

Topological Susceptibility with Three Flavors of Staggered Quarks

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As one test of the validity of the staggered-fermion fourth-root determinant trick, we examine the suppression of the topological susceptibility of the QCD vacuum in the limit of small quark mass. The suppression is sensitive to the number of light sea quark flavors. Our study is done in the presence of 2+1 flavors of dynamical quarks in the improved staggered fermion formulation. Variance-reduction techniques provide better control of statistical errors. New results from staggered chiral perturbation theory account for taste-breaking effects in the low-quark mass behavior of the susceptibility, thereby reducing scaling violations from this source. Measurements over a range of quark masses at two lattice spacings permit a rough continuum extrapolation to remove the remaining lattice artifacts. The results are consistent with chiral perturbation theory with the correct flavor counting.

1. INTRODUCTION

The continuum Euclidean partition function for QCD, restricted to topological charge sector ν with n_f degenerate flavors is

$$Z_\nu = \int [dU] \exp[S(U)] \det[M(U)]^{n_f}$$

$$\det[M(U)] = m^{|\nu|} \prod_n (\lambda_n^2 + m^2) \quad (1)$$

where U is the gauge field, $S(U)$, the Yang-Mills action, and λ_n are the nonzero eigenvalues of the Dirac operator $D(U)$, where $M(U) = D(U) + m$ [1]. As the quark mass m vanishes, nonzero values of ν are suppressed and the topological susceptibility

$$\chi = \langle \nu^2 \rangle / V = \sum_\nu \nu^2 Z_\nu / ZV$$

for Euclidean four-volume V vanishes. Chiral perturbation theory predicts

$$1/\chi = 2/\mu f^2 \sum_i n_i/m_i \quad (2)$$

for several flavors of degeneracy n_i and mass m_i .

The lattice partition function for staggered fermions has a similar form. But to compensate for the unwanted four-fold “taste” degeneracy, it is common to replace $\det[M(U)]^{n_f}$ by $\det[M(U)]^{n_f/4}$ (fourth-root trick) with the hope that the continuum limit recovers Eq. (1). Two principal lattice artifacts modify Eq. (2) [2]. First, topological charge counting becomes ambiguous on the scale of the lattice spacing. Second, the spectrum of eigenvalues of the fermion matrix $M(U)$ is modified so that would-be zero modes are shifted away from zero and the four-fold taste degeneracy required by the fourth-root trick is only approximate [3].

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$am_{u,d} / am_s$	$10/g^2$	L	lats.
quenched	8.00	20	408
0.05 / 0.05	6.85	20	425
0.04 / 0.05	6.83	20	351
0.03 / 0.05	6.81	20	564
0.02 / 0.05	6.79	20	484
0.01 / 0.05	6.76	20	658
0.007 / 0.05	6.76	20	493

$am_{u,d} / am_s$	$10/g^2$	L	lats.
quenched	8.40	28	396
0.031 / 0.031	7.18	28	496
0.0124 / 0.031	7.11	28	527
0.0062 / 0.031	7.09	28	592
0.0031 / 0.031	7.08	40	80

The staggered fermion artifacts can now be accommodated within the framework of staggered chiral perturbation theory [4], and, as we show from a numerical study with improved staggered fermions, improves the approach to the continuum limit, where we find a result consistent with Eq. (2).

2. STAGGERED CHIRAL PERTURBATION THEORY

We summarize the derivation of the leading $S\chi$ PT formula for the susceptibility and refer the reader to Ref. [4] for details. We start from the Aubin and Bernard generalization of the formalism of Lee and Sharpe [5] to obtain the effective chiral Lagrangian for staggered fermions to $\mathcal{O}(a^2)$:

$$\begin{aligned} \mathcal{L} = & \frac{f^2}{8} \text{Tr}(\partial_\mu U^\dagger \partial_\mu U) - \frac{\mu f^2}{4} \text{Tr}[\mathcal{M}(U^\dagger + U)] \\ & + \frac{m_0^2}{2} \phi_{0I}^2 + a^2 \mathcal{V}(U) \end{aligned}$$

The chiral field U describes N flavors of 4 tastes each and ϕ_{0I} is the flavor-singlet, taste-singlet field, \mathcal{M} is the taste-degenerate quark mass matrix, and $\mathcal{V}(U)$ is the dimension-six potential [5], which breaks the continuum taste symmetry. Only one Goldstone pion in the sixteen-member taste-multiplet survives. The ‘‘pion’’ with the highest mass is a taste singlet.

We work to quadratic order in the fields and in the mean-field approximation and follow Leutwyler and Smilga [1] to obtain the susceptibility with a degeneracy of n_i flavors of four tastes each:

$$\chi = \langle \nu^2 \rangle / V = \frac{f^2/16}{\sum_{i=1} n_i (1/m_{iI}^2 + 1/m_0^2)}$$

where the bare flavor-neutral, taste-singlet pseudoscalar masses are

$$m_{iI}^2 = 2\mu m_i + a^2 \Delta_I$$

in terms of the bare quark masses m_i and the taste splitting Δ_I . To eliminate the taste degeneracy, leaving only two degenerate u and d quarks and one s quark we put $n_{ud} = 1/2$ and $n_s = 1/4$, giving, finally

$$\chi = \frac{f^2 m_{\pi,I}^2 / 8}{1 + m_{\pi,I}^2 / 2m_{ss,I}^2 + 3m_{\pi,I}^2 / 2m_0^2} \quad (3)$$

This leading-order formula is identical to that of continuum chiral perturbation theory, except that it requires the taste-singlet meson masses. It interpolates [6] between the perturbative and quenched regimes and reproduces the Witten-Veneziano formula at infinite meson mass [7].

3. SIMULATION AND ANALYSIS

The topological charge density was measured as described in [2] on an ensemble of lattices (quenched and with up, down, and strange improved Asqtad staggered quarks) of two lattice spacings: ‘‘coarse’’ ($a \approx 0.125$ fm) and ‘‘fine’’ ($a \approx 0.09$ fm), as listed in the upper and lower tables respectively [8]. Included in this analysis is a new fine-lattice $40^3 \times 96$ ensemble with u and d quark masses at 1/10 the nominal strange quark mass.

To determine the susceptibility, we use a variance reduction technique [9], based on the observation that with large lattice volumes we gain statistically by writing the susceptibility as a local, intensive observable, rather than a global observable:

$$\langle \nu^2 \rangle / V = \sum_r C_{\text{meas}}(r),$$

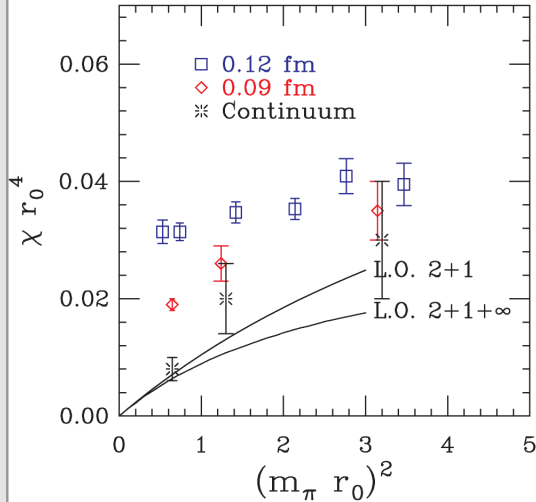


Figure 1. Result of [2]. The curves are from Eq. (3) with (L.O. 2+1+ ∞) and without (L.O. 2+1) the m_0^2 term.

where the measured topological charge density correlation function, is

$$C_{\text{meas}}(r) = \langle \rho(0)\rho(r) \rangle$$

and ρ is the topological charge density operator.

The correlator has the asymptotic form [10]

$$C_{\text{fit}}(r) = b_\eta D(m_\eta, r) + b_{\eta'} D(m_{\eta'}, r) + \dots$$

where $D(m, r)$ is the Euclidean scalar propagator.

We fit the correlator to this form for large $|r|$ and use it to model the large distance behavior, thereby reducing the variance due to contributions at large $|r|$:

$$\langle \nu^2 \rangle / V = \sum_{|r| \leq |r_{\text{cut}}|} C_{\text{meas}}(r) + \sum_{|r| > |r_{\text{cut}}|} C_{\text{fit}}(r).$$

4. RESULTS

As we have seen, at lowest order in chiral perturbation theory the staggered fermion artifact is removed by writing the susceptibility as a function of the taste-singlet pion mass, rather than the Goldstone pion mass. The improvement can be seen by comparing Figs. 1 and 2.

Residual lattice artifacts come from the lattice operator definition of topological charge density.

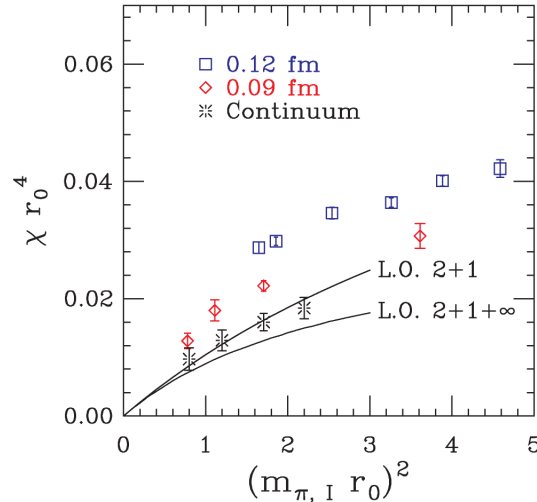


Figure 2. New result.

It is plausible that they scale as $\mathcal{O}(a^2)$. The continuum extrapolation of Fig. 2 at fixed abscissa is based on that assumption.

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REFERENCES

1. See, for example, H. Leutwyler and A. Smilga, Phys. Rev. D **46** (1992) 5607.
2. C. Bernard *et al.*, Phys. Rev. D **68** (2003) 114501
3. E. Follana, A. Hart and C. T. H. Davies, arXiv:hep-lat/0406010; S. Dürr, C. Hoelbling and U. Wenger, arXiv:hep-lat/0406027.
4. B. Billeter, C. DeTar, and J. Osborn, arXiv:hep-lat/0406032.
5. W. J. Lee and S. R. Sharpe, Phys. Rev. D **60** (1999) 114503; C. Aubin and C. Bernard, Phys. Rev. D **68** (2003) 034014 and 074011.
6. S. Dürr, Nucl. Phys. B **611** (2001) 281
7. E. Witten, Nucl. Phys. B **156** (1979) 269; G. Veneziano, Nucl. Phys. B **159** (1979) 213.
8. C. Aubin *et al.*, arXiv:hep-lat/0402030.
9. MILC collaboration (in preparation).
10. E. V. Shuryak and J. J. M. Verbaarschot, Phys. Rev. D **52** (1995) 295.