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METHODOLOGY FOR DETAILED LIBERATION ANALYSIS IN MINERAL PROCESSING TECHNOLOGY

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ABSTRACT

A transformation equation can be used to describe the relationship between the one, two and three dimensional information regarding the composition of mineral particles of specified size. Linear or areal grade distributions $f(g_i)$ can be transformed to an estimate of the volumetric grade distribution $p(g)$ via a transformation function $H(g_i|g,Nn---)$, a conditional probability function. The effect of the external particle structure (shape) and internal grain characteristics (grade, dispersion density, and grain size distribution) on the transformation matrix have been evaluated by computer simulation of randomly oriented, multiphase, irregularly shaped particles. Mean volumetric grade and dispersion density (number of grains per particle) are the most important variables which influence the transformation matrix. Least square minimization of fitted functions and the Phillips-Twomey inversion technique have been used to solve the transformation equation. Two examples, a computer simulated volumetric grade distribution and an experimental depth profile, provide evidence that such an approach can be useful for detailed liberation analysis.

Keywords: grade, liberation, multiphase.

INTRODUCTION

Data necessary for liberation analysis is frequently obtained as one or two dimensional information from the polished sections of multiphase mineral particles of specified size. The one or two dimensional information can be taken either as linear intercept measurements or as projected areal measurements, each of which is influenced to some extent by all values of the volumetric grade distribution of the dispersed mineral phase. Let $p(g)$ be the unknown volumetric grade distribution and let $H(g_i|g,Nn---)$ - a conditional probability function - represent the way in which particles of grade g reveal themselves as either linear or areal grades g_i. In this manner, the observed linear or areal grade distribution for particles of a specified size should be related to the volumetric grade distribution as follows;

$$f(g_i) = \int_0^1 H(g_i|g,Nn,---)p(g)dg \qquad (1)$$

where the subscript i designates the linear or areal case and Nn represents the dispersion density (number of grains per particle). Generally the linear or areal grade distributions can be obtained experimentally. Further, the population quantity for the measured linear or areal grade distribution is by their respective dimension (length or area) rather than by number. Equation 1 is known as the transformation equation.

The estimation of the volumetric grade distribution $p(g)$ from equation 1 requires a measure of the linear or areal grade distribution $f(g_i)$, and an estimation of the transformation matrix $H(g_i|g,Nn,---)$, in order to solve the transformation equation. This paper summarizes previous works (Sepulveda et al. 1985, Miller and Lin, 1985) and describes the characteristics of the transformation matrix, the solution of the tranformation equation and experimental verification by depth profile measurements.

TRANSFORMATION MATRIX

The solution of equation 1 requires that the transformation matrix, $H(g_i|g,Nn---)$, be known in advance. The experimental determination of this transformation matrix would involve, in each particular case, the preparation of several monosize, monograde particle fractions, which for all practical purpose, appears to be an impossible task. In this regard, a computer program for particle simulation and analysis, PARGEN, (Sepulveda et al. 1985, Miller and Lin, 1985) was developed to characterize and to construct the transformation matrix. Since the transformation matrix is a highly structure-dependent function, the PARGEN program was developed to represent real mineral particles and includes random variables in which both external structure and internal texture can be manipulated. Because particle generation is a random process repeated many times, and because an inexact solution is acceptable, Monte Carlo simulation is a suitable method to achieve the goal. From the simulation of randomly oriented, multiphase, irregularly shaped particles, the effect of the particles' structure and texture on the corresponding linear or areal grade distribution was determined and analyzed. From sets of these distributions an appropriate transformation matrix was established.

Volumetric Grade

Figure 1 illustrates the effect of volumetric grade on linear and areal grade distributions for ellipsoidal particles with a dispersion density of 3. The ordinate of Figure 1 and later figures represents the cumulative percent of the dispersed phase by the corresponding dimension (e.g. the percent of the total intercept length) less than the indicated grade. In this way the means of each distribution (linear, areal and volumetric) can be shown to be equivalent. It should be noted that although all the particles are locked particles of a fixed grade and dispersion density, in one or two dimensions they appear to be liberated to varying extents. For example, the linear grade analysis of particles containing 50 volume percent dispersed phase and dispersion density of 3 indicates that 10% of the continuous phase is free or liberated. Further, it is evident that the overestimation of the extent of liberation of the continuous increases as the volumetric grade decreases. The complete transformation matrix can be constructed, for a given degree of dispersion, from a family of such curves each curve representing monosize particles of a specified grade. Figure 1 presents such curves for a dispersion density of 3.

Dispersion Density

Besides a dependence on the volumetric grade, a distinct dependence on dispersion density (or number of grains per particle) has been observed for linear and areal grade distributions. This dependence is revealed in Figure 2 for particles containing 50% and 30% by volume of the dispersed phase for linear and areal grade distributions, respectively. It is important to note that the apparent degree of liberation for both dispersed and continuous phase is decreased as dispersion density increases. As expected instinctively and as revealed in Figures 1 and 2, the areal grade distribution provides more information and is a better estimate of the actual volumetric grade.

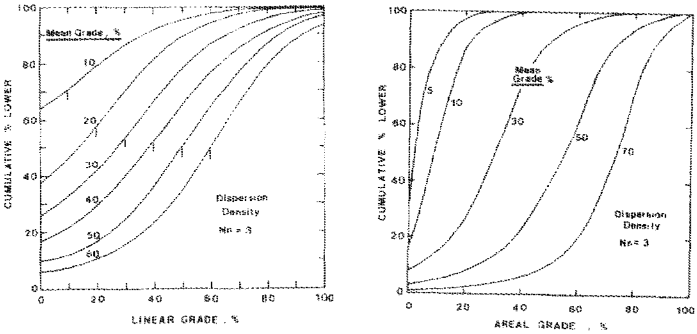

Figure 1. The effect of volumetric grade on the linear and areal grade distributions for simulated particles (ellipsoids) with a dispersion density of 3.

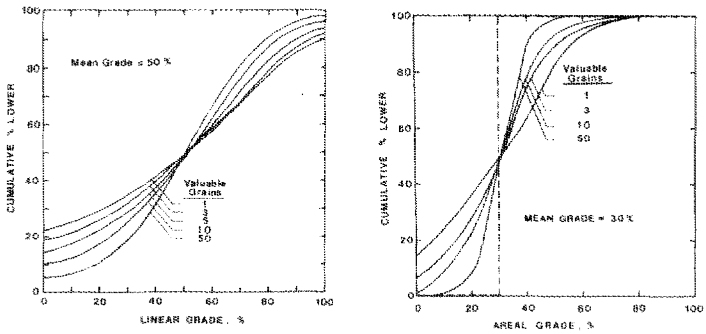

Figure 2. The effect of dispersion density on the linear and areal grade distributions for simulated particles (ellipsoids).

Particle Shape

Four types of particles; sphere, ellipsoid, oblate (or prolate) and "flattened" ellipsoid were generated by the PARGEN program to evaluate the influence of particle shape on linear and areal grade distributions. At a mean volumetric grade of 50% and a dispersion density of 3, Figure 3 illustrates the effect of particle shape on the linear and areal grade distributions. It is interesting to note that the variations observed are quite small except for the flattened ellipsoid particles.

Grain Size Distribution

All the results shown previously were based on a fixed grain size (determined by the volumetric grade and dispersion density). In this regard, four kinds of grain size distributions; uniform, exponential, normal and Weibull, have been evaluated. Results suggest that the grain size distribution has only a modest effect on linear and areal grade distributions as shown in Figure 4.

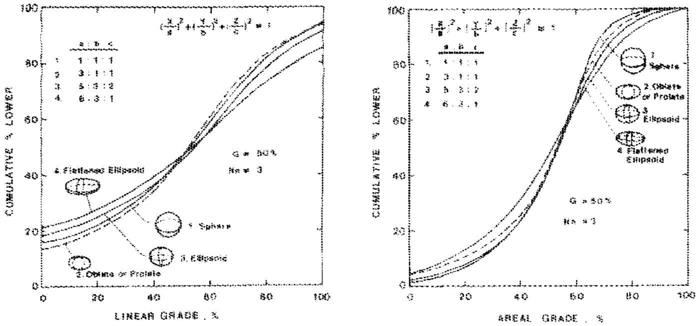

Figure 3. The effect of particle shape on the linear and areal grade distributions for simulated particles (ellipsoids) with a fixed volumetric grade of 50% dispersed phase and a dispersion density of three (Nn = 3).

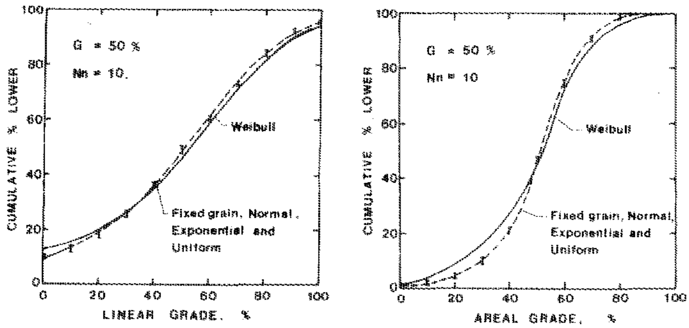

Figure 4. The effect of grain size distribution on the linear and areal grade distributions for simulated particles with a volumetric grade of 50% and a dispersion density of 10.

<u>Summary</u>

Based on these results the variables of volumetric grade and dispersion density appear to be sufficient to construct the transformation matrix for equation 1. From linear or areal grade distributions determined by image analysis and the corresponding transformation matrices, the volumetric grade distribution can be estimated by solving the transformation equation.

SOLUTIONS TO THE TRANSFORMATION EQUATION

The solution of the transformation equation for p(g) is not a trivial exercise and several approaches have been considered. Equation 1 can be discretized with respect to grade to obtain:

$$f_i = \sum_{j=0}^{n} h_{ij} p_j \qquad (2)$$

In matrix form, equation 2 becomes:

[illegible] (3)

from which,

$$\underline{p} = \underline{\underline{H}}^{-1}\,\underline{f} \qquad (4)$$

Unfortunately, due to the experimental error usually associated with the determination of the distribution of linear or areal grades $\underline{f}$ and the inexact characteristics of the transformation matrix $\underline{\underline{H}}$; equation 4, although valid, may lead to erroneous, unrealistic predictions of $\underline{p}$.

Least Squares Minimization

A least squares approach is one method which can be used to obtain the solution to equation 3. Such an approach involves the specification of a functional form for $\underline{p}$ in terms of a set of adjustable parameters $\underline{\beta}$, followed by a search for the optimum values of these parameters so as to minimize the least squares objective function:

$$\Phi = \min_{\underline{\beta}} \, (\underline{f} - \underline{\underline{H}}\,\underline{p})^T(\underline{f}-\underline{\underline{H}}\underline{p}) \qquad (5)$$

The assumed functional form for p(g) limits the usefulness of this method.

Phillips-Twomey Inversion

Another approach, which does not restrict the functional form of p(g), is that developed by Phillips (1962) and Twomey (1963) using a quadrature inversion to approximate the integral equation. If equation 3 is written to include experimental errors, $\underline{\varepsilon}$, the relationship is as follows:

$$\underline{f} + \underline{\varepsilon} = \underline{\underline{H}}\,\underline{p} \qquad (6)$$

According to Phillips and Twomey, the form of the error term $\underline{\varepsilon}$ should be left arbitrary except for the magnitude of $\underline{\varepsilon}$. Assuming $(\underline{\varepsilon})^2 \leq M^2$, then instead of a unique solution for p(g), we get a family of possible solutions, P. To obtain a suitable solution, p(g) is assumed to be smoothed and the constraint of minimizing the second-difference expression $\Sigma(p_{i-1}-2p_i+p_{i+1})$ is imposed. The solution for equation (6) under these circumstances is:

$$\underline{p} = (\underline{\underline{H}}^T\underline{\underline{H}} + \gamma\underline{\underline{Q}})^{-1}\underline{\underline{H}}^T\underline{f} \qquad (7)$$

where γ is a Lagrangian multiplier and

$$\underline{\underline{Q}} = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & & & & & & \\ -2 & 5 & -4 & 1 & 0 & 0 & & & \underline{\underline{0}} & & & \\ 1 & -4 & 6 & -4 & 1 & 0 & & & & & & \\ 0 & 1 & -4 & 6 & -4 & 1 \cdot & & & & & & \\ & & & & & & 1 & -4 & 6 & -4 & 1 & 0 \\ & & \underline{\underline{0}} & & & & 0 & 1 & -4 & 6 & -4 & 1 \\ & & & & & & 0 & 0 & 1 & -4 & 5 & -2 \\ & & & & & & 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix}$$

The matrix $\gamma\underline{\underline{Q}}$ serves to smooth the oscillation in p(g). The choice of γ is discussed in a separate paper (in preparation).

EXPERIMENTAL VERIFICATION

Examples of the application of the transformation equation to estimate known volumetric grade distributions from linear and areal grade distributions are presented for computer simulations and from experimental depth profile data.

Simulated Distribution of Volumetric Grades

Consider a particle sample with volumetric grade distribution, p(g), shown in Figure 5. For a dispersion density of 10, the corresponding linear and areal grade distributions with random error were evaluated. If both the linear and areal grade distributions so obtained are assumed to be the experimental measurements in a practical situation, the volumetric grade distribution is a set of unknown values which needs to be estimated. Both least square minimization of fitted functions and the Phillips-Twomey inversion technique have been tested. Figure 5 illustrates the comparison of the actual volumetric grade distribution with the best solutions obtained from both techniques. Since the volumetric grade distribution does not conform to the assumed fitting function, a poor result from the least squares method is expected. However, an excellent correspondence with the inversion method was found as shown in Figure 5.

Depth Profile Measurements

To test the solution of the transformation equation with actual experimental data, a middlings stream from an iron ore processing plant was analyzed. A sample of 74X105 micron particles, determined to have a mean volumetric grade of 34%, was mounted and sectioned. A depth profile at 18 micron intervals was done and each section was analyzed by image analysis. From these sections the volumetric grade distribution was established. Both linear and areal grade distributions were determined for the final section.

Based on both the least squares and inversion methods, the volumetric grade distribution was estimated from linear and areal grade distributions. Figure 6 illustrates the comparison between the experimental and estimated volumetric grade distributions. Satisfactory solutions were obtained with both methods.

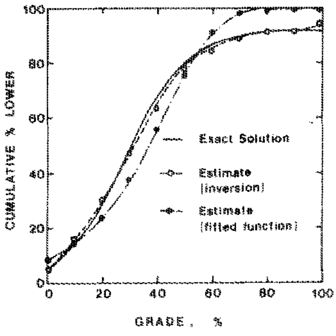

Figure 5. Comparison of exact solution of simulated volumetric grade distribution and estimated solutions obtained by least square and inversion methods.

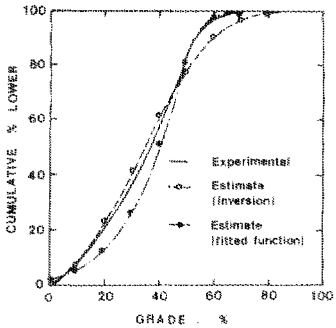

Figure 6. Comparison of estimated and experimental volumetric grade distributions. Sample: 74 X 105 microns fraction of iron ore.

SUMMARY AND CONCLUSIONS

For detailed liberation analysis, the volumetric grade distribution of multiphase mineral particles can be estimated from one or two dimensional information obtained by image analysis of polished sections. A transformation matrix was established by computer simulation (PARGEN program) to express the relationship between one, two and three dimensional information. Volumetric grade and dispersion density are the important variables which identify the appropriate transformation matrix. Two methods to solve the transformation equation and predict the volumetric grade distribution were evaluated. The Phillips-Twomey inversion techinque is considered to be the more suitable method to solve the transformation equation and estimate the volumetric grade distribution. Accurate estimation of the volumetric grade distribution will allow for detailed process engineering analysis of multiphase particulate systems.

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