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Integrable Open Spin Chain in Super Yang-Mills and the Plane-wave/SYM Duality

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Abstract: We investigate the integrable structures in an $\mathcal{N}=2$ superconformal $Sp(N)$ Yang-Mills theory with matter, which is dual to an open+closed string system. We restrict ourselves to the BMN operators that correspond to free string states. In the closed string sector, an integrable structure is inherited from its parent theory, $\mathcal{N}=4$ SYM. For the open string sector, the planar one-loop mixing matrix for gauge invariant holomorphic scalar operators is identified with the Hamiltonian of an integrable $SU(3)$ open spin chain. Using the K-matrix formalism we identify the integrable open-chain boundary conditions that correspond to string boundary conditions. The solutions to the algebraic Bethe ansatz equations (ABAE) with a few impurities are shown to recover the anomalous dimensions that exactly match the spectrum of free open string in the plane-wave background. We also discuss the properties of the solutions of ABAE beyond the BMN regime.

Keywords: AdS/CFT correspondence, Integrable Field Theories, Bethe ansatz.

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Contents

1. Introduction

Starting from BMN's proposal [1] on duality between the string theory on a plane-wave background and Super Yang-Mills (SYM), a number of aspects of this duality have been thoroughly investigated (e.g., see a recent review [2] and references therein). Compared with the well-known supergravity/CFT approach [3, 4] to AdS/CFT duality [5], the plane-wave/SYM duality has several remarkable features: 1) In the plane-wave limit of the $AdS_5 \times S^5$ background [6], the string theory is exactly solvable in light-cone gauge [7, 8]. This makes it possible to set up the explicit correspondence between string theory and certain sectors in large N gauge theory, beyond the gravity/CFT correspondence. 2) The plane-wave/SYM duality is perturbatively accessible from both sides. 3) On the SYM side, there exists a double scaling limit, in which one takes the rank N of the gauge group and the R-charge J of the BMN operators to infinity simultaneously, while keeping the effective coupling $\lambda' = \lambda/J^2$, rather than the 't Hooft coupling λ, fixed. This provide us a chance to study non-planar contributions in large N Yang-Mills, which correspond to string interactions in the plane-wave background.

More precisely, the tests of the duality were based on comparing the spectrum of string excitations with anomalous dimensions of the corresponding BMN operators. One difficult point in the study was that there exists operator mixing after taking quantum corrections into account. To construct generic BMN operators in SYM and evaluate their anomalous dimensions, one needed to consider one-loop mixing among a large number of gauge invariant operators [9]. A remarkable development in overcoming this difficulty was the observation made in [10] that the planar one-loop mixing matrix for anomalous dimensions in the scalar sector of $\mathcal{N}=4$ $U(N)$ SYM,

$$\Gamma_c = \frac{\lambda}{16\pi^2}\sum_{l=1}^{L}(K_{l,l+1} + 2 - 2P_{l,l+1}), \tag{1.1}$$

can be identified with the Hamiltonian of an integrable $SO(6)$ spin chain. Here $\lambda = g^2_{\rm YM}N$ is the 't Hooft coupling, K and P are the trace and permutation operator, respectively.

This Hamiltonian acts on the Hilbert space $\mathcal{H} = \otimes_{l=1}^{L} \mathcal{H}_l$, $\mathcal{H}_l = \mathbb{R}^6$ and satisfies periodic boundary conditions with $\mathcal{H}_{L+1} = \mathcal{H}_1$ (forming a closed chain). Then a powerful tool, the Bethe Ansatz [11] for one-dimensional (quantum) integrable systems, can be applied to find the spectrum of BMN operators and match them with string predictions. In addition, the algebraic Bethe ansatz equation (ABAE) can be solved even in the case with finite J. (For further developments using the dilatation operator of SYM, see ref. [12].)

More astoundingly, the integrable structure (1) in $\mathcal{N} = 4$ SYM has far-reaching implications for strings beyond the plane-wave limit. Alternatively, the BMN states could be viewed [13] as quadratic fluctuations of a semi-classical string in $AdS^5 \times S^5$. In the BMN case, the semi-classical solution is near-BPS. However, the BPS conditions turned out to be not essential, and there exist semi-classical sectors, far from BPS, which could also be used for precise tests of AdS/CFT correspondence. The duality between these so-called spinning strings and "long" scalar composite operators in gauge theory has been confirmed with the help of solving the ABAE of $SO(6)$ spin chain in the thermodynamic limit. (For a recent review, see [14] and references therein). Moreover, the integrable spin chains in Wess-Zumino models and orbifold gauge theories with less supersymmetries ($\mathcal{N} = 1, 2$), even away from the conformal points, have been studied by two of us in ref. [15].

On the other hand, it is very appealing to add D-branes and study the dynamics of open strings in the $AdS_5 \times S^5$ plus D-brane backgrounds. This effectively corresponds to adding fundamental flavors to the dual gauge theory. Then issues similar to the original BMN proposal for closed strings can be investigated for open strings too. A well-known model has been proposed in [16]. The gauge theory is a four dimensional $\mathcal{N} = 2$ $Sp(N)$ gauge theory with matter in $[2] \oplus (4 \times 2\mathbf{N})$ representations. The dual strings are in the plane wave limit of $AdS_5 \times S^5/Z_2$, arising as the near horizon limit of D3-branes at an O7-plane in type I' string theory. Various aspects of the plane-wave/SYM duality in this system have been investigated: the free plane-wave string and the corresponding BMN-like operators [16, 17], the plane-wave string interactions in terms of light-cone open-closed string field theory [18, 19, 20, 21], and the semi-classical open string solutions in $AdS_5 \times S^5/Z_2$ as well [22]. (Other studies on D-branes in the plane-wave background can be found in [23, 24, 25, 26, 27, 28].)

In this paper, we would like to explore the power of the integrable structures on the dual $Sp(N)$ gauge theory side. As a first step in this direction, we will concentrate on the test of (free) string spectrum, with the contributions corresponding to string interactions turned off. Then the sectors corresponding to closed string and open string can be treated independently. The closed string scalar sector in this theory has the same integrable structure as in the parent theory $\mathcal{N} = 4$ SYM; i.e. the planar one-loop anomalous dimension matrix (ADM) can be identified as the Hamiltonian of a closed $SO(6)$ spin chain. As for the open string sector, we restrict ourselves to the operators consisting of holomorphic scalars except the "quarks" at the two ends[1]. The ADM for such operators is of the form of the Hamiltonian of an open $SU(3)$ spin chain with boundary terms[2]. We have been able to identify the boundary conditions that are appropriate for those of open strings and

[1] For early study of higher-twist operators in QCD in terms of integrable open spin chains, see e.g. [29].

[2] In accordance with the conventions in the literature of the integrable systems, we use the symmetry of

show that they indeed make the open spin chain integrable. Then the open spin chain can be solved with the help of the Bethe ansatz. The solutions to the ABAE with a few impurities will be shown to exactly match the spectrum of a free open plane-wave string. These results are viewed as a preliminary step towards studying open string interactions in an approach that exploits integrable structures.

Though we will work in the context of the $Sp(N)$ gauge theory, the main techniques and results are expected to be applicable to other $\mathcal{N}=2$ SQCD [17].

The paper is organized as follows: In Section 2, we first give a short review of the $\mathcal{N}=2$ $Sp(N)$ theory and its duality to plane-wave strings; then we study planar one-loop operator mixing. In Section 3, we proceed to uncover the integrable structures in this model, paying particular attention to the sector dual to free open string. Then we solve the resulting integrable open spin chain with the Bethe ansatz, and show that the solutions to the ABAE agree with string theory predictions. Finally we conclude the paper with discussions in Section 4.

2. Operator Mixing in $\mathcal{N}=2$ $Sp(N)$ theory

The $\mathcal{N}=2$ $Sp(N)$ theory we study in this paper arises from the orientifold projection of 2N D3-branes spreading along x_1, x_2, x_3 directions, in the presence of four D7-branes and an O7-plane sitting at the origin $x_7 = x_8 = 0$. Due to the orientifold action, the strings become unoriented, and the gauge group on D3-branes changes from U(2N) to $Sp(N)$. The $Sp(N)$ gauge theory is superconformal when all 7-branes and D3-branes are sitting together. In this case, the near horizon geometry of D3-branes is $AdS_5 \times S^5/Z_2$. One then takes the plane-wave limit by boosting along a circle inside S^5/Z_2 in a world-volume direction of the D7-branes. The D7-branes together with 5-form flux break $SO(8)$ symmetry down to $SO(4) \times SO(2) \times SO(2)$, where two $SO(2)$ act on $z' = x_5 + ix_6$ plane and $w = x_7 + ix_8$ plane respectively. Therefore, the Neumann directions are along $\{x^1, \cdots, x^4, z', \bar{z}'\}$ and the Dirichlet directions are along $\{w, \bar{w}\}$. The light-cone spectrum of the free string in the plane wave limit of $AdS_5 \times S^5/Z_2$ is given by [16, 17]

$$\frac{2}{\mu} p^- = \Delta - J = \begin{cases} \displaystyle\sum_{n=-\infty}^{+\infty} N_n \sqrt{1 + \frac{4\pi g_s N n^2}{J^2}} & \text{for closed string,} \\ \displaystyle 1 + \sum_{n=-\infty}^{+\infty} N_n \sqrt{1 + \frac{\pi g_s N n^2}{J^2}} & \text{for open string.} \end{cases} \tag{2.1}$$

The dual $D=4$, $\mathcal{N}=2$ $Sp(N)$ gauge theory has $SU(2)_R \times U(1)_R$ R-symmetry and $SU(2)_L \times SO(8)$ global symmetry. The bosonic field content consists of the following $\mathcal{N}=2$ supermultiplets: the vector multiplet (V, W) in the adjoint representation, a hypermultiplet (Z, Z') in the antisymmetric representation, and four hypermutiplets $(\tilde{q}_A, q_A)$ in the

the bulk part of the spin Hamiltonian (i.e. the group associated with the Yang-Baxter R-matrix) to label the spin chain, which is $SO(6)$ and $SU(3)$ respectively for the closed and open chain, though the $Sp(N)$ gauge theory at hand does not possess these (global) symmetries.

fundamental representation ($A = 1, ..., 4$). In $\mathcal{N} = 1$ language, V is a vector multiplet, while W, Z, Z', $\tilde{q}_A$, q_A are all chiral multiplets. The superpotential reads

$$\mathcal{W} \sim q_A W \tilde{q}_A + \mathrm{tr}[(\Omega W)(\Omega Z)(\Omega Z')], \tag{2.2}$$

where Ω is the invariant rank-2 tensor of $Sp(N)$.

Now let us turn to the mixing matrix for anomalous dimension of composite operators in this $\mathcal{N} = 2$ $Sp(N)$ gauge theory. As in refs. [10, 15], we consider gauge invariant composite operators consisting of various chiral scalar matter fields without derivatives. The operator basis corresponding to closed string states are the single trace operators

$$\mathcal{O}^{close}_{i_1,...,i_L} = \mathrm{tr}[(\Omega\Phi_{i_1})\cdots(\Omega\Phi_{i_L})], \tag{2.3}$$

while the operator basis corresponding to open string states is of the form

$$\mathcal{O}^{open}_{i_1,...,i_L} = \lambda_{pq} Q^q \Omega(\Phi_{i_1}\Omega)\cdots(\Phi_{i_L}\Omega)Q^p. \tag{2.4}$$

Here the Q^p ($p = 1, ..., 8$) are linear combinations of the “quarks” $(\tilde{q}_A, q_A)$ forming an $SO(8)$ vector, λ_{pq} the Chan-Paton factors, and Φ_i linear combinations of Z, Z', W and their complex conjugates forming an “$SO(6)$ vector”[3]. In addition to having the “quarks” Q^p at the ends in operator basis (2.4), “anti-quarks” $\bar{Q}^p$ are also allowed to be put at the ends. At the leading order in $1/L$, the operator basis with Q^p at ends and that with $\bar{Q}^p$ at ends do not mix[4]. So we can treat them separately. For simplicity, we will consider only the operator basis (2.4) with Q^p at the ends.

Concerning the closed string sector, two facts are notable: 1) Chiral matters W, Z, Z' are obtained by a Z_2 orientifold projection from scalar fields in the $\mathcal{N} = 4$ $U(2N)$ theory. 2) All correlation functions of the orbifolded theory are known to coincide with those of its parent $\mathcal{N} = 4$ SYM [30, 31], except for a combinatoric factor. Consequently the closed-string BMN operators, i.e. linear combinations in the operator basis (2.3), can be obtained by a Z_2 orientifold projection from those in $\mathcal{N} = 4$ $U(2N)$ SYM. The planar one-loop anomalous dimension matrix (ADM) for these operators is expected to coincide with matrix (1.1) by replacing $\lambda \to 2\lambda$ [10, 15]. However, this statement is not exactly true for our $Sp(N)$ theory, because there is mixing between operators in $\mathcal{O}^{close}$ and in $\mathcal{O}^{open}$. For example, the two-point function of the operators $\mathcal{O}_1 \sim \mathrm{tr}[(\Omega W)(\Omega W^\dagger)(\Omega Z)^J]$ and $\mathcal{O}_2 \sim Q^q\Omega(Z\Omega)^J Q^q$ is non-zero even at the planar one-loop level. In fact, it is of

[3] Strictly speaking, their $SO(6)$ rotations do not form a global symmetry in this theory, because W and (Z, Z') belong to different representations of gauge group $Sp(N)$. However, in the planar diagrams, they give rise to symmetric contributions to the operator mixing matrix (or ADM), because of a formal SO(6) invariance for the second term of the superpotential (2.2). Therefore an $SO(6)$ symmetry will appear in the spin chain Hamiltonian that corresponds to the planar one-loop ADM. The fact that the symmetry of the integrable spin chain is not directly related to the global symmetry of the theory, rather it is determined by the formal invariance of the relevant superpotential, has been already observed in ref. [15].

[4] The mixing between the two operator bases may occur only when a $\bar{Z}'$ impurity appears in the neighborhood of boundary (anti-)quarks and, hence, is expected to be sub-leading in $1/L$[17, 21].

the order $\mathcal{O}(\sqrt{g_2})$ with $g_2 = J^2/N$, characteristic of open string interactions[5]. It reflects the fact that 7-7 strings interact with closed strings in the bulk. Since our goal in this paper is to explore the power of integrable structures in studying free string spectrum, these contributions in gauge theory representing string interactions are not relevant. So we temporarily turn them off, and hope to incorporate them back when we study string interactions in the future.

Once turning off the contributions dual to open-closed string interactions, the closed and open string sectors (2.3) and (2.4) are decoupled at the planar one-loop level, and we are allowed to treat the free closed string and free open string separately. Then the discussion for the closed string scalar sector is the same as in [10], so one has an $SO(6)$ closed spin chain, whose Hamiltonian corresponds to the ADM of single-trace BMN operators.

Let us turn to the open string sector. The key difference between open and closed string BMN operators is that the latter consists of single trace operators, while the former operators with fundamental "quarks" appearing at the two ends. So we naturally expects that open-string BMN operators correspond to an open spin chain with boundary. For simplicity, we will focus on the gauge invariant operators consisting of holomorphic scalars. Recall that the $\mathcal{N} = 4$ theory can be represented in terms of $\mathcal{N} = 1$ fields with manifest $SU(3)$-invariant superpotential $\mathcal{W} \sim \mathrm{tr}(Z_1[Z_2, Z_3])$, where $Z_i = \Phi_{2i-1} + i\Phi_{2i}$, and Φ_i form an $SO(6)$ vector. Then the gauge invariant composite operators consisting of only Z_i (or of only $Z_i^\dagger$) form the (anti-)holomorphic class, and the rest containing both Z_i and $Z_i^\dagger$ form the non-holomorphic class. The crucial point is that operators belonging to different classes do not mix each other under planar one-loop corrections. It has been shown [15] that the Hamiltonian restricted to the holomorphic class,

$$\Gamma_c = \frac{\lambda}{4\pi^2} \sum_{l=1}^{L} (1 - P_{l,l+1}), \tag{2.5}$$

describes an integrable spin chain with $SU(3)$ symmetry. One can do the similar classification in the $\mathcal{N} = 2$ $Sp(N)$ theory, for both closed string and open string sectors. Up to planar one-loop, the ADM for the open-string operators (2.4) in the holomorphic class can be easily computed using Feymann diagrams (Fig. 1), with the result

$$\Gamma_o = \frac{\lambda}{4\pi^2} \sum_{l=1}^{L-1} (1 - P_{l,l+1}) + \frac{\lambda}{4\pi^2} (\Sigma_1 + \Sigma_L). \tag{2.6}$$

Here the boundary terms are $\Sigma_1 = \Sigma(\otimes I_{3\times 3})^{L-1}$, $\Sigma_L = (I_{3\times 3}\otimes)^{L-1}\Sigma$, with $\Sigma = \mathrm{diag}\{0, 0, 1\}$. The "bulk" part (the first term) in Γ_o is the same as in the closed string sector. It is from three sources: flavor-blind gauge boson propagation, hopping of the impurity induced by the interactions between the chiral fields and self-energy corrections of the chiral fields.

[5]The details for power counting can be found in ref.[21]. It should be pointed out that this $\mathcal{O}(\sqrt{g_2})$ counting is not originated from non-planar diagrams, but from the difference in normalization between closed- and open-string operators: We normalize $\mathcal{O}_1$ by multiplying $g_{\mathrm{YM}}^{-(L+2)} N^{-(L+2)/2}$ and $\mathcal{O}_2$ by multiplying $g_{\mathrm{YM}}^{-(L+2)} N^{-(L+1)/2}$. Such normalization makes the two-point functions $\langle \mathcal{O}_1 \bar{\mathcal{O}}_1 \rangle$ and $\langle \mathcal{O}_2 \bar{\mathcal{O}}_2 \rangle$ both independent of the 't Hooft coupling at tree level. Then, the usual 't Hooft counting shows that $\langle \mathcal{O}_1 \bar{\mathcal{O}}_2 \rangle \sim \lambda/\sqrt{N}$.

(See [15] for details.) The last term in Γ_o exhibits the effects of the "quarks" at the ends. The nonzero element in Σ is a manifestation that only when the chiral field at the first or the last site is W, the ADM receives an extra contribution from the first term in the superpotential (2.2) (see fig. 1). This corresponds to the Dirichlet boundary condition for open string [16]. And the zero diagonal components in Σ correspond to the Neumann boundary condition.

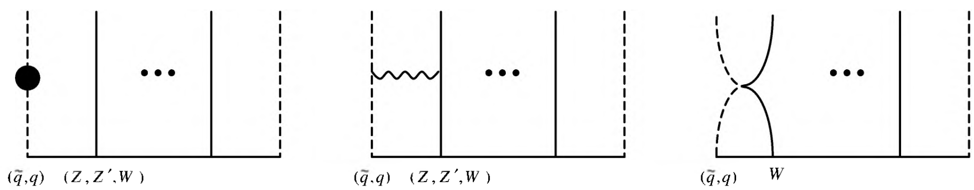

Figure 1: One-loop planar diagrams involving quarks at boundary. Here the solid lines denote chiral fields (Z, Z', W) and dash lines denote the chiral field $(\tilde{q}, q)$. It should be noticed that Z, Z' contribution is absent in the third graph.

The ADM Γ_o is of the form of the Hamiltonian of an open $SU(3)$ spin chain[6]. In next section we will prove that the boundary terms make the open spin chain integrable, and try to solve it by using the Bethe ansatz, showing that the solutions agree with the spectrum of the plane-wave open string.

3. Integrable Open Spin Chain and Its Solutions

Recall that the (quantum) integrability of closed spin chains is guaranteed, if there is an R-matrix satisfying the Yang-Baxter equations [32, 33]

$$R_{ab}(u)R_{ac}(u+v)R_{bc}(v) = R_{bc}(v)R_{ac}(u+v)R_{ab}(u), \tag{3.1}$$

where R_{ab} acts on the tensor space $\mathcal{H}_a \otimes \mathcal{H}_b$, with $\mathcal{H}_a$ the Hilbert space associated with the a-th lattice site, and u or v the spectral parameter. The explicit expression of the R-matrix depends on the symmetry group and the representation of the spin chain. For the $SU(n)$ chain (in the fundamental representation), it reads [32, 34]

$$R_{ab}(u) = \frac{1}{u+i}[uI_{ab} + iP_{ab}]. \tag{3.2}$$

For open spin chains, the integrability requires, in addition to an R-matrix satisfying eq. (3.1), also the existence of a boundary K-matrix satisfying the boundary Yang-Baxter equation [35, 36]

$$R_{ab}(u-v)K_a(u)R_{ba}(u+v)K_b(v) = K_b(v)R_{ab}(u+v)K_a(u)R_{ba}(u-v). \tag{3.3}$$

[6]Note that the $SU(3)$ symmetry of the "bulk" part of the open spin chain is not a symmetry of the gauge theory, for the same reason as pointed out in footnote 3. Moreover, as emphasized in footnote 4, this result holds only when we ignore the mixing with BMN operators with "anti-quarks" at the ends, which is of order of $1/L$.

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For an SU(3) open chain, the general diagonal K-matrix that solves Eq. (3.3) (with the R-matrix (3.2)) has been obtained in [37]:

$$K^{\pm}_{(l)}(u,\xi_{\pm}) = \mathrm{diag}\{\overbrace{a^{\pm},...,a^{\pm}}^{l},\overbrace{b^{\pm},...,b^{\pm}}^{n-l}\}, \tag{3.4}$$

where

$$\begin{aligned} a^{+} &= i(\xi_{+}-n)-u, \qquad & b^{+} &= i\xi_{+}+u, \\ a^{-} &= i\xi_{-}+u, \qquad & b^{-} &= i\xi_{-}-u, \end{aligned} \tag{3.5}$$

with arbitrary $\xi_{\pm}$ and any $l \in \{1,...,n-1\}$. Here we have used $\pm$ to label the two boundaries. The corresponding integrable Hamiltonian is given by [35, 38]

$$H_{open} = \sum_{m=1}^{L-1} H_{m,m+1} + \frac{1}{2\xi_{-}}\frac{d}{du}K^{-}_{1,(l)}(u,\xi_{-})\Big|_{u=0} + \frac{\mathrm{tr}_0[K^{+}_{0,(l)}(0,\xi_{+})H_{L,0}]}{\mathrm{tr}K^{+}_{(l)}(0,\xi_{+})}, \tag{3.6}$$

where $H_{m,m+1} = I_{m,m+1} - P_{m,m+1}$. $K_{i,(l)}$ denotes the K-matrix acting on Hilbert space $\mathcal{H}_i$; namely $K_{1,(l)}$ acts on the Hilbert space at the first site while $K_{0,(l)}$ on the auxiliary Hilbert space. The eigen-energies and momenta (which now label standing, rather than traveling, wave modes), defined relative to the pseudo-vacuum $\omega = (\vec{v}\otimes)^{L-1}\vec{v}$ (with $\vec{v} = (1,0,...,0)^T$), are given by [37]

$$\begin{aligned} E &= \sum_{j=1}^{n_1}\epsilon(\mu_{1,j}) + \epsilon_0(\xi_{+},\xi_{-}), \qquad & \epsilon(\mu) &= \frac{4}{\mu^2+1}, \\ P &= \sum_{j=1}^{n_1} p(\mu_{1,j}), \qquad & p(\mu) &= \frac{1}{i}\ln\frac{\mu+i}{\mu-i}. \end{aligned} \tag{3.7}$$

Here $\mu_{i,j}$ satisfy the algebraic Bethe ansatz equations (ABAE)

$$\begin{aligned} 1 = {} & [e_{l-2\xi_{-}}(\mu_{l,k})e_{2\xi_{+}-l}(\mu_{l,k})\delta_{l,q} + (1-\delta_{l,q})]\prod_{j=1}^{M_{q-1}} e_{-1}(\mu_{q,k}-\mu_{q-1,j})e_{-1}(\mu_{q,k}+\mu_{q-1,j}) \\ & \times \prod_{\substack{j=1,\\ j\neq k}}^{M_q} e_2(\mu_{q,k}-\mu_{q,j})e_2(\mu_{q,k}+\mu_{q,j}) \prod_{j=1}^{M_{q+1}} e_{-1}(\mu_{q,k}-\mu_{q+1,j})e_{-1}(\mu_{q,k}+\mu_{q+1,j}) \end{aligned} \tag{3.8}$$

for $k = 1,\cdots,M_q$ and $q = 1,\cdots,n-1$.

Here $M_0 = L$, $M_n = 0$, $\mu_{0,j} = \mu_{n,j} = 0$, and

$$e_n(\mu) = \frac{\mu+in}{\mu-in}. \tag{3.9}$$

In our present case, $n = 3$. We choose $l = 2$ (there is a symmetry between $l = 2$ and $l = 1$). Then we see that with $\xi_{+} = \xi_{-} = 1$, the Hamiltonian (3.6) reduces to

$$H_{open} = \sum_{l=1}^{L-1}(1-P_{l,l+1}) + (\Sigma_1+\Sigma_L) + \frac{1}{6}. \tag{3.10}$$

This is nothing but the ADM Γ_o given by eq. (2.6), up to a constant factor! So the open $SU(3)$ spin chain (2.6) is indeed an integrable system and can be solved exactly.

The parameters $\xi_\pm$ specify the boundary conditions in a general open spin chain model. In the case at hand, it breaks the bulk $SU(3)$ symmetry down to $SU(2)\times U(1)$, the same as the R-symmetry of the gauge theory at hand. Now the ABAE (3.8) reduce to

$$\begin{aligned} [e_1(\mu_{1,k})]^{2L} &= \prod_{\substack{j=1,\\ j\neq k}}^{M_1} e_2(\mu_{1,k}-\mu_{1,j})e_2(\mu_{1,k}+\mu_{1,j}) \prod_{j=1}^{M_2} e_{-1}(\mu_{1,k}-\mu_{2,j})e_{-1}(\mu_{1,k}+\mu_{2,j}), \\ 1 &= \prod_{\substack{j=1\\ j\neq k}}^{M_2} e_2(\mu_{2,k}-\mu_{2,j})e_2(\mu_{2,k}+\mu_{2,j}) \prod_{j=1}^{M_1} e_{-1}(\mu_{2,k}-\mu_{1,j})e_{-1}(\mu_{2,k}+\mu_{1,j}) \end{aligned} \tag{3.11}$$

Before we look for the solutions of ABAE, let us clarify a few points concerning the open spin chain. As we have seen, the "quarks" serve as the boundary fields on the chain, while each chiral scalar field in the composite operators, one of the $(\Omega Z, \Omega Z', \Omega W)$, corresponds to a site in the spin chain. The ground state associated with the Hamiltonian (2.6) is

$$G \sim Q^p\Omega(Z\Omega)^{J-1}Q^q. \tag{3.12}$$

It corresponds to the open string state with $\Delta - J = 1$ and is an eigenstate of the Hamiltonian (2.6) with zero eigenvalue. On the other hand, in the vector notation, it corresponds to the pseudo-vacuum ω used to construct the ABAE for the open spin chain. Therefore, the ground state of Hamiltonian (3.10) must also have zero energy. This fact, together with Eq. (3.7), implies that we should take

$$\epsilon_0(\xi_+ = \xi_- = 1) = -\frac{1}{6}. \tag{3.13}$$

For the present spin chain there are two types of impurities, labeled by two rapidities $\mu_{1,j}$ and $\mu_{2,j}$, which are associated with the two simple roots $\vec{\alpha}_1$ and $\vec{\alpha}_2$ of Lie algebra $SU(3)$. The states with impurities correspond to the excitations of Z' and W above the ground state G, i.e. the BMN operators with the replacements of some Z's by Z''s and/or by W's in G. Consider the highest weight $\vec{w}$, which generates the fundamental representation of $SU(3)$. Because the weight $\vec{w}-\vec{\alpha}_2$ is not equivalent to $\vec{w}$, a single μ_2-impurity without being bound to a μ_1-impurity on the same site is not allowed. The physical interpretation is the following: A single μ_1-impurity $(\vec{w}-\vec{\alpha}_1)$ creates a Z' replacement in the state G, while a $\mu_1-\mu_2$ bound impurity $(\vec{w}-\vec{\alpha}_1-\vec{\alpha}_2)$ creates a W replacement. But an individual μ_2-impurity $(\vec{w}-\vec{\alpha}_2)$ would kill the vacuum.

In contrast to closed spin chains, the so-called trace condition, which expresses the cyclic symmetry of closed-string BMN operators (2.3) and reflects the level matching condition of closed string, is absent for open chains. Hence the first non-trivial case for the open spin chain is a single μ_1-impurity in the ground state. It describes the BMN operators with a single Z'-insertion with $\Delta - J = 2$ (or $L - J = 1$ with L the total number of fields Z, Z' and W), or an oscillator excitation of the open string in the Neumann directions.

The ABAE now reduces to

$$\left(\frac{\mu_1+i}{\mu_1-i}\right)^{2L} = 1. \tag{3.14}$$

The solution is $\mu_1 = \cot n\pi/2L$ which, together with Eqs. (2.6), (3.7) and (3.10), gives the anomalous dimensions

$$\gamma_{Z'} = \frac{\lambda}{\pi^2}\sin^2\frac{n\pi}{2L} \xrightarrow{n\ll L=J+1} \frac{n^2\lambda}{4J^2} = \frac{\pi g_s N n^2}{2J^2}. \tag{3.15}$$

This result precisely agrees with the open string spectrum with one oscillator mode when $g_s N n^2/J^2 \ll 1$.

The BMN operators with a single W-insertion, or the open string state with a single oscillator mode in the Dirichlet directions, correspond to a $\mu_1 - \mu_2$ bound impurity in the open spin chain. The ABAE now reduce to

$$\begin{aligned}\left(\frac{\mu_1+i}{\mu_1-i}\right)^{2L} &= \frac{\mu_1-\mu_2-i}{\mu_1-\mu_2+i}\frac{\mu_1+\mu_2-i}{\mu_1+\mu_2+i},\\ 1 &= \frac{\mu_2-\mu_1-i}{\mu_2-\mu_1+i}\frac{\mu_2+\mu_1-i}{\mu_2+\mu_1+i}.\end{aligned} \tag{3.16}$$

The solution

$$\mu_1 = \cot\frac{n\pi}{2(L+1)}, \qquad \mu_2 = 0 \tag{3.17}$$

yields the anomalous dimensions

$$\gamma_W = \frac{\lambda}{\pi^2}\sin^2\frac{n\pi}{2(L+1)} \xrightarrow{n\ll L=J+1} \frac{\pi g_s N n^2}{2J^2}. \tag{3.18}$$

It is remarkable that the anomalous dimensions (3.18) coincide with those in Eq. (3.15) for very large L (or J), exactly as anticipated by the open string spectrum (2.1).

If all of the rapidities $\mu_{q,j}$ are real, it is convenient to take the logarithm of the ABAE (3.11):

$$\begin{aligned}2L\vartheta(\mu_{1,j}) = Q_{1,j}\pi &+ \sum_{k\neq j}^{n_1}[\vartheta(\frac{\mu_{1,j}-\mu_{1,k}}{2}) + \vartheta(\frac{\mu_{1,j}+\mu_{1,k}}{2})]\\ &- \sum_{k=1}^{n_2}[\vartheta(\mu_{1,j}-\mu_{2,k}) + \vartheta(\mu_{1,j}+\mu_{2,k})],\\ 0 = Q_{2,j}\pi &+ \sum_{k\neq j}^{n_2}[\vartheta(\frac{\mu_{2,j}-\mu_{2,k}}{2}) + \vartheta(\frac{\mu_{2,j}+\mu_{2,k}}{2})]\\ &- \sum_{k=1}^{n_1}[\vartheta(\mu_{2,j}-\mu_{1,k}) + \vartheta(\mu_{2,j}+\mu_{1,k})],\end{aligned} \tag{3.19}$$

where $\vartheta(x) = \cot^{-1}(x) \in (-\pi/2, \pi/2)$, and $Q_{q,j}$ are integers. Moreover there are no coinciding $Q_{q,j}$ for a given $q = 1$ or $q = 2$, because the discreteness of the Bethe roots requires them to be pushed to distinct branches of the logarithm function.

The above equations in general cannot be solved exactly. In the thermodynamic limit $L \to \infty$, however, some analytical results can be worked out. An interesting case is to consider a finite number of impurities, i.e., $n_1, n_2 \ll L$. If $Q_{q,j}/L \ll 1$, we have approximately $\vartheta(\mu_{1,j}) = Q_{1,j}\pi/2L$. Consequently the anomalous dimensions are approximately given by

$$\gamma = \frac{\lambda}{\pi^2}\sum_{j=1}^{n_1}\sin^2\frac{\pi Q_{1,j}}{2L} \xrightarrow{Q_{1,j}\ll L=J+n_1} \frac{\pi g_s N}{2J^2}\sum_j Q_{1,j}^2. \tag{3.20}$$

This result matches the free open string spectrum (2.1) for states with at most single occupancy of each excited oscillator mode when $g_sN/J^2 \ll 1$.

In contrast to the case with a finite number of impurities, we may also consider the operator with the largest number of impurities, i.e., $n_1, n_2 \sim L$. It corresponds to the highest excited eigenstate of the Hamiltonian (2.6), with the largest eigenvalue (anomalous dimension). In this case, we can replace $Q_{q,j}$ in the ABAE (3.19) by j, and j/L by a continuous variable x. Adopting the procedure presented in [39, 10, 15], we conclude that there are $2L/3$ μ_1-impurities and $L/3$ μ_2-impurities for the highest excited state. It implies that there are an equal number of Z, Z' and W fields in the corresponding composite operator, and they form an $SU(3)$ singlet. Its anomalous dimension is [15]

$$\gamma = -\frac{\lambda}{12\pi^2}L(\frac{\pi}{\sqrt{3}} + 3\ln 3). \tag{3.21}$$

This operator is far from BPS, and is beyond the BMN regime. Moreover, this one-loop result (3.21) makes sense even in the regime with $\lambda L \ll 1$. It corresponds to $R^4 \sim g_sN \to 0$ in $AdS_5 \times S^5$, just the opposite to the plane wave limit of $AdS_5 \times S^5$.

We also note that the ABAE (3.11) for the open spin chain allow complex solutions. As an example, let us consider the case with two μ_1-impurities. The ABAE now reduce to

$$\begin{aligned}\left(\frac{\lambda_1+i}{\lambda_1-i}\right)^{2L} &= \frac{\lambda_1-\lambda_2+2i}{\lambda_1-\lambda_2-2i}\frac{\lambda_1+\lambda_2+2i}{\lambda_1+\lambda_2-2i},\\ \left(\frac{\lambda_2+i}{\lambda_2-i}\right)^{2L} &= \frac{\lambda_1-\lambda_2-2i}{\lambda_1-\lambda_2+2i}\frac{\lambda_1+\lambda_2+2i}{\lambda_1+\lambda_2-2i},\end{aligned} \tag{3.22}$$

where $\lambda_1 = \mu_{1,1}$, $\lambda_2 = \mu_{1,2}$. When λ_1 acquires an imaginary part, the left hand side of the first equation in (3.22) grows or decreases exponentially as $L \to \infty$. Therefore, we make the ansatz

$$\lambda_1 = a + iu, \qquad \lambda_2 = a - iu, \tag{3.23}$$

where a and u are real and $a \sim L$, $u \sim L^k$, $(k < 1)$. The assumption $\lambda_2 = \bar{\lambda}_1$ ensures the energy to be real. Substituting Eq. (3.23) into the ABAE (3.22) and expanding it to the sub-leading order in $1/L$, we obtain

$$\begin{aligned}\left(1+\frac{2(u-1)}{a^2}+\frac{2i}{a}\right)^{2L} &\simeq \frac{u+1}{u-1}\frac{a+i}{a-i},\\ \left(1-\frac{2(u+1)}{a^2}+\frac{2i}{a}\right)^{2L} &\simeq \frac{u-1}{u+1}\frac{a+i}{a-i}.\end{aligned} \tag{3.24}$$

It can be reduced to

$$\left(1+\frac{4u}{a^2}\right)^L \simeq \frac{u+1}{u-1},$$
$$2L\vartheta(\frac{a}{2}) = \begin{cases} 2m\pi + 2\vartheta(a) + O(1/L) & k>0 \\ (2m+1)\pi + 2\vartheta(a) + O(1/L) & k<0 \end{cases} \tag{3.25}$$

For finite m, i.e., $m/L \ll 1$, we have $a \sim L$. Then the first equation in (3.25) gives $k = 1/2$. This result is similar to the closed chain case, in that Bethe roots behave as $\lambda = c_0 L + ic_1\sqrt{L} + ...$ in a large L expansion when they pick up an imaginary part. Finally we get the solution (with m an integer)

$$a \simeq 2\cot\frac{m\pi}{L}. \tag{3.26}$$

The energy (or anomalous dimension) of the solution (3.26) is

$$\gamma_b = \frac{\lambda}{2\pi^2}\sin^2\frac{m\pi}{L} \xrightarrow{m\ll L=J+2} \frac{\pi g_s N m^2}{J^2}, \tag{3.27}$$

with the right hand side corresponding to open string energy (2.1) with two excitations of mode number m along the Neumann direction z'. We expect that similar results should be true for complex roots corresponding to a finite number of oscillator excitations as $L \to \infty$.

4. Conclusions and Discussions

To conclude, in this paper we have investigated the integrable structures in an $\mathcal{N} = 2$ superconformal $Sp(N)$ gauge theory in four dimensions, which is known to be dual to an open+closed string system. We showed that the planar one-loop ADM associated with the gauge invariant composite operators (2.3) or (2.4) restricted to the holomorphic scalar sector can be identified with the Hamiltonian of either a closed or open integrable $SU(3)$ spin chain. In particular, we have established that the boundary terms in the open string chain in gauge theory that are appropriate for open string boundary conditions are indeed integrable ones. The Bethe ansatz method is exploited to solve the open spin chain. For solutions with a few impurities, the energy is shown to reproduce the anomalous dimensions of corresponding BMN operators, in perfect agreement with the free open string spectrum in a plane-wave background.

Our study raises several interesting questions. As shown in the last section, there are solutions with a large number of impurities, which is beyond the BMN regime and far from BPS. From the lessons in the closed spin chain, one may suspect that such solution could be related to the AdS/CFT test of certain semi-classical solutions in string theory; a possible candidate might be the open spinning strings [20]. A careful inspection shows that the solution (3.29) does not correspond to an open spinning string: its energy scales as L, while in general the energy of spinning strings scales as $1/L$. Moreover, there exist complex roots to the ABAE. We have seen in a simple example that in the limit $L \to \infty$, they reproduce the free string spectrum with, say, two excitations of mode number m. How to incorporate the complex roots in the Yang-Yang ansatz [39] for thermodynamics is still an open question. We leave further study of complex Bethe roots to future research.

In our study, we have been focusing on the gauge invariant operators in the holomorphic class, which corresponds to the replacement of Z'- and W-impurity in a pure Z-chain. As we know, in general $\bar{Z}'$- and $\bar{W}$-impurities are allowed in gauge theory, breaking the holomorphic nature of the chain. Such BMN operators have more involved ADM, and gives rise to more complicated closed or open spin chain. To clarify whether these spin chains are integrable is also an interesting question.

As mentioned in Section 2, even at the planar one-loop level, there exist the mixing between closed and open BMN operators. They naturally correspond to open/closed string interactions in the system. Though in this paper we have turned off these contributions in order to explore the power of the integrable structures in testing the free string spectrum, in particular in the hope of going beyond the BMN regime. However, the fact that the planar 1-loop ADM for the gauge invariant composite operators encodes information on string interactions is quite remarkable. (Indeed as shown in the light-cone string field theory [20, 21], open cubic and open-closed string interactions show up in the planar 1-loop calculation of two-point functions in SYM.) It would be fascinating to explore effects of string interactions on the gauge theory side by dealing with a more complicated or more complete ADM. Also it would be interesting to see whether there is a bigger integrable structure in the model at hand, such that the integrable structures we found in this paper separately for the free closed and open string sectors are only its substructures.

Of course to pin down the origin of the integrable structure(s), both on the SYM side and on the dual string theory side, is still a challenge. It has been revealed in [40, 41] that there is a relation between the infinite-dimensional non-local symmetry of type IIB superstring in $AdS_5 \times S^5$ [42, 43] and a non-Abelian and nonlinear infinite-dimensional Yangian algebra for weakly coupled SCYM. On the other hand, the present paper provides an example in which the symmetry of the bulk part of the integrable spin chain corresponding to the planar one-loop ADM in the BMN scalar sectors, $SO(6)$ (or $SU(3)$) for the closed (or open) string case, is *not* the global symmetry of the dual $\mathcal{N} = 2$, $Sp(N)$ gauge theory. (Rather, the spin chain symmetry is originated from a formal invariance of relevant terms in the superpotential, as observed in ref. [15].) This seems to be in line with the observation made in [44], that the existence of global symmetries in a field theory seems not an essential ingredient in its relation to an integrable model. Further study of relevant issues is needed to gain deeper insights.

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References

[1] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP-wave from $\mathcal{N} = 4$ super Yang-Mills*, JHEP **04** (2002) 013, hep-th/0202021.

[2] D. Sadri and M.M. Sheikh-Jabbari, *The plane-wave/super Yang-Mills duality*, hep-th/0310119.
J.C. Plefka, *Lectures on the Plane-Wave String/Gauge Theory Duality*, hep-th/0307101.

[3] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, Phys. Lett. **B428** (1998) 105.

[4] E. Witten, *Anti-de Sitter space and holography*, Adv. Theory. Math. Phys. **2** (1998) 253.

[5] J.M. Maldacena, Adv. Theor. Math. Phys. **2**(1998)231, hep-th/9711200.
O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**(2000)183.

[6] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, JHEP **01** (2002) 047, hep-th/0110242; *ibid* Class. Quant. Grav. **19** (2002) L87, hep-th/0201081.

[7] R.R. Metsaev, Nucl. Phys. **B625** (2002) 70, hep-th/0112044.

[8] R.R. Metsaev and A.A. Tseytlin, Phys. Rev. **D65** (2002) 126004, hep-th/0202109.

[9] N. Beisert, C. Kristjansen, J. Plefka, G.W. Semenoff and M. Staudacher, *BMN correlators and operator mixing in N=4 Super Yang-Mills theory*, Nucl. Phys. **B650** (2003) 125, hep-th/0208178;
N. Beisert, C. Kristjansen, J. Plefka and M. Staudacher, *BMN gauge theory as a quantum mechanical system*, Phys. Lett. **B558** (2003) 229, hep-th/0212269.

[10] J.A. Minahan and K. Zarembo, *The Bethe ansatz for $\mathcal{N} = 4$ super Yang-Mills*, JHEP **03** (2003) 013, hep-th/0212208.

[11] H. Bethe, Z. Phys. **71** (1931) 205; L.D. Faddeev and L.A. Takhtajan, J. Sov. Math. **24** (1984) 241; N.Y. Reshetikhin, Lett. Math. Phys. **7** (1983) 205; L.D. Faddeev, hep-th/9605187.

[12] N. Beisert, C. Kristjansen and M. Staudacher, *The dialtation operator of conformal $\mathcal{N} = 4$ Super Yang-Mills Theory*, Nucl. Phys. **B664** (2003) 131, hep-th/030306;
N. Beisert, *The complete one-loop dilatation operator of N=4 Super Yang-Mills theory*, Nucl. Phys. **B676** (2004) 3, hep-th/0307015;
N. Beisert and M. Staudacher, *The N=4 SYM integrable super spin chain*, Nucl. Phys. **B670** (2003) 439, hep-th/0307042l;
N. Beisert, S. Frolov, M. Staudacher and A.A. Tseytlin, *Precision spectroscopy of AdS/CFT*, JHEP **10** (2003) 037.

[13] S.S. Guber, I.R. Klebanova and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, Nucl. Phys. **B636** (2002) 99, hep-th/0204051.

[14] A.A. Tseytlin, *Spinning strings and AdS/CFT duality*, hep-th/0311139.

[15] X.J. Wang and Y.S. Wu, *Integrable spin chain and operator mixing in $\mathcal{N} = 1, 2$ supersymmetric theories*, hep-th/0311073.

[16] D. Berenstei, E. Gava, J. Maldacena, K.S. Narain and H. Nastase, *Open strings on plane waves and their Yang-Mills duals*, hep-th/0203249.

[17] Y. Imamura, *Open string: BMN operator correspondence in the weak coupling regime*, Prog. Theor. Phys. **108** (2003) 1077, hep-th/0208079.

[18] D. Berenstein and H. Nastase, *On lightcone string field from super Yang-Mills and holography*, hep-th/0205048.

[19] B. Chandrasekhar, A. Kumar, *D-branes in PP-wave light cone string field theory*, JHEP **06** (2003) 001, hep-th/0303223.

[20] B. Stefanski, *Open String Plane-Wave Light-Cone Superstring Field Theory*, hep-th/0304114.

[21] J. Gomis, S. Moriyama and J. Park, *Open+Closed String Field Theory from Gauge Fields*, hep-th/0305264.
J. Lucietti, S. Schafer-Nameki and A. Sinha, *On the exact open-closed vertex in plane-wave light-cone string field theory*, hep-th/0311231.

[22] B. Stefanski, *Open Spinning Strings*, hep-th/0312091.

[23] O. Bergman, M.R. Gaberdiel and M.B. Green, JHEP **0303**(2003)002.

[24] P. Lee and J.W. Park, *Open strings in PP-wave background from defect conformal field theory*, Phys. Rev. **D67** (2003) 026002, hep-th/0203257.

[25] K. Skenderis and M. Taylor, *Branes in AdS and PP-wave spacetimes*, JHEP **06** (2002) 025, hep-th/0204054.

[26] V. Balasubramanian, M.X. Huang and T.S. Levi and A. Naqvi, *Open strings from $\mathcal{N} = 4$ super Yang-Mills*, JHEP **08** (2002) 037, hep-th/0204196.

[27] S.G. Naculich, H.J. Schnitzer and N. Wyllard, *PP-wave limits and orientifolds*, Nucl. Phys. **B650** (2003) 43, hep-th/0206094.

[28] M. Sakaguchi, K. Yoshida, *D-branes of covariant AdS superstrings*, hep-th/0310228.

[29] V.M. Braun, S.E. Derkachov and A.N. Manashov, Phys. Rev. Lett. bf 81 (1998) 2020-2023; V.M. Braun, S.E. Derkachov, G.P. Korchemsky and A.N. Manashov, Nucl. Phys. **B553** (1999) 355; A.V. Belitsky, Phys. Lett. **B453** (1999) 59; A.V. Belitsky, Nucl. Phys. **B558** (1999) 259; A.V. Belitsky, Nucl. Phys. **B574** (2000) 407; S.E. Derkachov, G.P. Korchemsky and A.N. Manashov, Nucl. Phys. **B566** (2000) 203.

[30] M. Bershadsky, Z. Kakushadze and C. Vafa, *String expansion as large N expansion of gauge theories*, Nucl. Phys. **B523** (1998) 59;
Z. Kakushadze, *Gauge theories from orientifolds and large N limit*, Nucl. Phys. **B529**(1998) 157.

[31] M. Bershadsky, A. Johansen, *Large N limit of orbifold field theories*, Nucl. Phys. **B536** (1998) 141.

[32] C.N. Yang, Phys. Rev. Lett. **19** (1967) 1312.

[33] R.J. Baxter, Ann. Phys. **70** (1972) 193.

[34] B. Sutherland, Phys. Rev. **B12**(1975) 3795.

[35] E.K. Sklyanin, J. Phys. **A21** (1988) 2375.

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[36] I.V. Cherednik, Theor. Math. Phys. **61** (1984) 977; L. Mezincescu and R.I. Nepomechie, J. Phys. **A24** (1991) L17; S. Ghoshal and A.B. Zamolodchikov, Int. J. Mod. Phys. **A9** (1994) 3841.

[37] H.J. de Vega and A. González-Ruiz, Mod. Phys. Lett. **A9** (1994) 2207.

[38] A. Doikou and R.I. Nepomechie, *Bulk and boundary S-matrices for the $SU(N)$ chain*, Nucl. Phys. **B521** (1998) 547, hep-th/9813118.

[39] C.N. Yang nad C.P. Yang, *Thermodynamics of a One-Dimensional System of Bosons with Repulsive Delta-Functiuon Interaction*, J. Math. Phys. **10** (1969) 1115.

[40] L. Dolan, C.R. Nappi and E. Witten, *A Relation between Approachs to Integrabilities in Superconforla Yang-Mills Theory*, hep-th/0308089.

[41] G. Arutyunov, M. Staudacher, *Matching Higher Conserved Charges for Strings and Spins*, hep-th/0310182.

[42] I. Bena, J. Polchinski and R. Roiban, *Hidden symmetries of the $AdS_5 \times S^5$ superstring*, hep-th/0305116.

[43] L.F. Alday, *Non-local charges on $AdS_5 \times S^5$ and PP-waves*, hep-th/0310146.

[44] R. Roiban, *On spin chains and field theories*, hep-th/0312218.