

Thermodynamics with 2+1 and 3 Flavors of Improved Staggered Quarks *

 C. Bernard ^a, T. Burch ^b, S. Datta ^c, T.A. DeGrand ^d, C.E. DeTar ^e, Steven Gottlieb ^c, U.M. Heller ^f,
 K. Orginos ^g, R.L. Sugar ^h, and D. Toussaint ^b
^aDepartment of Physics, Washington University, St. Louis, MO 63130, USA

^bDepartment of Physics, University of Arizona, Tucson, AZ 85721, USA

^cDepartment of Physics, Indiana University, Bloomington, IN 47405, USA

^dPhysics Department, University of Colorado, Boulder, CO 80309, USA

^ePhysics Department, University of Utah, Salt Lake City, UT 84112, USA

^fCSIT, Florida State University, Tallahassee, FL 32306-4120, USA

^gRIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973-5000

^hDepartment of Physics, University of California, Santa Barbara, CA 93106, USA

We present preliminary results from exploring the phase diagram of finite temperature QCD with three degenerate flavors and with two light flavors and the mass of the third held approximately at the strange quark mass. We use an order $\alpha_s^2 a^2, a^4$ Symanzik improved gauge action and an order $\alpha_s a^2, a^4$ improved staggered quark action. The improved staggered action leads to a dispersion relation with diminished lattice artifacts, and hence better thermodynamic properties. It decreases the flavor symmetry breaking of staggered quarks substantially, and we estimate that at the transition temperature for an $N_t = 8$ to $N_t = 10$ lattice *all* pions will be lighter than the lightest kaon. Preliminary results on lattices with $N_t = 4, 6$ and 8 are presented.

1. INTRODUCTION

With the Relativistic Heavy Ion Collider (RHIC) now producing data, it has become even more important to understand the phase diagram of QCD at finite temperature, and to determine properties of the high temperature quark-gluon-plasma phase with confidence, *i.e.* with controlled lattice spacing errors.

It is fairly well established that QCD with two flavors of massless quarks has a second order finite temperature, chiral symmetry restoring phase transition. This transition is washed out as soon as the quarks become massive. QCD with three flavors of massless quarks has a first order finite temperature, chiral symmetry restoring phase transition, which is stable for small quark masses. Not well known is how large the quark masses can be before the phase transition turns second order and then into a crossover, both for degenerate quarks and especially for the physically relevant case of two light and one heavier strange quark.

In previous studies, the second question is particularly badly answered due to the flavor symmetry breaking in Kogut-Susskind quarks, usually used for this purpose: how can one study the influence of the strange quark when most of the (non-Goldstone) pions are heavier than the (Goldstone) kaon?

Adding a few terms to the conventional Kogut-Susskind action, namely three-link, five-link and seven-link staples and a third-neighbor coupling, removes all tree-level $\mathcal{O}(a^2)$ errors [1–3]. This “Asqtad” action shows improved flavor and rotational symmetry [2,4], and, at least in quenched QCD, good scaling properties [5]. This is illustrated in Fig. 1. There, and in other figures below, we plot results in units of r_1 , a scale defined in terms of the static $Q\bar{Q}$ potential by $r_1^2 F_{Q\bar{Q}\text{static}}(r_1) = 1$, which leads to $r_1 \sim 0.35$ fm [6].

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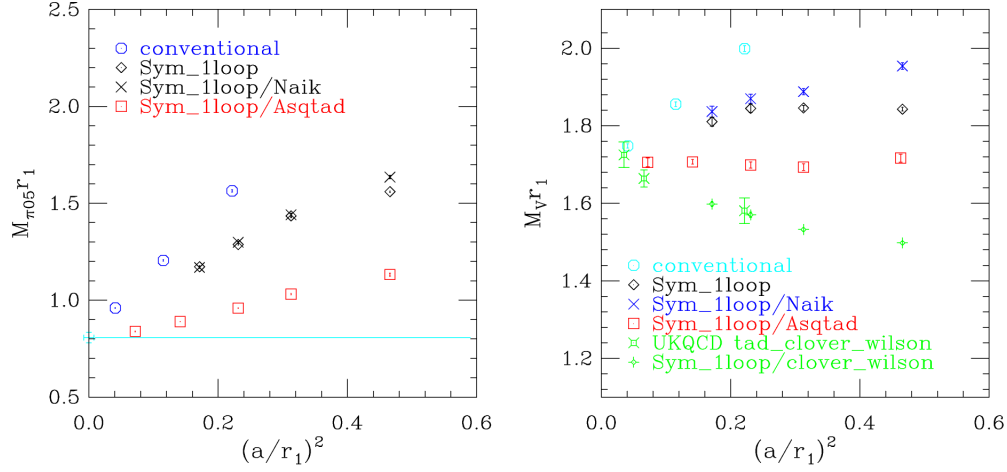


Figure 1. The mass of the second lightest pion, for fixed lowest pion mass in units of r_1 , shown as the horizontal line, as function of the lattice spacing (left). The “Asqtad” action gives the smallest flavor symmetry breaking. The mass of the vector meson, the rho, as function of the lattice spacing (right). The “Asqtad” action gives the best scaling.

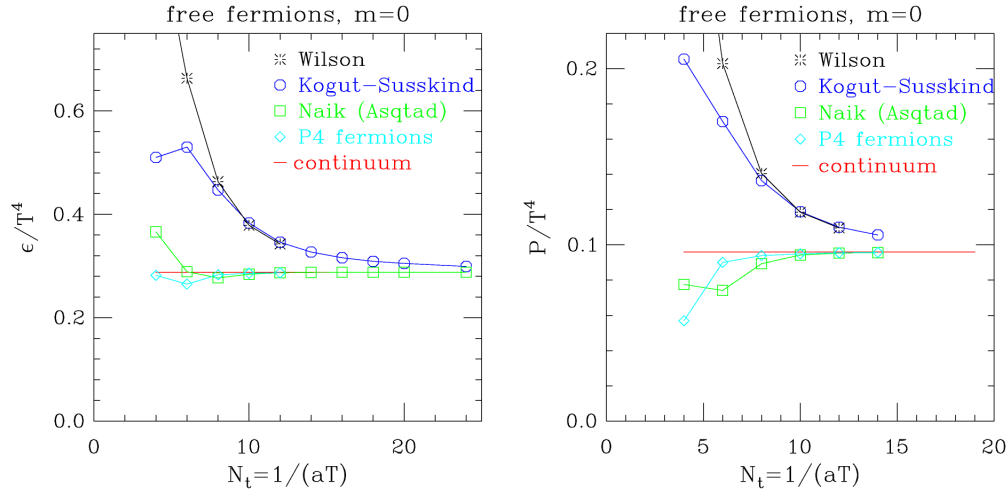


Figure 2. The energy density (left) and pressure (right) of free massless fermions as function of temporal lattice size N_t . For free fermions the “Asqtad” action reduces to the Naik action. The “p4” action is the improved staggered fermion action preferred by the Bielefeld group [7].

The three-link “Naik” term insures a good dispersion relation and thereby helps decrease the lattice artifacts in energy density and pressure at small temporal lattice size N_t , as can be seen in Fig. 2.

Based on the zero temperature simulations of Ref. [4] and the estimate of $T_c \sim 150 - 170$ MeV [8] we deduce that for $N_t = 8 - 10$ the kaon will be heavier than the heaviest non-Goldstone pion at the finite temperature transition.

We have zero temperature results, in particular, the value of the (bare) strange quark mass, at fixed lattice spacing $a \sim 0.13$ fm and $a \sim 0.2$ fm. Since we want to keep the physical quark masses approximately constant, when we vary the temperature which, at fixed $N_t = 1/(aT)$, means varying the gauge coupling β , we interpolated (extrapolated) between the values at the two lattice spacings.

Given a gauge coupling β and quark mass am_q , for the three-flavor simulations, or quark masses am_s and $am_{u,d}$, for 2 + 1 flavors, we determined the tadpole factor $u_0 = (\text{Tr}U_p/3)^{1/4}$ self-consistently in short

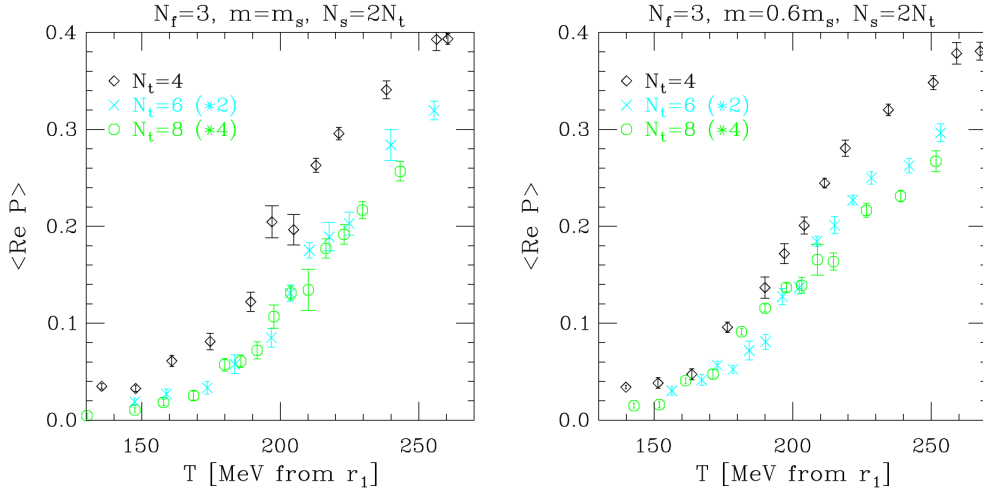


Figure 3. Real part of the Polyakov line for three flavors with $m_q \approx m_s$ (left) and $m_q \approx 0.6m_s$ (right). The data for $N_t = 6$ have been multiplied by two and the data for $N_t = 8$ by four. The spatial lattice sizes are $N_s = 2N_t$.

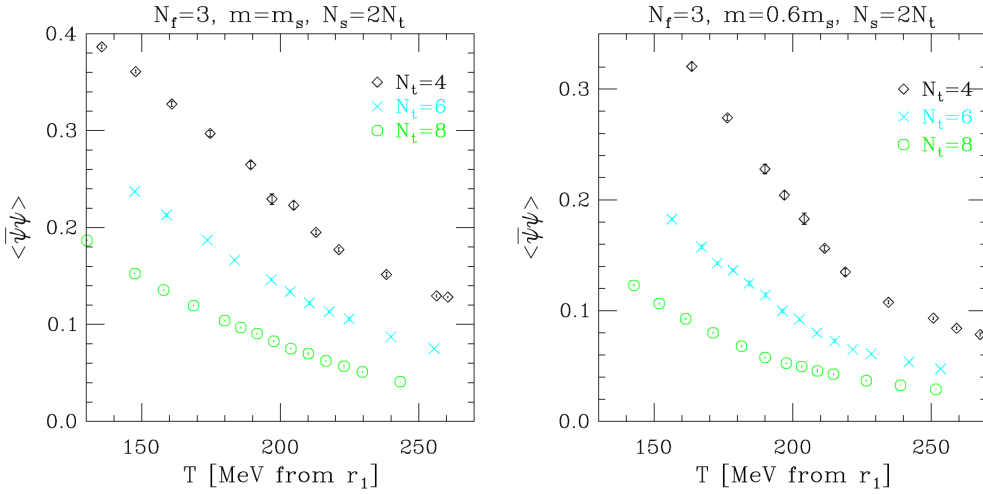


Figure 4. $\langle \bar{\psi}\psi \rangle$ for three flavors with $m_q \approx m_s$ (left) and $m_q \approx 0.6m_s$ (right).

zero temperature simulations on L^4 lattices.

To convert lattice scales to physical scales we interpolated a/r_1 with the form advocated by Allton [9]

$$a/r_1 = c_0 f(g_0^2) [1 + c_2 g_0^2 f^2(g_0^2)] , \quad f(g_0^2) = (b_0 g_0^2)^{-b_1/(2b_0^2)} \exp\left(-\frac{1}{2b_0 g_0^2}\right) . \quad (1)$$

For the (one-loop, tadpole) Symanzik improved gauge action the bare coupling g_0^2 is related to β by $g_0^2 = 10/\beta$. $b_{0,1}$ are the universal first two coefficients of the beta-function for three massless flavors, and $c_{0,2}$ are determined from the measured values of a/r_1 at $a \sim 0.13$ fm and $a \sim 0.2$ fm.

The Hybrid Molecular Dynamics R-algorithm was used for all simulations, with two noise vectors for the 2 + 1 flavor simulations.

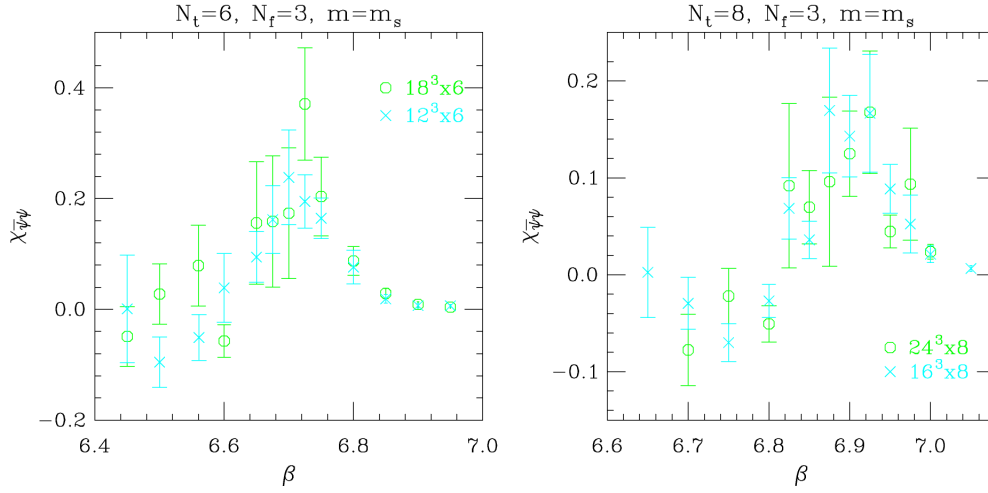


Figure 5. The $\bar{\psi}\psi$ susceptibility for three flavors with $m_q \approx m_s$ for lattices with $N_t = 6$ (left) and $N_t = 8$ (right).

2. RESULTS FOR THREE DEGENERATE FLAVORS

For our first simulation with $m_q \approx m_s$ we used a linear interpolation of $am_q(\beta)$ in β . Since the range of interpolation/extrapolation is quite large, we subsequently used a linear interpolation of $\log(am_q(\beta))$, compatible with the leading behavior of a (with m_q fixed) as function of β . In the region of the finite temperature transition or crossover for lattices with temporal extent $N_t = 6$ and 8, the difference between the two interpolation schemes is small enough that it will not affect our results significantly.

We show in Fig. 3 the real part of the Polyakov line and in Fig. 4 the condensate as function of the temperature, with the physical scale obtained via r_1 , for our simulations with three degenerate quarks of mass $m_q \approx m_s$ and $m_q \approx 0.6m_s$. The Polyakov line shows a crossover from confined behavior at low temperature to a deconfined behavior at high temperature. The condensate decreases as the temperature is increased, but shows no sign of a transition or sharp crossover.

In Fig. 5 we show the chiral susceptibility, $\chi_{\bar{\psi}\psi} = \partial\langle\bar{\psi}\psi\rangle/\partial m_q$ from the simulations with $m_q \approx m_s$ for lattices with $N_t = 6$ and 8. A peak is seen in both cases. But the peak height does not increase when we change the spatial lattice size from $N_s = 2N_t$ to $3N_t$, indicating that we are observing a smooth crossover from a chirally symmetric phase at high temperature (large β) to a phase of broken chiral symmetry at low temperature. The location of the peak coincides approximately with the steep increase of the Polyakov line in Fig. 3. The chiral susceptibility for $m_q \approx 0.6m_s$ looks similar to Fig. 5, but in that case we have, so far, results only for one spatial size, $N_s = 2N_t$.

3. RESULTS FOR 2+1 FLAVORS

We have also preliminary results from simulations with 2+1 dynamical flavors, with the heavier quark kept at the strange quark mass, and the lighter masses at $m_{u,d} = 0.6m_s$ and $0.4m_s$. The real part of the Polyakov line and the light quark condensate from these simulations are shown in Fig. 6. Again we see a crossover behavior, particularly for the Polyakov line. For these simulations we do not have the data yet to compute the chiral susceptibility.

4. CONCLUSIONS

We have made first simulations to explore the finite temperature phase diagram with an improved staggered fermion action, the ‘‘Asqtad’’ action, which reduces flavor symmetry breaking so that all pions are lighter than the kaon already at larger lattice spacing, and which improves rotational symmetry and the dispersion relation, leading to diminished lattice artifacts in energy density and pressure.

For three flavor simulations down to quark mass $m_q \approx 0.6m_s$, and for 2+1 flavor simulations with the

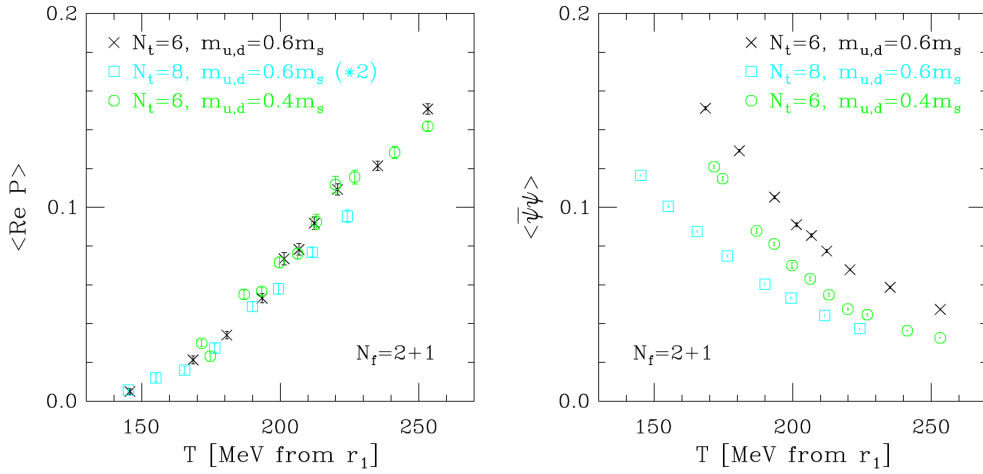


Figure 6. Real part of the Polyakov line (left) and $\langle \bar{\psi}\psi \rangle$ for the light quarks (right) for 2+1 flavors. The Polyakov line data for $N_t = 8$ has been multiplied by two.

light quark masses down to $m_{u,d} = 0.4m_s$ while the heavier quark is kept at the strange quark mass, we observed finite temperature crossover behavior, but so far no sign of a genuine phase transition. This result is compatible with other recent simulations [10,11] which found phase transitions only at quark masses lighter than those studied by us so far. The temperature at the crossover, with the scale set by r_1 , $T_c \sim 190 - 200$ MeV, is a little higher than expected from previous determinations [7,8]. This is presumably also due to our quark masses still being rather high.

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