Manifestations of Berry's Topological Phase for the Photon

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Recently, Berry recognized in quantum mechanics a topological phase factor arising from the adiabatic transport of a system around a closed circuit, which is essentially the Aharonov-Bohm effect in parameter space. Here we consider manifestations of this phase factor for a photon in a state of adiabatically invariant helicity. An interferometer is suggested to see this phase. Also, an effective optical activity for a helical optical fiber is predicted. These effects emerge on a classical level as topological features of Maxwell's theory.

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Quantum interference effects in the presence of topologically nontrivial electromagnetic fields often lead to spectacular phenomena in physics. Notable examples include the Aharonov-Bohm effect,[1] the Dirac monopole and the quantization of charge,[2] and the quantization of flux in superconductivity.[3] As Wu and Yang[4] have emphasized, at the heart of these phenomena lies the nonintegrable phase factor $\exp(ie\int_C A_l dx^l)$, which multiplies the wave function of the system after its transport around a closed curve C in the presence of a vector potential A_l in *ordinary space.*

Recently it has been recognized that in quantum mechanics there exists another analogous topological phase factor, namely Berry's phase.[5] This nonintegrable phase factor arises from the adiabatic transport of a system around a closed path in *parameter space,* which, according to Simon,[6] can be viewed as parallel transport in the presence of a gauge field in such spaces. This phase factor is amazingly universal. It has appeared theoretically in many contexts,[7] e.g., a treatment of the Born-Oppenheimer approximation,[8,9] fractional statistics,[10,11] and anomalies in gauge field theories.[12–15] Its domain of applicability thus apparently ranges from high-energy physics to low. Clearly it is important to look for this abstract phase factor experimentally. Therefore we would like to explore here some physical manifestations, more specifically, optical manifestations, of this phase factor.

A striking prediction of Berry[5] is that any spin-1 particle, a *boson,* can acquire a phase factor of -1 under certain rotations which return it to its original state classically. For example, the direction of a magnetic field can be slowly rotated through a cone of apex angle 120°, so that the spin's magnetic moment follows it adiabatically. After the magnetic field has returned to its initial direction, the spin's wave function has changed sign relative to that of an identical spin which has remained in an unchanged magnetic field. This sign change can manifest itself in the destructive interference between two beams of spins. Note the dependence of this phase on the spin's *history,* i.e., its nonintegrability.

The photon is a *massless* spin-1 boson. Its helicity $\mathbf{s}\cdot\mathbf{k}$, where $\mathbf{s}$ is the spin operator and $\mathbf{k}$ is the direction of its propagation, can only be $+1$ or -1. It is natural to ask whether the photon can acquire a Berry phase or not. In principle we can replace in the above example the direction of the magnetic field, (B_x,B_y,B_z), to which the photon does not couple, by the direction of the propagation of the photon, (k_x,k_y,k_z), which *can* be affected by slowly varying changes in the external environment (e.g., in the index of refraction), as the slowly varying parameters. Here parameter space is *momentum space,* or equivalently, *reciprocal space.* In contrast to the case of a *massive* spin-1 boson, the spin of the photon will always follow the direction of $\mathbf{k}$, since the masslessness of the photon guarantees that its helicity will remain either $+1$ or -1, if there is nothing to change the sign of the helicity. Thus the helicity quantum number is an adiabatic invariant.

We discuss below three cases in which $\mathbf{k}$ can change adiabatically: (1) when circularly polarized light propagates down a helically wound optical fiber, (2) when linear polarized light propagates down such a fiber, and (3) when microwaves propagate down a helically wound circular waveguide. In all three cases, it is required experimentally that there should be no sharp kinks (on the scale of a wavelength) in the fiber or waveguide, so that the helicity of the photon does not flip sign as it propagates. Also, we neglect any linear birefringence in the medium, and any ellipticity in the cross-sectional shape of the waveguide, which can cause conversion between states of opposite helicity.

We assume that the light propagates inside the twisting waveguide in a single mode. Let us parametrize its path by τ, the optical path length. Then the adiabatic invariance of the helicity of the photon implies that at each point τ, the photon's spin state $|\mathbf{k}(\tau),\sigma\rangle$ satisfies

$$\mathbf{s}\cdot\mathbf{k}(\tau)|\mathbf{k}(\tau),\sigma\rangle=\sigma|\mathbf{k}(\tau),\sigma\rangle, \quad (1)$$

where $\mathbf{k}(\tau)$ is the photon's propagation direction at τ and $\sigma=\pm1$ is its helicity quantum number, which is independent of τ. Formally, this is identical to the problem considered by Berry for a spin $\mathbf{s}$ in an adiabatically changing magnetic field $\mathbf{B}(t)$,

$$g\mathbf{s}\cdot\mathbf{B}(t)|\mathbf{B}(t),m_s\rangle=E|\mathbf{B}(t),m_s\rangle, \quad (2)$$

where g is related to the gyromagnetic ratio and m_s is the component of the spin along the direction of $\mathbf{B}(t)$. Here E is the energy eigenvalue, which is constant for the case where the magnetic field changes direction only, but is constant in magnitude. As the magnetic field is adiabatically changed, the parameters $\mathbf{B}(t)$ trace out a closed curve C on the surface of a sphere of radius B in parameter space (B_x,B_y,B_z). After one round trip in parameter space, the system must come back to its original state, apart from a dynamical phase factor $\exp(-iEt)$, which we temporarily ignore, and a geometrical phase factor $\exp[i\gamma(C)]$, where $\gamma(C)$ is Berry's phase. Berry showed that

$$\gamma(C)=-m_s\Omega(C), \quad (3)$$

where $\Omega(C)$ is the solid angle subtended by the curve C with respect to the origin $\mathbf{B}=0$. The right-hand side of Eq. (3) can be interpreted as the "magnetic flux" in *parameter space* through C in the presence of a "monopole" of strength $-m_s$ at the origin, which is a point of degeneracy. For the special case when C is a circle which subtends a cone with an apex at the origin with apex semiangle θ,

$$\gamma(C)=-2\pi m_s(1-\cos\theta). \quad (4)$$

For $\theta=60°$ and $m_s=1$, one obtains $\gamma=-\pi$, and thus the remarkable result that even bosons can acquire a phase factor of -1 after an azimuthal rotation of 360°, which *classically* restores the original state of the system. As noted above, this is an observable phase factor, which can cause destructive interference.

Now we extend these results to the photon. As it propagates smoothly down a helical waveguide, $\mathbf{k}$ is constrained to remain parallel to the local axis of this waveguide, since the momentum of the photon is in this direction. Since its helicity is adiabatically conserved, $\mathbf{s}$ is also constrained to remain parallel to the local axis of the waveguide. Hence the geometry of a helical path of a waveguide with a unity winding number constrains $\mathbf{k}$, and hence $\mathbf{s}$, to trace out a loop C on the surface of a sphere in parameter space (k_x,k_y,k_z). As a result of radial symmetry, the origin $\mathbf{k}=0$ of this space is singular. Berry's argument leads to a phase similar to that of Eq. (3).

$$\gamma(C)=-\sigma\Omega(C), \quad (5)$$

where $\Omega(C)$ is the solid angle subtended by the loop C with respect to $\mathbf{k}=0$. In the special case of a uniform helix, C is a circle and

$$\Omega(C)=2\pi N(1-\cos\theta), \quad (6)$$

where N is the winding number of the helix, and θ is the angle between the local waveguide axis and the axis of the helix, i.e., the pitch angle of the helix. Again, the phase $\gamma(C)$ can be viewed as the result of parallel transport along C in the presence of a Dirac monopole with strength $-\sigma$ at the origin $\mathbf{k}=0$.

The phase given by Eq. (5) can be seen in the following interference experiment: A circularly polarized laser beam is injected into a single input optical fiber. This fiber in turn couples an equal amount of the light into two helically wound optical fibers, each having N turns, but in *contrary* senses, which are adjusted to have equal optical path lengths. These two helices form the two arms of the interferometer. (With balanced arms, spurious effects, e.g., local optical activity inside the medium, can be canceled out.) The fibers are then brought together and coupled into a single output optical fiber, where interference occurs. The predicted interference pattern is

$$I=I_0\cos^2[2\pi N(1-\cos\theta)], \quad (7)$$

for uniform windings.

Next, let us inject a linearly polarized laser beam into a *single* helically wound optical fiber. Let the initial state be represented by

$$|x\rangle=2^{-1/2}(|+\rangle+|-\rangle), \quad (8)$$

where $|\pm\rangle$ are the eigenstates of $\sigma=\pm1$. After propagation through the helix, the final state at the output of the fiber, if we ignore for the moment dynamical phase factors, is

$$|x'\rangle=2^{-1/2}\{\exp(i\gamma_+)|+\rangle+\exp(-i\gamma_+)|-\rangle\}. \quad (9)$$

Here γ_+ is Berry's phase for $\sigma=+1$. Therefore $|\langle x|x'\rangle|^2=\cos^2\gamma_+$. By Malus's law, this implies that the plane of polarization has been rotated by an angle which is equal to γ_+. The sense of this rotation, when one looks into the output end of the fiber, is clockwise (i.e., dextrorotatory) for a left-handed helix. This allows a direct measurement of Berry's phase. It also gives a direct physical interpretation of this phase, namely, that it is an angle of optical rotation.

These effects are topological in nature. To see this, recall that there is a monopole at $\mathbf{k}=0$.[16] The circle C

associated with the helix can be continuously deformed into a closed curve C' of any shape without changing Berry's phase, provided that the solid angle subtended by C' with respect to the monopole is unchanged. Furthermore, the diameter of the circular cross section of the fiber can in principle be scaled arbitrarily, but adiabatically, as a function of τ without affecting this result.

It should be noted in passing that the optical-fiber experiments proposed above are different from the one proposed by Berry,[5] in which the cross-sectional *shape* of an optical fiber is slowly deformed in shape-parameter space. Here the circular cross-sectional shape is kept constant, but the *direction* of light propagation is slowly changed. Hence the parameter spaces are different. Our proposed experiments are, in our opinion, simpler to carry out than Berry's.

These results also apply to microwaves propagating down a helically wound circular waveguide. Note that the interior of the waveguide can now be the vacuum, so that all medium-related effects vanish. However, Maxwell's equations with appropriate boundary conditions should yield these results classically. Hence Berry's phase manifests itself from high energies to low, from quantum to classical regimes: The monopole at $\mathbf{k}=0$ has an influence which never disappears, no matter how far or close one is to it.

Now we return to the question of the dynamical phase factor. The evolution of the spin of the photon is governed by a Hamiltonian that has a form similar to that of the spin magnetic moment in a magnetic field:

$$H(\tau)=H_0+\kappa\mathbf{s}\cdot\mathbf{k}(\tau), \tag{10}$$

where $H_0|\mathbf{k}(\tau),\sigma\rangle=E_0|\mathbf{k}(\tau),\sigma\rangle$ defines background propagation. This is the most general Hamiltonian which can be formed from the two vectors $\mathbf{s}$ and $\mathbf{k}(\tau)$ in a straight waveguide, for a massless spin-1 particle in an isotropic medium with an isotropic cross-sectional boundary. For gradual windings, it is expected that additional terms arising from the winding will be negligibly small. The coupling constant κ is to be determined by experiment. The equation of motion for the spin state of the photon is

$$i(d/d\tau)|\mathbf{k}(\tau),\sigma\rangle=H(\tau)|\mathbf{k}(\tau),\sigma\rangle. \tag{11}$$

Clearly the resulting dynamical phase factor $\exp(-iE\tau)$ depends in general on the form of H. However, the dynamical phase factors of the two arms of the interferometer, which have equal optical path lengths, are the same, and do not enter into the intensity pattern given by Eq. (7). Hence this pattern is independent of the specific form of H, and depends only on the geometrical phase factor. The separate roles of the dynamical and geometrical phase factors *can* be seen in the experiment with linearly polarized light in a single helical fiber. After propagation through the helix, the final state at the output of the fiber is, in general,

$$|x'\rangle=2^{-1/2}\{\exp[-i(E_0\tau+\kappa\tau-\gamma_+)]|+\rangle+\exp[-i(E_0\tau-\kappa\tau+\gamma_+)]|-\rangle\}. \tag{12}$$

Therefore $|\langle x|x'\rangle|^2=\cos^2(\kappa\tau-\gamma_+)$. By Malus's law, this implies that the plane of polarization has been rotated by an angle which is equal to $\kappa\tau-\gamma_+$. If the fiber were straight, γ_+ would be zero. The rotation of the plane of polarization would then be $\kappa\tau$, and must therefore arise from the optical activity of the medium, with κ being related to its optical activity coefficient, which can be measured and subtracted experimentally. Note that Berry's phase γ_+ is independent of the size of κ. Hence in cases where κ is negligible, the dynamical phase factors can be ignored and the rotation angle is just $-\gamma_+$. In summary, a fiber made out of nonoptically active material, when wound gradually into a helix, acquires an effective *global* optical activity.

Under what experimental conditions will the adiabatic theorem[17] be applicable? Firstly, the validity of Eq. (1) needs that the photon's propagation direction, $\mathbf{k}(\tau)$, be well-defined everywhere along the waveguide, and that it changes adiabatically. This leads to

$$L,R_c,R_t \gg d, \tag{13}$$

where d is the diameter of the cross section, L the total length, R_c the radius of curvature, and R_t the radius of torsion, of the path of the waveguide. Secondly, and more importantly, there is a question about the adiabatic conditions for Eq. (11). The nondiagonal matrix elements which would lead to a violation of the adiabatic theorem are given by

$$\langle\mathbf{k}(\tau),-|\partial/\partial\tau|\mathbf{k}(\tau),+\rangle=(2\kappa)^{-1}\langle\mathbf{k}(\tau),-|\partial H/\partial\tau|\mathbf{k}(\tau),+\rangle=(2nR_c)^{-1}\langle\mathbf{k}(\tau),-|\mathbf{s}\cdot\mathbf{n}(\tau)|\mathbf{k}(\tau),+\rangle, \tag{14}$$

where n is the index of refraction and $\mathbf{n}(\tau)$ is the principal normal vector of the path. Since $\mathbf{s}$ is a vector operator, by the Wigner-Eckart theorem,

$$\langle\mathbf{k}(\tau),-|\mathbf{s}\cdot\mathbf{n}(\tau)|\mathbf{k}(\tau),+\rangle=0, \tag{15}$$

i.e., $\Delta\sigma=\pm2$ is *forbidden.* Thus there is no violation of the adiabatic theorem, no matter how small κ is. This is a special situation due to the form of the Hamiltonian, Eq. (10), and the special properties of the photon (i.e., its massless, spin-1 nature). This also solves a paradox in the understanding of the applicability of the adiabatic theorem here. Normally, this theorem requires that the system evolves very slowly. But in our case the photon travels at the speed of light, so that it takes little time to go through the entire path of the waveguide. However, there is another much faster time scale, namely the time for light to cross the diameter of the waveguide. This is the time it takes for the local isotropy of the system, including its boundaries, to be communicated throughout the system locally. Also, this is the time scale on which the direction of $\mathbf{k}(\tau)$, and hence $\mathbf{s}$, is established. In short, Eq. (13) is the only relevant condition. Since the diameter of the core of an optical fiber is of the order of microns, this condition is easily satisfied experimentally.

An astute reader may point out that our optical effects could be explained in principle entirely classically in terms of Maxwell equations plus appropriate boundary conditions. Intuitively, one expects classically that the mutually orthogonal triad of vectors $\mathbf{k}$, $\mathbf{E}$, and $\mathbf{B}$ will adiabatically propagate by parallel transport inside a gradually wound isotropic fiber, thus leading to the above results. While this is correct, we point out, as a matter of principle, that the classical theory fails for low photon number when fluctuations set in, whereas our quantum theory still holds. (This is similar to the situation in Young's two-slit experiment: Classical and quantum interference patterns agree, but fluctuations are absent classically.) Also, the derivation of parallel transport starting from Maxwell's equations in the adiabatic limit for an isotropic medium, with isotropic cross-sectional boundary conditions, is not trivial. (In this connection, note that the ray-optics limit, for which parallel transport in an isotropic but inhomogeneous medium has been derived, does not apply to a single-mode optical fiber, since the diameter of the fiber can be comparable to the wavelength of light.) Furthermore, it would be nontrivial to exhibit the topological nature of these effects in such a classical treatment for more complicated geometries, such as for a nonuniform helix, or for a variable-diameter waveguide. Fundamentally, it is the *Bose* nature of the photon which permits these optical manifestations of Berry's phase to emerge on a macroscopic, classical level. Thus we would rather think of these effects as topological features of classical Maxwell theory which originate at the quantum level, but survive the correspondence-principle limit ($\hbar \rightarrow 0$) into the classical level. (This situation has an analog in quantum field theory: Namely, chiral gauge anomalies, which are known to be topological in origin, if present at the fundamental, constituent level, must survive with exactly the same amount at the composite level,[18] or in a certain decoupling limit in which some mass parameters tend to infinity.[19])

It would be interesting to see these optical effects experimentally verified.

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Note added.—After this paper was written, the second of the above predicted effects, namely, *global* optical activity in a helically wound optical fiber, was experimentally verified.[20]

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