

# Quiver Mechanics for Deconstructed Matrix String

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## Abstract

In this paper we propose a quiver model of matrix quantum mechanics with 8 supercharges which, on a Higgs branch, deconstructs the worldsheet of Matrix String Theory. This discrete model evades the fermion doubling problem and, in the continuum limit, enhances the number of supersymmetries to sixteen. Our model is motivated by orbifolding the Matrix Model, and the deconstruction *ansatz* exhibits a duality between target space compactification and worldsheet deconstruction.

## 1 Introduction

Despite many great progresses in the last decade, a nonperturbative formulation of string theory, which is easy to work with, remains a hot topic of research. As alternatives to string field theory, a number of attempts have been tried by resorting to discrete descriptions. Among them the string bit (or bit string) model [1] is conceptually appealing in that relativistic string is treated as a composite of point-like entities – string bits. Recently there are

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revived interests [2] in string bit models, because the BMN correspondence [3] between type-*IIB* string theory in a pp-wave background [4, 5] and  $\mathcal{N} = 4$  supersymmetric Yang-Mills (SYM) theory in four dimensions is suggestive of a string bit picture. However, since a string bit model discretizes string worldsheet into a one-dimensional spatial lattice, depending on the way to do discretization, it may suffer from problems such as fermion doubling and/or absence of sufficient supersymmetries to ensure correct string interactions (for discussions of these problems, see [6, 7]). In a sense, the BFSS Matrix Model [8] is in line with the spirit of string bits, but in the context of 11-dimensional M-Theory. Nonetheless, within this framework nonperturbative type-*IIA* strings, say, are described still by a two dimensional *field theory*, the Matrix String Theory [9, 10, 11, 12], not by quantum mechanics.

In this paper we propose a different approach to a discrete formalism for nonperturbative string theory. We insist on providing a (quantum) *mechanical* framework to describe dynamics of string bits. Essentially what we propose is to *deconstruct*, rather than to discretize, the string worldsheet. On one hand, it is known that to deconstruct a continuous dimension [13] into a one-dimensional lattice, one needs to look up a quiver/moose field theory [14] with product gauge group and with bi-fundamental matter fields. Then assigning non-vanishing vacuum expectation value (VEV) to bi-fundamental bosonic fields gives rise to the kinetic energy (or hopping) as well as gauge connections in the latticized direction. On the other hand, in certain cases deconstruction allows us to build lattice theories with exact supersymmetry free from the fermion doubling problem [15]. This is possible because a deconstructed lattice gauge theory allows at least some of the matter fields, in particular fermions, to live on *links*. (Recall that in usual discretization, only the gauge fields live on links, while matter fields, either bosonic or fermionic, all live on sites. For deconstruction (or large quiver theories) in other contexts of string theory, see Refs. [16, 17, 18, 19, 21, 22, 23]). We believe that the success of formulating superstring theory in terms of discrete models would make well-developed notions and techniques for one-dimensional many-body models on a lattice, exactly solvable or not, accessible to string/M theory.

Motivated by these ideas, we devise a supersymmetric quiver model of matrix quantum mechanics with 8 supercharges, by orbifolding the BFSS Matrix Model in Section 2. Then

we show that on a Higgs branch it deconstructs string worldsheet with no fermion doubling (Section 3). Moreover, it is shown in Section 4 that in the continuum limit the discrete model recovers the Matrix String Theory, i.e. the two dimensional Super Yang-Mills (SYM) theory that provides the DLCQ description of second quantized Type-*IIA* superstrings. The Yang-Mills coupling is related to the string coupling in the just right manner as required by correct scaling for string interactions. A physical picture is given in Section 5 for our (de)construction of string world sheet in terms of the *D0*-brane description of Matrix Theory, which explicitly exhibits a duality between worldsheet deconstruction and target space compactification in M Theory. Finally we address the features and advantages of our quiver model for the dynamics of string bits (Section 6). Appendix A is devoted to some details of the transformation of gamma matrices and fermions at each stage of our (de)construction procedure.

## 2 Orbifolding

To develop a quiver mechanics that describes the dynamics of “string bits” from deconstructing the worldsheet of Matrix String, we will start from the BFSS Matrix Theory, and consider it in an orbifold background  $\mathcal{C}^2/\mathcal{Z}_N$ . The physical motivation is the following: We view the deconstructed closed string as a closed chain formed by  $N$  beads and directed links (in both directions) connecting the neighboring beads. From the point of view of deconstruction, this closed chain is identical with the theory space (or quiver diagram [14]) with a discrete  $\mathcal{Z}_N$  symmetry. With the dynamical degrees of freedom living on this space desirably being those of matrix mechanics, it is natural to start with the Matrix Theory that describes quantum mechanics of *D0*-branes. This leads to the idea of orbifolding the Matrix Theory on  $\mathcal{C}^2/\mathcal{Z}_N$ . The reason to choose  $\mathcal{C}^2$  is to make surviving supersymmetry as large as possible. The hope is that in a deconstruction phase and in a large- $N$  (continuum) limit, Matrix String Theory will be recovered. (For convenience, later we will call beads as sites, using the more familiar terminology of lattice theory.)

Start with  $U(KN)$  BFSS Hamiltonian, which reads (in temporal gauge  $Y^0 = 0$  and in

units of the 11-dimensional Planck length  $l_p$ ):

$$\mathcal{H} = R \text{Tr} \left\{ \frac{1}{2} \Pi_I^2 - \frac{1}{4} [Y^I, Y^J]^2 - \frac{i}{2} \theta^T \gamma^I [Y_I, \theta] \right\}. \quad (1)$$

Here  $R$  is the radius of the compactified light cone;  $Y^I$  ( $I = 1, \dots, 9$ ) are  $KN$ -by- $KN$  bosonic hermitian matrices, whose superpartners are 16 fermionic hermitian matrices,  $\theta$ , which transform as  $SO(9)$  Majorana spinor. Moreover,  $\text{Tr}$  is the trace over  $KN$ -by- $KN$  matrices. (The transpose  $T$  of  $\theta$  acts only on the spinor index  $\alpha = 1, \dots, 16$ , which we have suppressed.)

Now we choose  $\mathcal{C}^2$  to be the hyperplane with  $I = 6, 7, 8, 9$ , and define

$$Z^1 = \frac{1}{\sqrt{2}}(Y^6 + iY^7), \quad Z^2 = \frac{1}{\sqrt{2}}(Y^8 - iY^9). \quad (2)$$

The orbifolding conditions by the action of  $\mathcal{Z}_N$  read

$$\begin{aligned} \mathbf{U}^\dagger Y^I \mathbf{U} &= [\omega^{M_{89} - M_{67}}]_I^J Y^J, \\ \mathbf{U}^\dagger \theta^\alpha \mathbf{U} &= [\omega^{\sigma_{89} - \sigma_{67}}]_\beta^\alpha \theta^\beta. \end{aligned} \quad (3)$$

Here the embedding of  $\mathcal{Z}_N$  into  $U(KN)$  is given by  $\mathbf{U} = U \otimes \mathbf{1}_K$  with the  $N$ -by- $N$  clock matrix  $U = \text{diag}(\omega, \omega^2, \dots, \omega^N)$  and  $\omega = \exp(i2\pi/N)$ .  $M_{67}$ ,  $M_{89}$ ,  $\sigma_{67}$  and  $\sigma_{89}$  are rotation generators in the (6, 7)-plane and (8, 9)-plane for a vector and a spinor, respectively.

By now it is well-known that with  $Y^I$  and  $\theta$  written as  $N$ -by- $N$  block-matrices with each block being  $K$ -by- $K$ , the above orbifolding conditions (3) force many blocks to vanish. The outcome is a quiver theory with a lattice interpretation. More concretely,  $Y^i$  ( $i = 1, 2, 3, 4, 5$ ) are block-diagonal; the non-vanishing diagonal blocks are the variables living on *sites* of the circular quiver diagram.  $Z^a$  ( $a = 1, 2$ ) have non-vanishing blocks only just above each diagonal block and at the left-bottom corner; these non-vanishing blocks, as well as their hermitian conjugate, are variables live on the *directed links* connecting the neighboring sites in the quiver diagram. Similarly for fermions, after a change of spinor basis, the 16 components of  $\theta$  are sorted into two groups: eight of them,  $\Psi^\alpha$  ( $\alpha = 1, \dots, 8$ ), live on sites, while the remaining eight,  $\tilde{\Psi}^\beta$  ( $\beta = 1, \dots, 8$ ), live on links connecting neighboring sites. (Their expressions in terms of the original  $\theta$  are given in Appendix.)

Therefore, after orbifolding, the BFSS Hamiltonian on  $\mathcal{C}^2/\mathcal{Z}_N$  is of the following form:

$$\begin{aligned}
 \mathcal{H} = & R \text{Tr} \left\{ \frac{1}{2} \Pi_i^2 + \Pi_a \Pi_a^\dagger - \frac{i}{2} (\Psi^\dagger \sigma^i [Y_i, \Psi] + \tilde{\Psi}^\dagger \bar{\sigma}^i [Y_i, \tilde{\Psi}]) \right. \\
 & - \frac{i}{\sqrt{2}} (\Psi^\dagger \sigma_-^a [Z_a, \tilde{\Psi}] + \Psi^\dagger \sigma_+^a [Z_a^\dagger, \tilde{\Psi}] + \tilde{\Psi}^\dagger \sigma_+^{a\dagger} [Z_a, \Psi] + \tilde{\Psi}^\dagger \sigma_-^{a\dagger} [Z_a^\dagger, \Psi]) \\
 & - \frac{1}{4} [Y^i, Y^j]^2 - [Y^i, Z^a][Y^i, Z^{a\dagger}] \\
 & \left. + \frac{1}{2} [Z^a, Z^{a\dagger}]^2 - [Z^1, Z^2][Z^{1\dagger}, Z^{2\dagger}] - \frac{1}{2} [Z^1, Z^{2\dagger}][Z^{1\dagger}, Z^2] \right\}. \quad (4)
 \end{aligned}$$

A representation of the 8-by-8  $\sigma$ - and  $\bar{\sigma}$ -matrices, which are deduced from the gamma matrices in Eq. (1), is given in Appendix.

If we had not taken the temporal gauge, the gauge potential  $Y^0$  in the BFSS Matrix Theory should be orbifolded too, turning into a site variable in accordance with Eq. (3) with  $I = 0$ . The Lagrangian of the orbifolded system (4), with  $Y^0$  recovered, has an unbroken gauge group  $U(K)^N$ , one factor at each site, which is the block-diagonal subgroup of the original  $U(KN)$ . The site variables  $Y^0$ ,  $Y^i$  and  $\Psi^\alpha$  transform as the adjoint representation under the unbroken gauge group at each site. As for link variables  $Z^a$  and  $\tilde{\Psi}^\beta$ , they transform as the bi-fundamental representation of the two  $U(K)$ 's at neighboring sites connected by the link. In each term of Eq. (4), there is no ambiguity for what site indices should be taken for each factor: the unbroken gauge invariance dictates the site indices of each factor.

Our system (4) has a supersymmetry with 8 supercharges parameterized by a site spinor  $\epsilon$ :

$$\begin{aligned}
 \delta_\epsilon Y_0 &= \epsilon^\dagger \Psi, & \delta_\epsilon Y^i &= \epsilon^\dagger \sigma^i \Psi, & \delta_\epsilon Y^m &= 2\epsilon^\dagger \sigma^m \tilde{\Psi}, \\
 \delta_\epsilon \Psi &= -2i\sigma^{0i} [\nabla_0, Y_i] \epsilon - \sigma^{ij} [Y_i, Y_j] \epsilon - \sigma^{mn} [Y_m, Y_n] \epsilon, \\
 \delta_\epsilon \tilde{\Psi} &= -2i\sigma^{0m} [\nabla_0, Y_m] \epsilon - 2\sigma^{im} [Y_i, Y_m] \epsilon, \quad (5)
 \end{aligned}$$

where  $\nabla_0 = \partial/(R\partial t) - i[Y^0, \cdot]/2\pi$ .

### 3 Deconstructing

The above orbifolded action (4) does not contain hopping terms, i.e. no bilinear terms involving neighboring sites on the lattice. To generate desired string oscillations, we need to

introduce hopping terms by carrying out another key step in the procedure of deconstruction: to consider the Higgs branch with non-vanishing VEV for bosonic link variables. The necessity to do so is natural from the Matrix Theory point of view: After orbifolding, the Hamiltonian (4) describes  $K$   $D0$ -branes at the orbifold singularity, i.e. at the origin of  $\mathcal{C}^2$ . We need to pull these  $D0$ -branes away from the singularity, by giving the variables  $Z^a$  ( $a = 1, 2$ ) a non-vanishing VEV.

Let us reparameterize  $Z^a$  ( $a = 1, 2$ ) in terms of polar coordinates:

$$Z^1 = (\langle Z^1 \rangle + \rho_1) e^{i(\vartheta + \varphi)}, Z^2 = (\langle Z^2 \rangle + \rho_2) e^{i(\vartheta - \varphi)}. \quad (6)$$

The orbifolding conditions (3) are then equivalent to

$$\vartheta \sim \vartheta + \frac{2\pi}{N}, \quad (7)$$

keeping other variables unaffected. (Thus the action of the  $\mathcal{Z}_N$  symmetry is implemented as a discrete subgroup of translations in the angular  $\vartheta$ -direction.)

For convenience, we choose the following moduli conditions for the VEV of bosonic link variables:

$$\langle Z^1 \rangle = -\frac{N R_9}{\sqrt{2}} \mathbf{V}, \quad \langle Z^2 \rangle = 0. \quad (8)$$

(A choice of the classical moduli different from Eq. (8) is expected to differ merely by irrelevant operators in the continuum limit, in the sense of renormalization group when flowing to the infrared region.) Here the minus sign on the right-hand side is a convention.  $R_9$  is the length scale introduced due to the VEV of  $Z^1$ ; its physical meaning is the radius of the M-circle in Matrix String theory via the “9-11 flip”, which is related to the  $IIA$  coupling by  $R_9 = g_s l_s$ .  $\mathbf{V}$  is the shift matrix given by  $V \otimes \mathbf{1}_K$ ,

$$V := \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix}.$$

Then we expand the variables around their VEV:

$$Z^1 = \langle Z^1 \rangle + \frac{1}{\sqrt{2}}(X^6 + iA_1), \quad Z^2 = \frac{1}{\sqrt{2}}(X^7 - iX^8), \quad (9)$$

and substitute these expansions into the orbifolded Hamiltonian (4), we get a quiver model of matrix mechanics, which will be interpreted as a model that deconstructs worldsheet of Matrix String Theory.

To facilitate comparison with the conventions used in Matrix String Theory, we specify the trace to be

$$\text{Tr} = \frac{1}{NR_9} \sum_{n=1}^N \text{tr} \quad (10)$$

where  $\text{tr}$  is the trace over  $K$ -by- $K$  matrices, and  $n$  labels the sites in the quiver diagram. Then the quiver model can be viewed as a lattice Hamiltonian, with a lattice constant determined by the inverse VEV of  $Z^1$ ,

$$a = \frac{2\pi}{NR_9}. \quad (11)$$

In this way, the overall factor  $1/N R_9$  in Eq. (10) essentially plays the same role as the lattice constant usually in front of the summation over sites in a discretized Hamiltonian. More concretely, one may understand the factor  $1/N$  as coming from orbifolding, while the factor  $1/R_9$  later will be seen to control the size of string worldsheet.

One of the advantages of our model is that, as a discrete model, it evades the problem of fermion doubling. To show this, let us consider the spectrum of fermions in our model. Write  $\Psi = (\chi^+, \chi^-)^T$ ,  $\tilde{\Psi} = (\eta^-, \eta^+)^T$ . The free fermionic part of the Hamiltonian reads

$$\begin{aligned} \mathcal{H}_{f.f.} = & \frac{R}{4\pi} a \sum_{n=1}^N \text{tr} \left\{ \chi_n^{+\dagger} \alpha^1 \frac{\eta_n^- - \eta_{n-1}^-}{a} + \eta_n^{-\dagger} \alpha^1 \frac{\chi_{n+1}^+ - \chi_n^+}{a} \right. \\ & \left. - \chi_n^{-\dagger} \alpha^1 \frac{\eta_n^+ - \eta_{n-1}^+}{a} - \eta_n^{+\dagger} \alpha^1 \frac{\chi_{n+1}^- - \chi_n^-}{a} \right\}. \end{aligned} \quad (12)$$

Here we use the convention that fermion at site  $n$  is denoted as  $\chi_n^\pm$ , while fermions on the link connecting sites  $n$  and  $n+1$  as  $\eta_n^\pm$ , and  $\alpha^1 = \mathbf{1}_2 \otimes \tau_2$  with  $\tau_2$  the usual 2-by-2 Pauli matrix. We observe that the Hamiltonian (12) is just that for Susskind's staggered fermions [25]. Therefore as is well-known, there is no fermion doubling in the present context. The natural emergence of staggered fermions is due to the existence of link fermions  $\tilde{\Psi}$  in addition to the site fermions  $\Psi$ ; they combine together to form staggered fermions. We note that link fermions were not considered before in usual lattice field theory models. Their appearance is due to the lattice interpretation of the quiver diagram resulting from orbifolding [15].

## 4 Constructing

Now we proceed to consider the continuum limit of the quiver model that was developed in the last two sections. The continuum limit is defined to be  $N \rightarrow \infty$ ,  $a \rightarrow 0$  with  $Na = 2\pi/R_9$  fixed. In this limit, the distinction between site and link variables disappears, and the quiver diagram (or the theory space) turns into a continuous circle, which combines with the light cone time  $t$  to form closed string worldsheet.

The fixed value of  $Na$  is now understood as the worldsheet length. Let us define the string worldsheet coordinate  $\sigma$  such that its range is  $[0, 2\pi]$ , after rescaling the variables as

$$Y^i \rightarrow \frac{X^i}{\sqrt{R_9}}, \quad (i = 1, \dots, 5), \quad X^m \rightarrow \frac{X^m}{\sqrt{R_9}}, \quad (m = 6, 7, 8), \quad (13)$$

in accordance with the conventions used in the Matrix String Theory [9]. Moreover, to cast the kinetic term for the gauge field in the canonical form, we rescale  $A_0 = Y_0/R_9$  and  $A_1 \rightarrow R_9 A_1$ . Adopting the normalization in which the total light-cone momentum  $P_+ \equiv K/R = 1$ , it is straightforward to show that the orbifolded Hamiltonian (4), with the help of Eqs. (8), (9), (10) and (13), in the continuum limit reproduces a  $d = 1 + 1$ ,  $\mathcal{N} = 8$  SYM theory, whose action is (in string with  $l_s = 1$ )

$$S_{C.L.} = \frac{1}{2\pi} \int d\tau \int_0^{2\pi} d\sigma \operatorname{tr} \left\{ -\frac{1}{2} f_{\mu\nu} f^{\mu\nu} - \frac{1}{2} [\nabla_\mu, X^I] [\nabla^\mu, X^I] + \frac{g_{YM}^2}{4} [X^I, X^J]^2 \right. \\ \left. + \psi^T [\nabla_0, \psi] - \psi^T \tau_3 [\nabla_1, \psi] + ig_{YM} \psi^T \beta^I [X_I, \psi] \right\}. \quad (14)$$

Here the worldsheet indices  $\mu, \nu = 0, 1$ , the target space index  $I$  now runs from 1 to 8, and

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_{YM} [A_\mu, A_\nu],$$

$$[\nabla_\mu, X^I] = \partial_\mu X^I - ig_{YM} [A_\mu, X^I].$$

(The overall factor  $R$ , the light cone radius in DLCQ Matrix Theory, has been absorbed into the definition of the worldsheet time, together with a factor of the inverse  $R_9$  into  $\sigma$ .)

In Eq. (14), we have diagonalized the covariant kinetic energy terms of the fermions ( $\chi^\pm, \eta^\pm$ ) in terms of a new fermion field  $\psi$  and separated the two-dimensional left- and right-moving spinors. The details of the transformations and the resulting  $\beta^I$ -matrices in

the Yukawa coupling terms can be found in Appendix. Now it is easy to check that the continuum limit recovers the rotational symmetry for the eight transverse coordinates  $X^I$ , and the left- and right-moving parts of  $\psi$  transform as  $\mathbf{8}_s$  and  $\mathbf{8}_c$ , respectively.

Thus the action (14) is exactly that of the SYM obtained before by compactifying Matrix Theory on a circle [26, 9, 10, 11, 12], which describes type-*IIA* superstrings (or type-*IIB* *D1*-strings [12]). Moreover, the Yang-Mills coupling  $g_{YM}$  is related to the type-*IIA* string coupling  $g_s$  by

$$g_{YM}^2 = \frac{1}{g_s^2 l_s^2}, \quad (15)$$

exactly the same as that in the Matrix String Theory, where this relation indicates that the string coupling  $g_s$  scales inversely with worldsheet length [9]. From the M theory point of view, this means that the Yang-Mills coupling should be inversely proportional to the M-circle radius  $R_9$ , to give correct scaling for string interactions.

Extended supersymmetry is enhanced to 16 supercharges in the action (14). (Similar enhancement of supersymmetries has been found in deconstructed models before, say in Refs. [15, 19]). Actually the above action is precisely the dimensional reduction of  $d = 1 + 9$ ,  $\mathcal{N} = 1$ ,  $U(K)$  SYM to  $d = 1 + 1$ ; the existence of 16 supersymmetries is evident.

## 5 M Theory Picture

The deconstruction procedure we have presented in above sections has an explicit physical picture in M theory. The orbifolding in Section 2 (see Eq. (3)) is actually to gauge the discrete translations in the angular  $\vartheta$ -direction, defined in Eq. (6), as manifestly shown by Eq. (7). The orbifolded Hamiltonian (4) describes  $K$  *D0*-branes at the  $\mathcal{C}^2/\mathcal{Z}_N$  orbifold singularity. The assignment in Eq. (8) of non-vanishing VEV to the link variables  $Z^1$  means that the classical configuration of *D0*-branes is now away from the orbifold singularity by a distance  $NR_9$  in the wedge  $0 \leq \vartheta \leq 2\pi/N$ . With the two edges identified in the orbifold picture, the *D0*-branes are now in a very thin cone and far away from the tip, so essentially they can be considered to live on a *cylindrical geometry*, whose transverse circle is of radius  $R_9$ . What one has achieved is the compactification of M theory on a circle; our choice of

the moduli implies that it is in the 7th direction. This is the so-called M-circle that relates M theory to type-*IIA* string theory. Note that the identification of the VEV of bosonic link variables with a compactification radius makes sense *only* in string/M theory; in usual deconstruction in non-string-theory context, it is the inverse VEV that is proportional to the lattice constant in the (de)constructed extra dimension [13, 15].

The worldsheet in Matrix String Theory is known to live on the dual circle of the M-circle. To (de)construct it, we have to assign to the circular quiver diagram a lattice constant  $a$ , such that the circumference is just that of the dual circle,  $2\pi/R_9$ . The lattice constant  $a$  given by the deconstruction *ansatz* (11) is just right. Moreover, this choice of  $a$  also turns the coordinate  $Y^7$  in Eq. (9) into a discretized version of a covariant derivative, just as the compactification in M theory required. So in the M theory picture the Matrix String worldsheet is of radius  $1/R_9$ . (In the conventions for Matrix String Theory, the lattice constant is essentially  $2\pi/N$  with the normalization  $Na = 2\pi$ .) Therefore, what the quiver diagram deconstructs is indeed the Matrix String worldsheet, and the deconstructed lattice constant (11) exhibits explicitly a duality between worldsheet (de)construction and target space compactification in M theory. Previously in Matrix Theory this duality appears as an *ansatz* for solving the quotient conditions for compactification. Here in this note the duality naturally appears due to deconstruction of worldsheet, which also gives rise to correct scaling for string interactions.

## 6 Discussions

In this note, we have proposed to deconstruct string worldsheet, resulting in a quiver model of supersymmetric matrix quantum mechanics, which provides the DLCQ description of type-*IIA* superstring bits. The Matrix String Theory is recovered exactly in the continuum limit. In the spirit of providing a lattice formulation of string theory, our proposal is similar to string bit models. But there are several important differences arising from deconstruction in contrast to naively discretizing the string action.

First, in addition to gauge fields, there are other dynamical variables living on links. On one hand, there are link fermions. This makes it possible to evade the famous fermion dou-



bling problem in the usual lattice theory. Also the fact that the theory has eight extended supersymmetry which is enhanced in the continuum limit to sixteen supercharges is closely related to the existence of link fermions. On the other hand, unlike the Wilson lines in lattice gauge theory, the bosonic link variables are non-unitary. Since the geometric significance of link variables is parallel transport, the non-unitary portion in them implies a dynamical effects on geometry. We believe the introduction of the non-unitary links should have profound effects on quantum gravity, hence on string/M theory as well as on holography.

Secondly, the lattice spacing  $a$  is now related to the expectation value of bosonic link variables and, therefore, becomes dynamical in the context of deconstruction, as a characteristics shared by all deconstruction models. This is a wonderful feature, distinct from usual discretization in which lattice spacing is introduced by hand merely as a means of regularization. In other words, the lattice spacing may ultimately be a physical quantity determined by the underlying nonperturbative dynamics, while the continuum geometry emerges effectively in the infrared regime. It is in this sense we think the quiver matrix quantum mechanics may perhaps be more fundamental/microscopic than Matrix String Theory is.

Normally the idea of deconstruction is used to deal with a higher dimensional *target space* in terms of a lower dimensional theory. In this note what we have attempted to do is to deconstruct *string worldsheet*. The success seems to imply a *worldsheet/target-space duality*. Now we have two different ways to obtain the Matrix String Theory. The standard routine is to compactify the BFSS matrix quantum mechanics on a circle and obtain the Matrix String Theory as an SYM on the dual circle. In this note, we orbifolded the BFSS model, and consider the deconstruction phase of the resulting quantum mechanics; after taking the continuum limit, we recover Matrix String Theory. So what we have done is actually the deconstruction of the dual space of the compactified circle in Matrix Theory. In this way, one may say that we have a duality between worldsheet deconstruction and target space compactification in string/M theory. The well-known *IIA/M* duality can be viewed as a prototype of this duality; namely the eleventh dimension in M theory is originated from trading the worldsheet spatial dimension to target space.

Our quiver mechanics model of strings is motivated by orbifolding the BFSS Matrix

model and taking the deconstruction phase. However this proposal can be taken directly as the starting point of an alternative to string field theory. The hope is that the quiver matrix mechanics (with finite  $N$ ) could be shown to be mathematically more manipulable and less singular than string field theory. One further advantage of studying the quiver mechanics seems to be that it opens the door for numerical study of string theory. One can start with the  $K = 1$  case, which provides a well-behaved, discretized toy model for superstring theory. Another possible advantage of the quiver mechanical approach is to provide a framework for studying renormalization group flow or even phase diagrams or phase transitions in string/M theory.

In this note, we have studied the simplest case of quiver mechanical models in string theory. It is obvious that more complicated quiver mechanical models can be constructed for higher dimensional extended objects, e.g.  $Dp$ -branes, or for strings in other backgrounds, such as pp-waves and some noncommutative geometric backgrounds. It is expected that the success of formulating superstring theory in terms of discrete models should make well-developed notions and techniques for one- or two-dimensional many-body models on a lattice, exactly solvable or not, accessible to string/M theory.

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*Note Added* When this work was completed and being written up, a preprint (hep-th/0306147) by Danielsson *et al.* appeared [27], in which the use of staggered fermions in a string bit model was also proposed to avoid the fermion doubling problem.

## A Representations of Gamma Matrices and Fermions

In this Appendix, we give some technical details about how fermions are transformed at each step of our (de)construction. There are good reasons to believe that the sixteen fermions in the continuum action (14) must possess the correct chirality properties under the transverse  $SO(8)$  rotations to describe type-*IIA* strings. However, it is non-trivial to see how this happens explicitly, since the  $SO(8)$  symmetry is violated by orbifolding and gets recovered only

in the continuum limit. This is also crucial for the supersymmetries broken by orbifolding to get recovered.

In Eq. (1) the fermionic coordinates  $\theta$  is an  $SO(9)$  Majorana spinor of 16 real components, which can be labelled by four indices  $s_1, s_2, s_3, s_4$ , each taking two values only. The gamma matrices  $\gamma^I$  ( $I = 1, \dots, 9$ ) can be read off from the standard  $\Gamma$ -matrices in ten dimensions [28], and are represented by a direct product of four 2-by-2 matrices:

$$\begin{aligned}
 \gamma^1 &= \epsilon \otimes \epsilon \otimes \epsilon \otimes \epsilon \quad , \quad \gamma^2 = \tau_1 \otimes \mathbf{1} \otimes \epsilon \otimes \epsilon, \\
 \gamma^3 &= \tau_3 \otimes \mathbf{1} \otimes \epsilon \otimes \epsilon \quad , \quad \gamma^4 = \epsilon \otimes \tau_1 \otimes \mathbf{1} \otimes \epsilon, \\
 \gamma^5 &= \epsilon \otimes \tau_3 \otimes \mathbf{1} \otimes \epsilon \quad , \quad \gamma^6 = \mathbf{1} \otimes \epsilon \otimes \tau_1 \otimes \epsilon, \\
 \gamma^7 &= \mathbf{1} \otimes \epsilon \otimes \tau_3 \otimes \epsilon \quad , \quad \gamma^8 = \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \otimes \tau_1, \\
 \gamma^9 &= \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \otimes \tau_3 \quad .
 \end{aligned} \tag{16}$$

Here we used  $\mathbf{1}$  and  $\tau_i$  to denote the 2-by-2 unit and Pauli matrices; and  $\epsilon = i\tau_2$ .

The change from  $\theta$ -spinors to the site fermions  $\Psi$  and the link fermions  $\tilde{\Psi}$  in Eq. (4) only involves the indices  $s_3, s_4$ :

$$\begin{pmatrix} \theta^{s_1 s_2 ++} \\ \theta^{s_1 s_2 +-} \\ \theta^{s_1 s_2 -+} \\ \theta^{s_1 s_2 --} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} (\theta^{s_1 s_2 11} + i\theta^{s_1 s_2 12}) + i(\theta^{s_1 s_2 21} + i\theta^{s_1 s_2 22}) \\ (\theta^{s_1 s_2 11} - i\theta^{s_1 s_2 12}) + i(\theta^{s_1 s_2 21} - i\theta^{s_1 s_2 22}) \\ (\theta^{s_1 s_2 11} + i\theta^{s_1 s_2 12}) - i(\theta^{s_1 s_2 21} + i\theta^{s_1 s_2 22}) \\ (\theta^{s_1 s_2 11} - i\theta^{s_1 s_2 12}) - i(\theta^{s_1 s_2 21} - i\theta^{s_1 s_2 22}) \end{pmatrix}, \tag{17}$$

Then we have  $\Psi = (\theta^{s_1 s_2 ++}, \theta^{s_1 s_2 --})^T$ ,  $\tilde{\Psi} = (\theta^{s_1 s_2 -+}, \theta^{s_1 s_2 +-})^T$ .

This transformation gives rise to the following matrices in Eq. (4):

$$\begin{aligned}
 \sigma^1 &= \tau_2 \otimes \tau_2 \otimes \mathbf{1} \quad , \quad \bar{\sigma}^1 = -\sigma^1, \\
 \sigma^2 &= -\tau_1 \otimes \mathbf{1} \otimes \mathbf{1} \quad , \quad \bar{\sigma}^2 = -\sigma^2, \\
 \sigma^3 &= -\tau_3 \otimes \mathbf{1} \otimes \mathbf{1} \quad , \quad \bar{\sigma}^3 = -\sigma^3, \\
 \sigma^4 &= \tau_2 \otimes \tau_1 \otimes \tau_3 \quad , \quad \bar{\sigma}^4 = \sigma^4, \\
 \sigma^5 &= \tau_2 \otimes \tau_3 \otimes \tau_3 \quad , \quad \bar{\sigma}^5 = \sigma^5, \\
 \sigma^6 &= i\mathbf{1} \otimes \tau_2 \otimes \mathbf{1} \quad , \quad \sigma^7 = \mathbf{1} \otimes \tau_2 \otimes \tau_3,
 \end{aligned}$$

$$\begin{aligned}\sigma^8 &= -\mathbf{1} \otimes \mathbf{1} \otimes \tau_2, & \sigma^9 &= \mathbf{1} \otimes \mathbf{1} \otimes \tau_1. \\ \sigma_{\pm}^1 &= \frac{\sigma^6 \pm i\sigma^7}{2}, & \sigma_{\pm}^2 &= \frac{\sigma^8 \mp i\sigma^9}{2}.\end{aligned}\quad (18)$$

In Eq. (12) we have  $\chi^+ = \theta^{s_1 s_2 ++}$ ,  $\chi^- = \theta^{s_1 s_2 --}$ ,  $\eta^+ = \theta^{s_1 s_2 +-}$ ,  $\eta^- = \theta^{s_1 s_2 -+}$ . Hence  $\chi^{-\dagger} = \chi^+$ ,  $\eta^{-\dagger} = \eta^+$ .

The continuum limit irons out the distinction between site and link fermions, and the fermionic part of the action reads

$$S = \frac{1}{2\pi} \int d^2\sigma \operatorname{tr} \{ \psi^T [\nabla_0, \psi] - \psi^T \gamma^7 [\nabla_1, \psi] \quad (19)$$

$$+ i g_{YM} (\psi^T \gamma^i [X_i, \psi] + \psi^T \gamma^8 [X_7, \psi] + \psi^T \gamma^9 [X_8, \psi]) \}. \quad (20)$$

Here  $\psi$  is just the image of  $\chi$  and  $\eta$  under the inverse transformation of Eq. (17), with positions of indices  $s_2$  and  $s_3$  interchanged only for convenience. To separate the left- and right-moving components we need to do orthogonal transformations to diagonalize  $\gamma^7$  to be of the form  $\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \otimes \tau_3$ . Observe that  $\gamma^7 = \mathbf{1} \otimes \epsilon \otimes \tau_3 \otimes \epsilon$ . We first try to do an orthogonal transformation  $N_1$  involving only the second and the fourth factors in the product, such that

$$(\epsilon \otimes \epsilon) N_1 = N_1 (\mathbf{1} \otimes \tau_3). \quad (21)$$

Then we do a second orthogonal transformation  $N_2$  involving only the third and the fourth factors in the product, such that

$$(\tau_3 \otimes \tau_3) N_2 = N_2 (\mathbf{1} \otimes \tau_3). \quad (22)$$

The explicit form of  $N_1$  and  $N_2$  are

$$N_1 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \end{pmatrix}, \quad N_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (23)$$

In the continuum action (14) we use the same notation  $\psi$  for the transformed fermions, with the gamma matrices other than  $\gamma^7$  changing into eight  $\beta$ -matrices, and with  $\gamma^7$  into  $\beta^9$ :

$$\beta^1 = \epsilon \otimes \tau_1 \otimes \mathbf{1} \otimes \epsilon, \quad \beta^2 = \tau_1 \otimes \epsilon \otimes \epsilon \otimes \tau_1,$$

$$\begin{aligned}
 \beta^3 &= \tau_3 \otimes \epsilon \otimes \epsilon \otimes \tau_1 \quad , \quad \beta^4 = -\epsilon \otimes \tau_3 \otimes \mathbf{1} \otimes \epsilon, \\
 \beta^5 &= \epsilon \otimes \mathbf{1} \otimes \epsilon \otimes \tau_1 \quad , \quad \beta^6 = \mathbf{1} \otimes \epsilon \otimes \mathbf{1} \otimes \epsilon, \\
 \beta^7 &= \mathbf{1} \otimes \mathbf{1} \otimes \tau_1 \otimes \tau_1 \quad , \quad \beta^8 = \mathbf{1} \otimes \mathbf{1} \otimes \tau_3 \otimes \tau_1.
 \end{aligned} \tag{24}$$

The explicit expression  $\beta^9 = \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \otimes \tau_3$  shows that the left- and right-moving components of  $\psi$  have opposite chirality under the transverse  $SO(8)$  rotations. So the action (14) describes type-*IIA* strings.

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