamical and does not incorporate interactions among glueballs. The expression (4) for the closed bosonic string neglects glueball decay. Therefore, one might expect in reality that the spectral density (4) is substantially altered by interactions that result in the decay of highly excited glueballs into lower-lying glueballs. In fact, these decays are suppressed in the large-N limit.[13] If we accordingly count the glueball states lying in the glueball continuum, then a Hagedorn mechanism may still be responsible for the phase transition in the pure gluon plasma, even in the presence of glueball interactions.

2. Z(N) domains and glueball modes at high temperature

If glueballs are the most practical large-scale degrees of freedom for $T < T_c$, what are the corresponding excitations for $T > T_c$? To answer this question we appeal to a high-temperature approximation to lattice gauge theory.[14] The high-temperature lattice partition function is reduced to a sum over Wilson lines (Polyakov loops) W in the fundamental representation of the color group on a three-dimensional spatial lattice:

$$Z(\beta) = \int \prod dW_i \exp\left[\beta \sum_{(ij)} \mathrm{Re}(\mathrm{Tr} W_i \mathrm{Tr} W_j^\dagger)\right] , \tag{5}$$

where the sum is over nearest-neighbor pairs and $\beta = aT/2g^2$ for lattice spacing a and coupling g^2. In SU(2) this sum becomes

$$Z(\beta) = \int \prod d\mu_i(\theta) \exp\left[4\beta \sum_{(ij)} \cos\theta_i \cos\theta_j\right] , \tag{6}$$

with $d\mu_i(\theta) = \sin^2\theta_i d\theta_i$, practically the partition function for a three-dimensional Ising model. For large β the sum can be reexpressed in terms of two basic structures: *domains,* consisting of regions in which the phase takes the value $\theta \approx 0$ or $\theta \approx \pi$ and *spin waves,* corresponding to small amplitude fluctuations in the color-singlet plasmon field θ^2 about the background domain value. There is a similar result for SU(3) with $Z(3)$ domains and spin waves. The spin waves in each case are color-singlet plasmons. The domain structure is suppressed at high temperature, since the cost in internal energy required to produce a domain of surface area S at temperature T is (in lattice units)

$$E(S) \propto T^2 S . \tag{7}$$

However, as the temperature is decreased toward T_c, the domain structure becomes increasingly complex, and eventually the phase transition occurs. The domains can be regarded as solitons in the singlet plasmon field. As the phase transition is approached it becomes increasingly difficult to distinguish them from the singlet plasmons themselves, and it is questionable to treat them as separate plasma components.

In numerical simulations of full lattice QCD the asymptotic behavior of the Wilson line correlation can be interpreted as measuring the size of the domain structures (assuming these structures are larger in scale than the glueball modes in the same channel). In the pure gluon plasma in SU(3) it is found that close to the phase transition the asymptotic Wilson line correlation length is a few times larger than the plaquette correlation length in SU(3), the latter being more naturally associated strictly with glueball modes.[15]

Although it may not be possible to disentangle the plasmon modes and domain structures, we can at least draw some suggestions from the $Z(N)$ model: (1) The plasma very likely supports domainlike structures that play a crucial role in the confinement phase transition in the pure gluon plasma. (2) These domain structures may give an important contribution to the plasma energy density at temperatures just above T_c. Furthermore, because the $Z(N)$ domains can account for the confinement phase transition, a dense accumulation of glueball plasma modes in the Hagedorn sense is not required as T_c is approached from above.[16]

B. Plasma with gluons and heavy quarks

1. *Glueballs and mesons at low temperature*

For SU(2) color a single massive dynamical quark species of any mass m obliterates the phase transition.[14,17,18] For SU(3) color it is now strongly suspected that the phase transition is also absent for a finite range of bare-quark masses.[19] Let us focus on the SU(2), massive quark case, although the discussion is readily adapted to SU(3) for masses for which no phase transition occurs. We first discuss the obliteration of the phase transition in terms of energy eigenstates.

The eigenstates of the Hamiltonian are now glueball states and heavy $q\bar{q}$ mesons of course. As we mentioned above, the flux tube and closed string models suggest an exponentially increasing glueball spectral density in the absence of dynamical quarks. However, with quarks present, a sufficiently high-energy glueball state decays into $q\bar{q}$ mesons. We assume this decay is rapid enough that glueball states above threshold for such a decay are suppressed in the plasma and play no significant thermodynamic role. (In the $1/N$ expansion with a large number of flavors the total width for glueball decay to mesons is ~ 1.)[13] Thus, the exponential increase is terminated by the opening of the two meson threshold at about $4m$, and the level density of glueball states is approximately

$$\rho_1(E) \approx \exp(E/T_0)\theta(4m-E) . \tag{8}$$

The heavy $q\bar{q}$ meson has radial and rotational bands, mixed with vibrational excitations of the flux tube joining the quarks. The vibrational excitations of the flux tube alone provide an exponentially rising level density with the same limiting temperature as the glueball spectrum.[20] However, the discrete meson spectrum is also terminated at approximately the two-meson threshold. Thus in SU(2) the disappearance of the phase transition can be explained by a truncation of the excitation spectrum of both mesons and glueballs. For SU(3) in the Patel model sufficiently light quarks have the same effect.

2. *Pseudodeconfinement at high temperature*

How are these energy eigenstates affected by the heat bath? Since there is a smooth analytic path through the phase diagram from the low- to high-temperature regime, whatever happens to the spectrum, there must be a one-to-one correspondence between poles and branch points in the thermal propagators (2) at low and high temperature.[7] Since all such states are color singlets at low temperature, they are color singlets at high temperature as well. In particular, there should be no branch point associated with a $q\bar{q}$-ionization threshold. As for the poles associated with color-singlet states, shifts in frequency and changes in stability are expected. It is plausible that these shifts are roughly comparable to the temperature, and for $T \ll m$ they do not alter the level densities appreciably. However, the fine structure in the spectrum may be blurred by thermal broadening. Figure 1 illustrates a possible change in the spectral function $\rho_{q\bar{q},q\bar{q}}(\mathbf{k},\omega)$ for the thermal $q\bar{q}$ meson propagator in passing from low to high temperature. An increasingly dense set of discrete states in the range from approximately $2m$ to $4m$, consisting of vibrational, rotational, and radial excitations, are broadened thermally. The more crowded states merge into a pseudocontinuum.

The spectrum of thermal propagators is physically important, since it determines, among other things, the resonant peaks in plasma emission cross sections. However, a different language may be more economical for thermodynamic calculations. Since the mesons in the high-temperature plasma are so readily excited, it may be technically correct to say that every quark is connected by a tube of flux to an antiquark elsewhere in the medium, making a color singlet; but because it is easy to pass from one excited level to another at finite temperature, the paired antiquark can occur nearly anywhere, there is a large volume entropy associated with its production, and there is negligible cost in free energy associated with its formation and pairing.[10] In this way a low value of $F_0(T)$ in Eq. (1) is compatible with pair production despite the rather high-quark pair mass. Thus, for practical purposes, the quark may behave as though it is deconfined in the high temperature plasma, even though in reality it is part of a color singlet. We call this situation "pseudodeconfinement." Such a state of affairs could show up in the spectrum of the thermal meson propagator, as in the figure, in that the pseudocontinuum might be practically indistinguishable from a $q\bar{q}$ continuum, except at low frequencies, where the absence of a true branch point would expose the illusion.

Although the heavy quarks may appear to be deconfined, the glueball modes are not deconfined in the same sense, however. For heavy quarks the false continu-

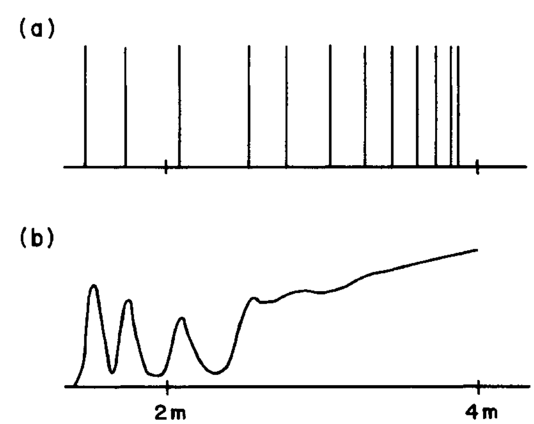

FIG. 1. A sketch of a possible form for the spectral function (3) for $q\bar{q}$ meson propagation as a function of energy in QCD with one quark species of very large mass m. Glueball modes are not shown. (a) represents low temperature and (b) represents high temperature. The latter shows a possible pseudocontinuum below the two meson threshold.

um makes its appearance well below the two-meson threshold; but the gluon is massless, so a two-gluon pseudocontinuum cannot be distinguished from the true two-glueball continuum. Nonetheless, for a different reason we may find an approximate gluon-parton continuum: If we consider the spectral function $\rho_{GG}(\mathbf{k},\omega)$, where G is a local operator producing a glueball state, then we would expect for very high frequency or high wave number to be able to approximate this spectral function in terms of a gluon pseudocontinuum in much the same way we do the zero-temperature cross section for e^+e^- annihilation to hadrons. Thus, in this unphysical plasma consisting of only heavy quarks and gluons, a practical set of high-temperature intermediate-range degrees of freedom may be low-lying glueball modes, domain structures, a couple of low-lying mesonic modes, and "pseudofree" heavy quarks. How many low-lying modes are sufficiently narrow to be counted separately is a quantitative question we do not address here. Finally, to complete this scenario the short-range "gluon" component may account for the thermodynamic behavior.[6]

C. Plasma with gluons and light quarks

Light quarks in both SU(3) and SU(2) color attack the flux tubes and prevent them from growing. In our scenario they therefore suppress the exponentially growing excitation spectrum, and replace the heavy-heavy $q\bar{q}$ pseudocontinuum with a true heavy-light-quark meson continuum. Only a few radial and rotational excitations remain, leaving no room for the pseudodeconfinement mechanism.

Viewing the phase transition from the high-temperature side now, we see that light quarks also inhibit the formation of $Z(N)$ domains. In the $Z(N)$ model, they give rise to a "magnetic" field,[14,17] i.e., in the formation of a $Z(N)$ domain, a cost in volume energy is now added to the cost in surface energy. This inhibition accounts for the absence of a true confinement transition as the plasma is cooled.

A phase transition related to the restoration of chiral symmetry also occurs if some quark species are sufficiently light. The relevant structural change is the dissolution of the low-temperature chiral condensate, attributable to strong interactions in the π- and σ-meson sector. Thus, a treatment in terms of weakly interacting elementary degrees of freedom must fail at the phase transition. Above the phase transition the thermodynamic behavior of the plasma can be explained largely by a "parton" component for wavelengths shorter than a fraction of $1/T$ (Ref. 7).

Whether the low-lying mesonic states survive as narrow peaks in the correlation functions is a quantitative question, depending crucially on the range of screening of the strong interactions in the plasma. We have seen that as the quark mass is lowered the screening mechanism changes from a fluctuation in the flux-tube orientation to a dissolution of the flux tubes by pair production. Thus with light quarks one expects the same screening processes to occur at both low and high temperature. Matsui and Satz[21] have suggested taking the color-singlet Debye screening mass in the pure gluon plasma as a measure of the screening range for the color-octet exchange, and have concluded that the J/ψ has no room to be bound at moderately high temperature.[22] However, screening the short-range octet potential at the range of the color-singlet plasmon is suspect, since the same logic would forbid the formation of the J/ψ at very low temperature, where the corresponding Debye screening mass is much larger still. Moreover, as we have noted the screening mechanism with light quarks is qualitatively different. One could argue instead that, in view of the relatively small changes in the simulated screening mass of the ρ mesonic mode,[5] the range of the strong interactions is not likely to be drastically changed in the crossover region, and it is possible that the low-lying mesons and baryons, made of light or heavy quarks, including the J/ψ, remain relatively narrow states ($\Gamma \sim T$). In any event, since it is experimentally impossible to tell whether the J/ψ dissolves into a quark continuum or a $D\bar{D}$ continuum, the broadening or disappearance of the J/ψ, although a possible signal for interesting thermal changes in the spectrum, is not a signal for deconfinement.

As for explaining the results of light-quark thermodynamic simulations, if we are to pursue the quasi-ideal gas approach and allow a few low-lying hadronic plasma modes to survive as narrow excitations, then the higher-lying modes must conspire so as to give the dominant contribution to the thermodynamic quantities. In one crude approach the thermodynamic simulations can be reproduced approximately, if these higher lying modes are represented by an effective parton contribution for short wavelengths.[6,7] One may justify this approach as in the corresponding approximation to the high-temperature pure gluon plasma and in the traditional approximation to the zero-temperature cross section for e^+e^- scattering: spectral functions for local meson operators at high energy or momentum receive contributions from many combinations of color singlet channels with broad resonances, these contributions being approximated by a quark psuedocontinuum. Thus in the ideal-gas thermodynamics, the higher lying resonances are discarded in favor of the partons that give rise to the pseudocontinuum.

III. SUMMARY AND CONCLUSION

We have argued that the apparent contradiction between thermodynamic simulations of the plasma, suggesting deconfinement, and simulations of linear response functions, suggesting confinement, can be resolved by understanding that poles in the linear response functions need not be the most useful thermodynamic degrees of freedom particularly when their widths are comparable to their spacing. For the case of SU(2) color with a single heavy quark, we showed a possible complementarity between a description in terms of confined excitations and in terms of "pseudodeconfined" quarks. For the light quark case a similar pseudodeconfinement mechanism is not required, at least for the low lying plasma modes, but at least at short wavelengths a parton component is needed to account for the thermodynamic behavior of the plasma.

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