Discrete spatial filtering with SQUID gradiometers in biomagnetism

A. C. Bruno, P. Costa Ribeiro, and J. P. von der Weid
Department of Physics, Pontificia Universidade Catolica, Rio de Janeiro, Brazil
O. G. Symko
Department of Physics, University of Utah, Salt Lake City, Utah 84112

(Received 4 April 1985; accepted for publication 6 December 1985)

First-, second-, and third-order gradiometers used in detecting biomagnetic signals are analyzed as spatial filters. Their transfer functions independent of the source to be measured are presented and both the magnitude and phase characteristics of the transfer functions are analyzed. The distortion introduced by the gradiometer can be estimated from these characteristics. In order to treat the signal in that approach, the spatial Fourier transform of a magnetic signal produced by a current dipole at a given distance is discussed.

INTRODUCTION

For a number of years SQUID magnetic sensors[1] have been used in biomagnetism research and have been instrumental in significant achievements in the field. Usually the signal is coupled to the sensor by a flux transformer which has the detection coil in a gradiometer configuration for spatial discrimination between signal and noise.[2] This technique is also used in other fields of research involved in the detection of relatively weak magnetic fields, the monopole search being one of them.[3,4]

A gradiometer can be basically described as an array of coils, the lowest one, called the "pick-up" coil or "face loop,"[5] being primarily sensitive to the near source of signal and the others acting as "compensating coils" to eliminate the far sources of noise. Such a simplification has to be abandoned when a quantitative result is needed, as in the case for the localization of brain sources; the influence of the "compensating coils" on the signal should then be taken into account.[6]

Usually each laboratory working on biomagnetism has its own gradiometer, described as being of first,[7] second,[8,9] or third order[10,11] with given distances between coils, number of turns, and area of each coil. The choice for the actual values of these parameters has been rather qualitatively dictacted by the kind of local background noise and signal to be measured. Usually in a comparison of coils, the assessment is based on noise level achieved by these different systems and is presented as a value in $\mathrm{fT}/\sqrt{\mathrm{Hz}}$ for the equivalent field spectral density or power spectrum of the noise in the pick-up coil.

With such an approach it is difficult to know the exact noise rejection of each system (the actual background noise is not known), thus making it difficult to compare them. Also, if the source is not just a current dipole (as is the case of the heart), the signal spatial dependence is not known. Such shortcomings have been partially overcome by the approach of Wikswo[5] and Storey.[12] They used the reciprocity principle to make a comparison between different gradiometer designs for noise rejection in the presence of magnetic dipole sources only. The sensitivity of various gradiometer configurations to only current dipole sources has also been calculated.[5]

We present a more general approach[13] to this problem by describing analytically the gradiometer in terms of its transfer function irrespective of the kinds of signal or noise. Our approach is such that gradiometers are high-pass discrete spatial filters whose transfer function enables a comparison to be made between the different gradiometer configurations. Such transfer functions will help to select the best design for a given signal and noise, as well as to determine the effect of the "compensating coils" on the signal.

The fundamental idea in the use of gradiometers to discriminate the signal from the noise is that in a given region of space, the spatial magnetic field dependence can be fitted by a polynomial (Taylor series). Far sources (noise in our applications) are fitted to only one to three terms of the polynomial, since just the field and its first derivatives are predominant, while near sources (the signal in our case) will require a larger number of terms. A gradiometer makes the spatial discrimination by canceling the first terms of the polynomial.

Although it is not customary procedure, the monotonic magnetic field spatial dependence can be fitted just as well by a Fourier expansion. In such an approach, spatial discrimination is reached by using the fact that a distant source will have low frequency components of its *spatial* frequency spectrum, while near sources will have low and high frequency components. The spatial discrimination is then achieved by using a spatial high-pass filter.

The use of spatial filters is well known in other fields, such as optics[14] or tomography,[15] and can be instructive and useful to apply to magnetic field studies in biophysics. This is the goal of this paper. The gradiometer is presented as a nonrecursive spatial filter with its own transfer function; then this approach is compared to the conventional Taylor series approach. Finally, the spatial Fourier transform of the signal is considered, to show how the various gradiometers (i.e., filters) will respond to it.

THE GRADIOMETER SENSOR: A TAYLOR SERIES APPROACH

The general configuration of an Nth order gradiometer is shown in Fig. 1. It consists of an array of $N+1$ coils connected in series, each with n_i turns, and separated by the

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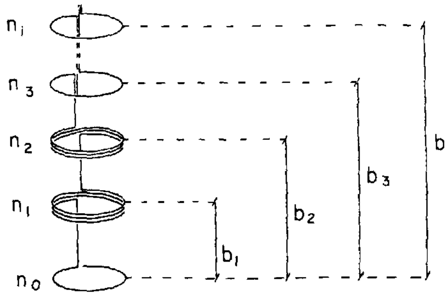

FIG. 1. Gradiometer of Nth order.

distance b_i from the lowest coil. For simplicity, we consider the case where they all have the same area A. The equations that characterize such a gradiometer are derived by considering the total flux $\phi(z_0,t)$ applied to the array, with the lowest coil at z_0 from the source,

$$\phi(z_0,t) = f(z_0)g(t) = A\left(\sum_{i=0}^{N} n_i B(z_i)\right)g(t), \quad (1)$$

where $B(z_i)$ is the magnetic field component normal to the plane of the coil, z_i is the distance from the ith coil to the origin, and $g(t)$ is the time dependence of the flux, expressed separately since biomagnetic fields are quasistatic.

Expanding $B(z)$ in a Taylor series about the position z_0 of the lowest coil and letting $b_i = z_i - z_0$, Eq. (1) can be written as

$$\phi_{\rm total}(z_0,t) = A\{n_0 B(z_0) + n_1[B(z_0) + B^{(1)}(z_0)b_1 + B^{(2)}(z_0)(b_1^2/2) + \cdots] + n_2[B(z_0) + B^{(1)}(z_0)b_2 + B^{(2)}(z_0)(b_2^2/2) + \cdots] + \cdots\}g(t). \quad (2)$$

This can be arranged in matrix form as

$$\phi_{\rm total}(z_0,t) = A\,[111\ldots]\begin{bmatrix} n_0+n_1+n_2+\cdots & & & 0 \\ & n_1b_1+n_2b_2+\cdots & & \\ & & n_1\frac{b_1^2}{2}+n_2\frac{b_2^2}{2}+\cdots & \\ & & & \ddots \\ 0 & & & \end{bmatrix}\begin{bmatrix} B(z_0) \\ B^{(1)}(z_0) \\ B^{(2)}(z_0) \\ '' \\ '' \\ '' \end{bmatrix} g(t). \quad (3)$$

The above diagonal matrix expresses the effects of the array of coils on the field and its derivatives $B^{(1)}, B^{(2)}, \ldots$ at z_0, the pick-up coil position. A gradiometer of Nth order will result when the *first N diagonal terms* are canceled, that is imposing the condition,[16]

$$\sum_{i=0}^{N} n_i \frac{b_i^\alpha}{\alpha!} = 0, \quad \alpha = 0,\ldots,N-1. \quad (4)$$

Such conditions are fulfilled by all existent gradiometers, even by the so-called asymmetric ones that use different areas for the sets of coils, and where n_i is not limited to be an integer.

THE GRADIOMETER SENSOR: A NONRECURSIVE SPATIAL FILTER APPROACH

A gradiometer detects a continuous time signal, sampling it at discrete points in space b_i which correspond to the coils' positions. The net detected signal, at each instant of time, is a weighted sum of all those sampled values. Thus it seems appropriate to use a digital mathematics formalism for describing the gradiometer sensor.

In such an approach, the output of a so-called moving average or nonrecursive filter is generally expressed as[17]

$$y_m = \sum_{i=-\infty}^{\infty} h_i x_{m-i}. \quad (5)$$

This procedure defines a new set of numbers y_m from the set of numbers x_m which correspond to the sampled values at constant intervals. The weighting factors h_i depend on the filter.

The discrete Fourier transform (DFT) of the signal (see for example Ref. 17) at some instant of time can then be expressed as

$$X(k) = \sum_{m=-\infty}^{\infty} x_m e^{-jm\lambda_s k}, \quad (6)$$

where k is the wave number and λ_s is the distance between two successive sampling points.

For our case of the gradiometer, assume that the distances b_i can be expressed as multiples of λ_s,

$$b_i = \beta_i \lambda_s, \quad (7)$$

where β_i has integer values and λ_s are the maximum common divisors of all the b_i. The signal detected by the gradiometer at position z_m over all its space will be

$$f(z_m) = \sum_{i=0}^{N} n_i \phi(z_m + b_i),$$

or considering the digital nature of the detection scheme,

$$f_m = \sum_{i=0}^{N} n_i \phi_{m+i}, \quad (8)$$

where $N+1$ is the number of coils, and $\phi(z_m + b_i) = \phi_{m+i}$ is the magnetic flux at the ith coil at some instant of time and at a given discrete position in space, labeled by the index m.

The transfer function of the filter (gradiometer) in k space defined as

$$H(k) = F(k)/\Phi(k),$$

where $F(k)$ and $\Phi(k)$ are, respectively, the DFT of f_m and ϕ_m, can be obtained by taking the Fourier transform of the sequence of filter coefficients

$$H(k) = \sum_{i=0}^{N} n_i e^{-jb_i k}. \quad (9)$$

Spatial discrimination is accomplished by making the moving average filter (gradiometer) into a high-pass filter. The correctness of such an approach is shown by the fact that the design conditions for the successive higher-order gradiometers obtained by canceling the successive elements of the square matrix in Eq. (3) are obtained, as well, by imposing increasing flatness in the stop-band region of the high-pass filter transfer function. For instance, the design condition for a first-order high-pass filter is obtained by making Eq. (9) equal to zero for $k = 0$. This yields

$$\sum_{i=0}^{N} n_i = 0, \quad (10)$$

which is the same condition as for a first-order gradiometer in the Taylor series approach. For a more efficient high-pass filter the design conditions are obtained by making $H(0) = 0$ and also $|\partial H(k)\partial k| = 0$ at $k = 0$, i.e., the extra condition of

$$\sum_{i=0}^{N} n_i b_i = 0, \quad (11)$$

which is a second-order gradiometer in the Taylor series approach. Generalizing, the gradiometer condition given by Eq. (4) in the Taylor series approach corresponds to imposing

$$\left| \frac{\partial^\alpha}{\partial k^\alpha} H(k) \right|_{k=0} = 0 \quad (12)$$

in the spatial Fourier technique.

The equivalence of both approaches is illustrated by considering the Fourier transform for a spatially constant field and a linearly varying field in space.

For a spatially uniform field, normalized so as to have unit value over all space,

$$\phi_m = 1, \quad -\infty < m < \infty, \quad (13)$$

its Fourier transform becomes[18]

$$\Phi(k) = (2\pi/\lambda_s)\delta(k), \quad (14)$$

$\delta(k)$ being a delta function.

Each discrete measurement of the filter at a given position $m\lambda_s$ will be

$$f_m = \frac{\lambda_s}{2\pi} \int_{-\pi/\lambda_s}^{\pi/\lambda_s} F(k) e^{jm\lambda_s k}\, dk, \quad (15)$$

which becomes

$$f_m = H(0). \quad (16)$$

Thus the condition $H(0) = 0$ eliminates the detection of a spatially constant field.

For a linearly varying field in space,

$$\phi_m = am\lambda_s, \quad -\infty < m < \infty, \quad (17)$$

its Fourier transform will be

$$\Phi(k) = \sum_{m=-\infty}^{\infty} am\lambda_s e^{-jm\lambda_s k}. \quad (18)$$

It can be written as

$$\Phi(k) = ja \frac{\partial}{\partial k} \sum_{m=-\infty}^{\infty} e^{-jm\lambda_s k}, \quad (19)$$

i.e., the Fourier transform is the derivative of the delta function. Hence the output of the filter will be

$$f_M = -ja \left.\frac{\partial H(k)}{\partial k}\right|_{k=0} + am\lambda_s H(0). \quad (20)$$

Thus to discriminate against a spatially linear field we should impose the condition of the transfer function and its first derivative at $k = 0$ to be equal to zero. A similar procedure would be used for higher orders.

Figure 2 shows the magnitude of the transfer functions for conventional first-, second-, and third-order geometries. All the gradiometers have the same λ_s and K_0 is π/λ_s. The behavior in the DFT approach is the inverse of the Fourier series: the space domain is discrete and the frequency domain is periodic with a period of $2\pi/\lambda_s$. Since it is an even function, it is only necessary to represent the half-period dependence, from 0 to π/λ_s.

PHASE CHARACTERISTICS

So far we have dealt with the amplitude response $|H(k)|$ of the gradiometer. There is also a phase response $\theta(k)$, although at first glance it is not as easy to interpret. Figure 3

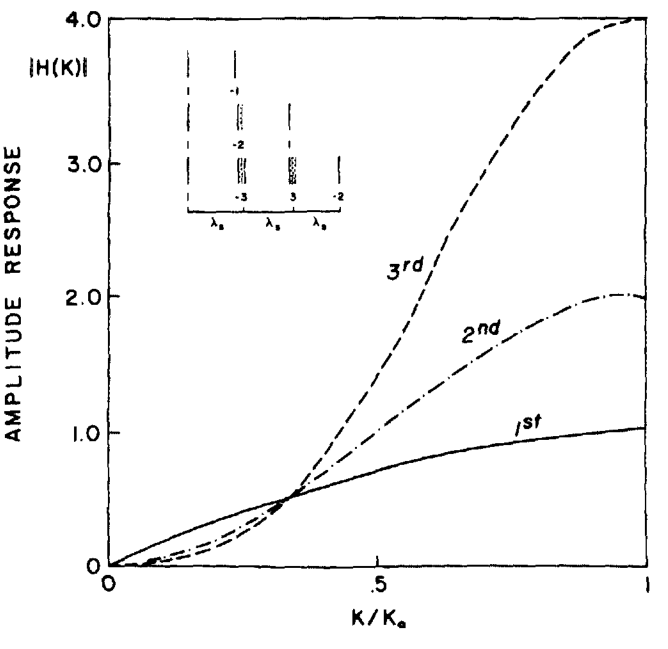

FIG. 2. Amplitude response of first-, second-, and third-order gradiometer in k space ($K_0 = \pi/\lambda_s$).

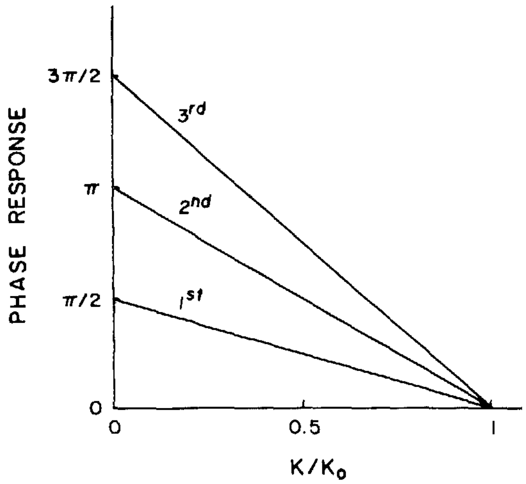

FIG. 3. Spatial phase response of first-, second-, and third-order gradiometers in k space ($K_0 = \pi/\lambda_s$).

shows the calculated phase characteristics for the first-, second-, and third-order gradiometers whose amplitude transfer functions are presented in Fig. 2. These graphs were obtained from the definition of the phase between input and output signals:

$$\phi = \tan^{-1}[\operatorname{Im} H(k)/\operatorname{Re} H(k)]. \qquad (21)$$

Since we are dealing with a spatial frequency response of the system, the phase information leads to a spatial lag between the output signal and the input signal. Indeed, as can be seen in Fig. 3, a gradiometer is a linear phase spatial filter. Such characteristics are important for the evaluation of distortion produced by our filter. The largest phase lag occurs at the frequency where the gain of the system changes the most with respect to frequency. The spatial lag that will occur, especially at low frequencies, can be used to estimate the error introduced by the gradiometer in determining the depth of a current dipole.[6]

SPATIAL FOURIER TRANSFORM OF THE SIGNAL

The noise output of gradiometers of different orders has been discussed above, using the gradiometers' transfer functions and characterizing the noise by their dominant low-order terms in a Taylor expansion. It is important now to discuss the Fourier transform of a given signal from a near source, in order to understand more clearly the meaning of Fig. 2.

The handling of a continuous signal by a digital mathematical formalism is a twofold problem. First, a sampling frequency must be chosen, and second, the infinite sequence generated must be truncated in order to apply the DFT.

Since the sampling frequency must be larger than twice the highest frequency component of the signal (Nyquist theorem),[17,18] we have to estimate the highest frequency component. For the case of magnetic or current dipole, care has to be taken in making such estimates as both are spatially monotonic functions. Certainly, the highest frequency component will be related to the distance from the dipole to the first point of measurement, i.e., first coil position. The DFT is periodic with a period of $2\pi/\lambda_s$. If the sampling frequency chosen is smaller than the Nyquist limit, there will be an overlap between the periodic replicas of the spectrum (aliasing effect).[19]

As to the truncation problem, consider an extreme case where the truncated sequence, representing the spatial dependence of the signal, has only two terms. Assume that the field decreases by a factor Q from the first term to the second term, i.e.,

$$\phi_1 = 1, \phi_2 = (1/Q). \qquad (22)$$

The DFT of such a sequence will be

$$\Phi(k) = 1 + (1/Q)e^{-jk\lambda}. \qquad (23)$$

Its modulus is

$$|\Phi(k)| = [1 + (1/Q^2) + (1/Q)\cos k\lambda\,]^{1/2}. \qquad (24)$$

For a spatially constant field $Q = 1$ (noise),

$$|\Phi(k)| = 2\cos k(\lambda/2), \qquad (25)$$

while for a field which decays spatially very fast $Q = \infty$ (the signal),

$$|\Phi(k)| = 1. \qquad (26)$$

Those two cases and intermediate ones are shown in Fig. 4.

Hence, if a constant sequence such as the one produced by the noise ($Q = 1$) is defined by only two terms, its DFT is not a delta function as in Eq. (14). Also, if the field decays very fast in space, as would be the case for a signal from a very near source that would be detected just by the pick-up coil ($Q = \infty$), a two-term truncation leads to a constant pattern in the frequency domain. Thus, the truncation can produce as well as the aliasing effect, spurious high frequency components in the spectrum making it difficult to interpret the plots of the filter transfer function (Fig. 2), and the signal and noise frequency spectrum (Fig. 4).

Fortunately the problems of choosing a sampling frequency and the number of terms of Eq. (6) can be avoided because one can have the analytical expression for the spatial dependence of the signal source. Therefore, one can use it to

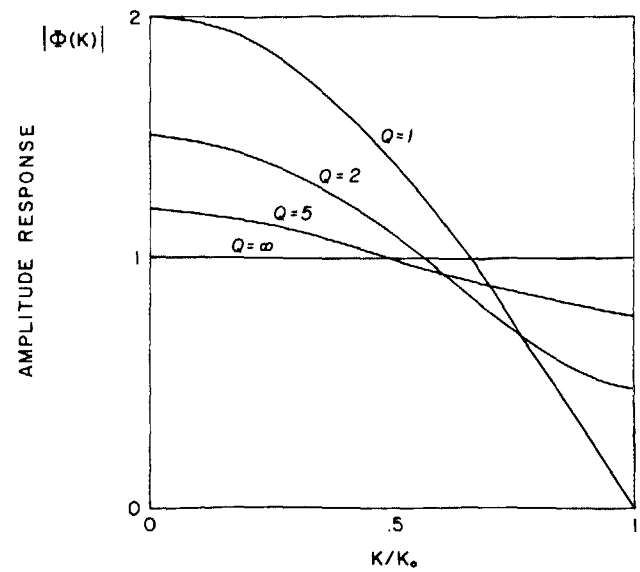

FIG. 4. Amplitude response for a truncated signal with only two terms differing by a factor of Q.

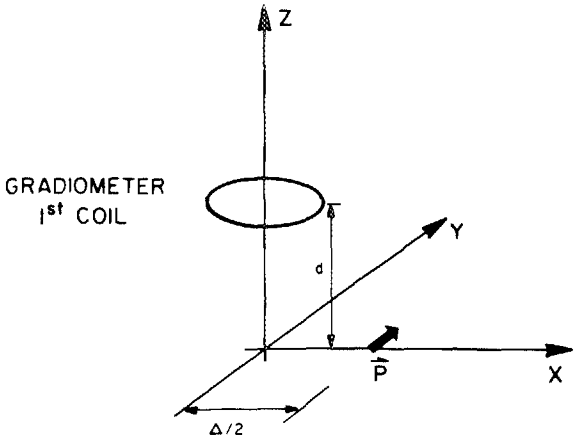

FIG. 5. Parameters used in the calculation of the field produced by a current dipole.

calculate the continuous Fourier transform of the signal. The continuous Fourier transform and the half-period of the DFT are identical for a sampling frequency and number of terms chosen in a proper way.[17]

The analytical spatial dependence of a near source signal, for instance a current dipole, can be obtained by applying the Biot–Savart law in the situation illustrated in Fig. 5. Let us assume that the field is initially measured at the point z_0 in a plane distant d from the current dipole. The current dipole $\mathbf{p}$ is located in the plane xy, oriented in the $+y$ direction,[20] at a distance $x = \Delta/2$ from the z axis, Δ being the distance between field extremes in the $z_0 = d$ plane.

Thus the field can be expressed as

$$B_z(z) = \frac{\mu_0 |\mathbf{p}| d}{4\pi\sqrt{2}} \frac{1}{(d^2/2 + z^2)^{3/2}}. \quad (27)$$

Consequently, one can have the Fourier transform by applying the definition for each spatial frequency k,

$$X(k) = \int_{-\infty}^{\infty} B_z(z) e^{-jkz}\, dk, \quad (28)$$

which can be evaluated numerically.

Now we may determine a maximum frequency of interest in the Fourier transform of the signal, in order to give a visual meaning to Fig. 2. In this case, we may choose $k_{\max}$, so that most of the energy, say 95%, lies inside the frequency range $(0, k_{\max})$. In the Fourier series, the sum of $|a_n|^2$ yields the average power.[18] In the Fourier transform, the integration of $|X(k)|^2$ yields the total energy of the signal.[18] We can choose $k_{\max}$ using the following criteria:

$$\int_0^{k_{\max}} |X(k)|^2\, dk = 0.95 \int_0^{\infty} |X(k)|^2\, dk. \quad (29)$$

The values of the maximum frequency $k_{\max}$ determined by Eqs. (27)–(29) are depicted in Fig. 6 as a function of distance d from 3–10 cm. As can be seen one can use approximately π/d as $k_{\max}$.

Finally, one can obtain the first design criteria using this new model. As was said before, the gradiometers' transfer function should be represented in k space up to π/λ_s, where λ_s is the minimum distance between two adjacent coils. Thus if one equals $k_{\max}$ to π/λ_s, one can obtain the distance between coils that the gradiometer should have so as to give the signal's maximum detectable energy. The minimum distance between coils as a function of distance d is shown in Fig. 7. It should be stressed that this result is in accordance with the Taylor approach. If a gradiometer is to act as a magnetometer for near sources, the base line should be of at least the order of the distance between the source and the gradiometer.[5,6] It must be remembered, nevertheless, that the final criteria used to choose the gradiometer's base line must also take into account the noise as well. The combination of amplitude response and phase response can be used to determine the amount of distortion introduced by the gradiometer; this distortion will, for example, affect the determination of the depth of the current dipole source and its strength.

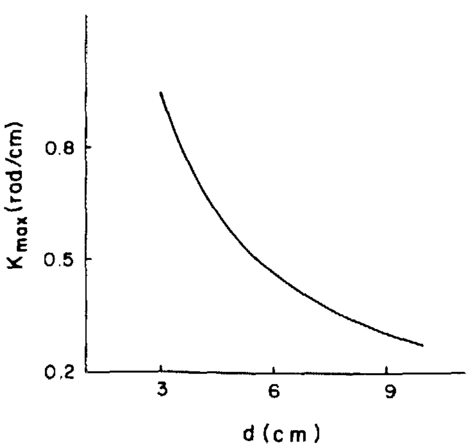

FIG. 6. Spatial frequency limit ($K_{\max}$) as a function of the distance between the current dipole and the gradiometer first coil (d).

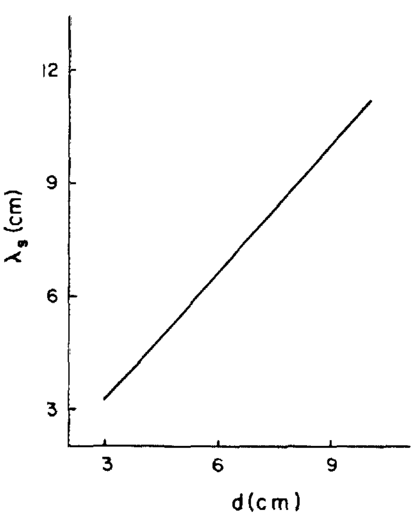

FIG. 7. Minimum distance between two adjacent coils of a gradiometer as a function of the distance between the dipole and the gradiometer first coil (d).

CONCLUSIONS

This paper presents a novel approach to understanding and designing of gradiometer coils for the detection of weak biomagnetic signals in the presence of noise. The gradiometer is considered as a spatial filter whose characteristics are determined by its transfer function independent of the model used for the source. The transfer function yields the amplitude response and the phase response. The amplitude response gives results in agreement with the Taylor series approach but which are more general; it is a system analysis approach. As an example in the selection of gradiometer characteristics based on the amplitude response of various gradiometers, the spatial Fourier transform of a dipolar signal is presented. Using the approach described in this paper, it will be possible to design gradiometers making trade offs between signal and noise energies for a specific spatial situation of weak magnetic fields and their source localization. The dynamic error[21] introduced by the gradiometer can be estimated by inspection of the amplitude response (Fig. 2) and the phase response (Fig. 3).

ACKNOWLEDGMENTS

We gratefully acknowledge discussions with Professor A. Mesquita. This work was supported by CNPq, Brazil, and the National Science Foundation (International Division).

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