The Trouble with Superselection Accounts of Measurement*

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A superselection rule advanced in the course of a quantum-mechanical treatment of some phenomenon is an assertion to the effect that the superposition principle of quantum mechanics is to be restricted in the application at hand. Superselection accounts of measurement all have in common a decision to represent the indicator states of detectors by eigenspaces of superselection operators named in a superselection rule, on the grounds that the states in question are states of a so-called *classical* quantity and therefore not subject to quantum interference effects. By this strategy superselectionists of measurement expect to dispense with use of projection postulates in treatments of measurement. I shall argue that superselection accounts of measurement are self-contradictory, and that treatments of infinite systems, if they can avoid the contradiction, are not true superselection accounts.

1. Introduction.

> We have assumed that every bounded Hermitian operator represents a measurable quantity. This assumption is acceptable for many simple physical systems, but there are systems for which it is wrong. (Jordan [14, 89])

In contemporary quantum mechanics we have had to come to terms with the proposition that the quantum state of a modeled system, at

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any given moment in time, is a collection of probability assignments to a class of potential events, or *prospects,* associated with this system. The prospects for each physical system stand to each other in the structure of an algebra, each node of which signifying that a measurable quantity, also called an *observable,* recognized by the theory as belonging to the system being treated, takes on a magnitude lying in a specified range. The mathematical representation of the quantum state is therefore a function from nodes in an algebra representing those prospects to real numbers in the interval [0, 1]. Thus rather than saying of a system treated by quantum mechanics—as we might say of a system treated by classical physics—that its state representation is a collection of *value* or *magnitude* assignments made to its quantities, it is better to say that the state representation of quantum mechanics is a report on the *conditions* of the system's quantities.

We begin modeling, as Jordan in the above-quoted passage remarks, with the hypothesis that every bounded Hermitian operator defined on the chosen representation space of functions from magnitudes to intervals, shall stand in a unique representation relation to a single measurable quantity, but quickly discover that this proposal cannot be adhered to universally. It can be adhered to, as will be discussed here at some length, only on condition that superselection operators do not apply in the model being employed. Systems represented in Hilbert spaces in connection with which the initial hypothesis holds true are called *simple.* Predication of this simplicity to systems absolutely is infelicitous, since by definition the term refers primarily to a *relation* between physical systems and entities serving to model them—a relation between objects like free spinless particles and certain abstract clinics in which they typically receive treatment at the hands of theoreticians. In applying the term 'simple' to systems absolutely, we give the impression that simplicity is a quality of systems in themselves. But this is misleading. Simplicity, as here defined, is concerned with a relationship between model and system modeled. While I will not urge new terminology, nevertheless I will suggest that we should not forget this point, as relationships between models and systems modeled are at the center of the superselection proposal to treat quantum measurement, and these relationships will thus be the primary focus of this essay.

A vast number of *nonsimple* systems exist. For nonsimple systems, the preliminary hypothesis to which Jordan draws attention must be restricted as follows: each bounded Hermitian operator *that commutes with every superselection operator in the model chosen for the system* stands in a unique representation relation to a single measurable quantity.

A superselection operator exists when the space representing it is, in the following sense, too rich: there is more structure in the representing space than is acknowledged to exist in the physical system being treated. Under these conditions a superselection *rule* is said to operate, for the sake of preventing illicit use, by theoreticians, of this excess structure. A superselection rule tolerates the existence of excess structure so long as (1) the theory framed in the model does not make representational use of elements which describe no physical situation of any kind, and (2) the theory does not make discriminations among elements of the model which describe physically identical situations, relative to the modeling purposes to which the model is put. Excess structure is to be tolerated because it can sometimes be used to advantage, and without violating either of (1) or (2), for representing quantities or states that do not participate in quantum (sometimes called *interference*) effects.

Superselection theories of the process of quantum measurement put excess structure to work for the sake of treating interactions between measuring and measured within an (otherwise) canonical framework. Superselection treatments of measurement, which are intended to replace the use of projection postulates, have enjoyed considerable attention since the early 1980s. A cross section of these includes Araki [1], Beltrametti and Casinelli [2], Machida and Namiki [16], Bub [8], [6], [7], [9], Wan [22]. It is the core principle of superselection accounts—which I hereby designate the *superselection policy*—that the indicator states of detection devices shall be represented by the eigenspaces of superselection operators designated by a superselection rule.

I shall argue that superselection accounts of measurement exploit excess structure illegitimately, and in the process become self-contradictory.

2. Measurement Tensions in Quantum Theory. In the life of any physical theory there arises a set of theoretical questions having to do with measurement, which may be posed to that theory concerning each quantity it recognizes. The characteristic question of this group concerns *ideal* measurement. Suppose the theory recognizes a quantity Q as belonging to a system S, which it portrays as lying in a state ψ. The characteristic question concerning ideal measurement is: Does the theory in question recognize the possibility of an ideal *process* of measuring Q in the state ψ by a suitable measuring device M? In an ideal process, the interaction between S and M terminates in a state which includes the joint conditions of S and M, such that the condition of some quantity belonging to M—say Q_M—unambiguously reveals the (original) condition of Q in ψ because the magnitudes of this Q_M are perfectly correlated with those of Q. Classical mechanics has no general difficulties with justifying affirmative answers to questions of this form, while quantum theory is plagued by its *systematic* inability to offer affirmative answers. Ironically, measurement of systems treated by quantum mechanics is ubiquitous in practice, but (apparently) impossible in theory. Let us examine a very simple, but nevertheless generic example.

Suppose a system S is selected, to which belongs a quantity A whose operator representation has a set of eigenvectors which I shall designate by $\{\phi_i | i = 1, \ldots, n\}$, and that the eigenvalues of the operator (also A) representing this quantity are not multiply degenerate.[1] S, let us suppose, has been prepared in a state $\psi(0) = \Sigma_r\, c_r \phi_r$, with r ranging over members of the index set of the eigenvectors $\{\phi_i\}$. We choose a nondestructive measurement process,[2] in which the measured system S and the measuring device M, initially independent of each other, interact for such a time as to form a compound system, then separate again in a fashion that afterwards it continues to be sensible to speak of a final state of S and a final state of M as distinct systems. And we perform a measurement on S by apparatus M of the quantity A, without making note of the results.

Then the *measurement problem of quantum mechanics* is constituted by tensions among the following:

P1 (Comparability): Indicator states of detector M can be represented via Hilbert space vectors and operators on them, just as the conditions of microquantities are routinely represented. Let A_M, in a Hilbert space $\mathscr{H}_M$, represent the indicator quantity, and let σ_0, σ_1, σ_2, . . . , be its eigenvectors, representing states in which A_M takes on definite magnitudes, with σ_0 representing the state of null indication. The initial state of the composite $S + M$ is therefore $\psi_0 \otimes \sigma_0$ (in operator form, $P_{\psi_0} \otimes P_{\sigma_0}$).

P2 (Definiteness): Subsequent to a measurement transaction, the composite $S + M$ must lie in a state in which the attribute A_M is, at least for a brief interval of time, value-definite; it must therefore be a mixture of the form $W = \Sigma_r |c_r'|^2\, P_{\varphi_r} \otimes P_{\sigma_r}$, with P_{φ_r} representing

1. I will follow the convention that Greek characters shall range over mathematical representations of quantum states: vectors in a Hilbert space. Each such vector in turn represents a function from attributes or prospects to real numbers in the interval [0, 1]. A capital Roman character shall designate an operator as well as the physical quantity the operator is intended to signify; the context will determine which of these is the intended denotation for a particular token. The operator P_ϕ shall designate the operation of projection onto the Hilbert subspace spanned by the vector ϕ. I limit attention for the purposes of this discussion to operators having eigenvectors and point spectra.

2. To borrow a notion from Beltrametti and Cassinelli [2].

states of S. (No Hilbert space vector representation of this condition is available.)

P3 (Insolubility): There exists no unitary time transformation from an initial state represented by a formula $P_{\psi_0} \otimes P_{\sigma_0} \in \mathscr{H}_S \otimes \mathscr{H}_M$, into a final state represented by the formula W of (P2), if minimal restrictions on the relations between c and c' are imposed.[3]

(P1) is the proposition that the quantities of S and the quantities of M, whether these are microscopic or macroscopic, are owed comparable treatment. (P2) articulates a certain constraint on what shall count as an episode of measurement. Unavoidably it adheres to some, if modest, principles of interpreting state functions, as any constraint on what shall count as a measurement must give at least a partial interpretation of these. (And of course all matters of interpretation are subject to challenge.) According to a canon of interpretation to which advocates of (P2) subscribe—though of course one which is not universally accepted—neither that state represented by a superposition of the form $\Sigma_r c_r\sigma_r$ nor that represented by $\Sigma_r c_r\phi_r \otimes \sigma_r$ is one in which the magnitude of A_M is definite.[4] But the terminal state of a composite having undergone measurement cannot be any state whatever: it must be one in which the magnitude of A_M is, without question, definite; for the process to which S falls victim could not otherwise count as a measurement. A measurement, if it is nothing else, is a process at the conclusion of which a detection device registers a definite (and usually stable) reading.

Among (P1)–(P3), (P3) (Insolubility) is inescapable. Its demonstration requires assumption of the following, minimal, presumptions: (a) the terminal state of $S + M$ shall be representable in the form $\Sigma_r |c_r'|^2 P_{\varphi_r} \otimes P_{\sigma_r}$, with no restrictions whatever on φ_r, but with restrictions on c_r' and σ_r that capture certain very minimal conditions which justify regarding the process that S and M undergo as a *measurement* process;[5] and (b) the time transitions countenanced shall be consistent

3. The proposal to look at the possibility of transitions to a final state in which $c_r' \neq c_r$ and $\varphi \neq \phi_r$, for some r, was first considered by Wigner [24], who argued that no interaction satisfying very stringent criteria could eventuate in a nontrivial mixture of eigenstates for the composite. Subsequent work by A. Fine [12], whose proof was later improved by A. Shimony [18], has established generalizations of Wigner's negative results by weakening his proposed criteria. Brown [5] gives an overview of these results. H. Stein [19] presents a simpler and more elegant proof of these matters.

4. The convention has come to be known as the *eigenstate-eigenvalue principle*, according to which a quantity Q has a definite magnitude if and only if the system's state representation is a mixture (trivial or not) of eigenvectors of Q.

5. These restrictions are discussed in detail in Stein [19]. These do *not* assert that in ideal measurement the *measured* system comes to have a definite magnitude of the quantity measured on it. This condition has been relaxed for quantum mechanics.

with the time-dependent Schrödinger equation. It is then shown that according to canonical quantum theory, no process satisfying (a), (b), (P1) and (P2), exists, for the time-dependent Schrödinger equation simply does not admit of transformations satisfying (P1), (P2) and (a), even if representations of the condition of M are allowed to range over the set of convex sums of projection operators on $\mathcal{H}_M$ (and hence to range over all probability states) representable in terms of the macroquantity defined in (P1). Surrender of either (P1) or (P2) will, of course, dissolve the tension. As will surrender of (a) or (b). The measurement problem exists only so long each of the four conditions are retained.

The measuring device is wanted in a state of definite indication—at all times to be sure, but at the conclusion of measurement processes especially; for nothing else is recognized as legitimate by our experience. The process by which this coming to definiteness takes place is sometimes called *reduction of the wave packet* or *wave function collapse.* But canonical quantum theory does not and, to all appearances, *cannot* acknowledge the existence of any such process. Addition of an axiom called a *projection postulate* was once intended as an antidote to this defect in the theory. A projection postulate is an assertion to the effect that the process in question exists. With one stroke a projection postulate confers on certain unobserved entangled states (ones in which the magnitude of a certain quantity of the measuring device is not definite, hence states very *un*like W) an equal right to exist, while demanding their hasty retreat in favor of the more familiar W. But since the problem faced by the quantum theory resides in the lack of *consistency* among (P1), (P2), (a) and (b), simple addition of postulates cannot remove the defect.

3. Superselection Rules, and the Modeling of Physical Systems. We say that a subspace $\mathcal{M}$ *reduces* a linear operator B if $B\psi$ lies in $\mathcal{M}$ for every vector ψ in $\mathcal{M}$ and $B\psi$ lies in $\mathcal{M}^{\perp}$ for every vector ψ in $\mathcal{M}^{\perp}$. This condition holds just in case for every ψ in $\mathcal{M}$, P_{ψ} commutes with B [15, 37]. And we say that a Hilbert space $\mathcal{M}$ is *invariant* (or *simple*) *under a set of (Hermitian) operators S* when $\mathcal{M}$ reduces every member of S. Then we can say that a subspace $\mathcal{M}$ is invariant under a set of (Hermitian) operators S when, of all those operators defined on it, only multiples of the identity operator commute with every member of S.[6] A nontrivial operator that commutes with every self-adjoint operator in an algebra of operators is called a *superselection operator* or *superselector,* relative

6. By Schur's Lemma, cf. [14, 67ff].

to that algebra of operators. And each such superselector marks a set of *superselection subspaces,* also known as *coherent* subspaces or *supersectors.*

A simple example will serve to illustrate these definitions. Select a coordinate frame for a system S, letting x, y and z name the three spatial dimensions, and let the operator J_3 represents the system's z-directed component of angular momentum. J_3 is therefore also a representation of S's orientation along the z axis. Consider the operator $R = e^{-i2\pi J_3}$. Application of R will therefore bring about a rotation, through an angle of 2π about the z axis, of the orientation of J_3. R may thus represent the process of rotating the system S through an angle of 2π. If S is the only system being treated, then we can also conceive of this rotation as a process that leaves S alone, but rotates the *coordinate axes,* the orientation of which is selected for convenience's sake, through an angle of -2π about the original z axis. If we conceive of the operation to which R corresponds in the second way, then we shall say that it has no effect on S—for the entity (the coordinate axes) on which R represents an operation is not one that may interact with any physical system, S included. Some would say, in fact, that it is no entity of any kind. The two conceptions of the maneuver—the first being that R corresponds to an operation on S, and the second being that R corresponds to an operation on the coordinate axes—are equivalent, significantly enough, so long as our world exhibits a rotational symmetry of 2π about the z axis—which it would appear to do. Since the first process is equivalent to the second, no measurable quantity in our system should be affected by it. Measurable quantities are unaffected by human institutions of coordination, the objects they call into existence (coordinate axes) or operations thereon. Thus R represents a process only by a stretching of terminology. And operators representing measurable quantities should commute with R, since application of R should have no effect on application of operators representing measurable quantities. This condition can be imposed as follows: Let B be a bounded operator representing a measurable quantity. Then

$$\forall B \text{ (if } B \text{ is a measurable quantity, then } e^{i2\pi J_3} B e^{-i2\pi J_3} = B). \quad (3.1)$$

R in this case has eigenvalues 1 and -1 since (as I shall simply stipulate) J_3 can take on integer and half odd integer values. Let ψ be a vector such that $R\psi = \psi$, and ϕ be such that $R\phi = -\phi$. ϕ and ψ are thus eigenstates of R, with $(\phi, B\psi) = (\phi, e^{i2\pi J_3} Be^{-i2\pi J_3}\psi) = (e^{-i2\pi J_3}\phi, Be^{-i2\pi J_3}\psi) = -(\phi, B\psi)$. And so $(\phi, B\psi) = 0$. Hence, if a system is such that it may exhibit either integral and half-odd-integral eigenvalues of angular momentum, every operator B that represents a measurable quantity on that system will be such that $B\psi$ lies in the same eigenspace of R as ψ. The subspace spanned by ψ is therefore invariant: the operation of a measurable quantity B on any object lying in the subspace results in a object that lies, once again, in that subspace. Once (3.1) is adopted, therefore, a set of invariant subspaces—*supersectors*—emerges.

We can give a physical interpretation of the invariance property for a supersector S as follows: S represents a group of states which have in common the magnitude of R, and no measurable quantity can interact with either R, or any other measurable quantity, in such a way as to alter the magnitude of R for a physical system. This formal condition, we might say, corresponds to the fact that states represented in different invariant subspaces are prevented by nature from combining with each other. R is thus a superselector relative to the algebra of measurable quantities exemplified by systems subject to that rule of nature.

Superselectors are normally *discovered,* in relation to function spaces already in view, and algebras of operators on them: a superselector is identified when an operator is discovered to commute with every operator in a chosen algebra. Another procedure is to bring an algebra of measurable quantities to attention, as a distinguished set, through specification of superselectors: for example, a von Neumann algebra is said to be *generated* by a specified set S of projection operators, when it includes all and only those operators formed by taking scalar multiples, sums, products and weak limits (starting with bounded functions) of members of S. Thus one means of *restricting* membership in an operator algebra of measurable quantities is to include only operators which commute with a designated operator, or set of operators. A membership restriction of this form is called a *superselection rule.* In the presence of a superselection rule, an operator B will be admitted into a target algebra only if it commutes with every operator named by the rule.

There are important effects, on the model being used, of adopting a superselection rule, for example the following: in the presence of a superselection rule on a space $\mathscr{H}$, the set of all density operators defined on $\mathscr{H}$ becomes, in a very specific sense, overlarge: there exist sets of density operators, such that any two members of the same set, though they might have different mathematical properties, will nevertheless agree on all probability measure assignments made to the prospects associated with physical quantities. This equivalence is of great significance, since the set of physical quantities for a given system S, which I will call $\mathscr{A}_S^*$, comprises the only quantities whose conditions matter from the point of view of modeling S. Superselection rules are, therefore, a gauge of excess model structure. What may be surprising, however, is the *formal relations* between operators that represent identical probability assignments. An illustration is worth a thousand words.

Consider a unit vector ϕ_k in a supersector $\mathscr{M}_k$ and a second unit vector $\phi_{k'}$ in another superselection subspace $\mathscr{M}_{k'}$ such that $k \neq k'$, and form the superposition $\psi = \lambda\phi_k + \lambda'\phi_{k'}$ with $|\lambda|^2 + |\lambda'|^2 = 1$. Now let $D = |\lambda|^2 P_{\phi_k} + |\lambda'|^2 P_{\phi_{k'}}$. If we then choose a basis for the composite Hilbert space $\mathscr{H}$ of which ϕ_k and $\phi_{k'}$ are both members, and let P be any projection operator reduced by the coherent subspaces, we have: $\mathrm{Tr}(PP_\psi) = (\phi_k, PP_\psi\phi_k) + (\phi_{k'}, PP_\psi\phi_{k'}) = |\lambda|^2 (\phi_k, P\phi_k) + |\lambda'|^2 (\phi_{k'}, P\phi_{k'})$.[7] But now compare $\mathrm{Tr}(PD) = (\phi_k, P[|\lambda|^2P_{\phi_k} + |\lambda'|^2P_{\phi_{k'}}]\phi_k) + (\phi_{k'}, P[|\lambda|^2P_{\phi_k} + |\lambda'|^2P_{\phi_{k'}}]\phi_{k'}) = |\lambda|^2(\phi_k, P\phi_k) + |\lambda'|^2(\phi_{k'}, P\phi_{k'})$. The traces are equal. Now since every observable in the algebra $\mathscr{A}_S^*$ is reduced by the coherent subspaces, every observable will be a linear function of projectors reduced by the coherent subspaces. Consequently the two density operators P_ψ and D will agree on probability assignments for every operator reduced by the coherent subspaces, even though other of their mathematical properties differ; P_ψ and D make the same assignments of probability to every prospect in the algebra $\mathscr{A}_S^*$. Since in contemporary quantum mechanics we accept the proposition that a state of a system S is nothing more than a collection of probability assignments to that special class of prospects associated with the algebra of quantities $\mathscr{A}_S^*$, we are obliged to accept both P_ψ and D as (perhaps syntactically different) representations of the *same state*. It turns out, in fact, that in Hilbert spaces with superselection rules there exist infinitely large equivalence classes of density operators, each member of a given class equally suitable for representing the same state.

Despite this excess, it can be shown that in the presence of superselection rules there is exactly one operator in each of the equivalence classes with the feature that it commutes with every member of the set of superselectors, and as a result with every member of $\mathscr{A}_S^*$. In each equivalence class this is the mixture whose parts are projection operators onto the designated supersectors. (Demonstration of the uniqueness of the canonical mixture is lightly sketched in Beltrametti and Cassinelli [2, Ch. 5].) Beltrametti and Cassinelli [2] designate this the *canonical mixture* associated with the corresponding pure state; I shall follow their assignation. Thus whilst the set of physical states does not correspond one-to-one with the set of density operators, even so the set of physical states corresponds one-to-one with the set of density operators that are *also* members of $\mathscr{A}_S^*$.

The argument that P_ψ and D are representations of the same physi-

7. A detailed proof appears in van Fraassen [20, 190].

cal state, together with demonstration that *D* is a preferred representation, provide grounds for the proposition that superpositions of vectors drawn from distinct eigenspaces do not represent new states, but instead *mixtures*—that is to say logical combinations—of eigenstates already recognized. No new physical states can be manufactured by superposite combination of these eigenstates because nature, as it is said, keeps the eigenstates *separated.* It is this idea—that there are laws of nature, expressible in the form of superselection rules, which give rise to separated states—that motivates superselection-based treatments of, for example, nuclear processes. In nuclear physics the fundamental nucleons (protons and neutrons) are often treated as if they were merely different states of the same kind of physical system: the so-called nucleon. The circumstances in which such treatment does not lead to error are those in which the quantities (for example, charge), which would in different circumstances distinguish the behaviors of systems to which they belong, are of either little or no consequence in the circumstances treated by the proposal, because the electrical forces in those circumstances are either nonexistent or negligible in comparison with the strong nuclear forces that dominate; as a result, proton and neutron behave identically under these circumstances. Thus there need be no treatment of quantities that alter under the exertion of electrical forces. In such cases the invention of the nucleon simplifies the treatment of the problem. But the simplification is purchased at the price of introducing excess structure, the exploitation of which must be vigilantly prohibited: a superposition of pure proton and pure neutron is not recognized as representing a genuine condition of a nucleon. Well-established terminology now has it that a law, expressible via a superselection rule and called the *law of conservation of electric charge,* is at work keeping the nucleon states separated.[8] We have an illustration here, therefore, of how a fact that concerns relations between objects and mathematical representations can be conceived of as a law of nature, and expressed in the form of a superselection rule.

One moral I shall draw at this point is that supersectors are structurally equivalent to each other, in relation to a distinguished algebra of operators—an algebra distinguished by our intentions to treat certain quantities and leave aside others. We may thus treat these supersectors as capable of representing exactly the same entities and situations in the world, each supersector being a perfect duplicate of the others, relative to the matters we are interested in treating. Or we may permit the use of different supersectors to represent different entities or situations at different times—so long as the entities we choose to

8. A similar case is made for baryonic charge conservation.

represent are, for the purposes of the treatment, structurally identical—simply by inscribing our intentions in the legend of the model, as it were. And we shall be permitted to do the latter provided that we do not recognize real-time transitions between the fictionally identical states: we must refuse to acknowledge transitions to and from these fictionally identical states as straightforward descriptions of real-time goings-on.

What we may *not* do, however, is pretend that the legend inscription is a part of the model itself. If we elect to represent protons and neutrons as different states, identical in mathematical form, of a fictitious observable (nucleon), we cannot claim that there is a piece of structure in our model that corresponds to proton and a *different* piece of structure that corresponds to neutron. For the assignment of a specific supersector to the neutron can be nothing but a matter of convention, not a matter of ensuring that the supersector selected has properties suited to the representation of neutron by contrast with proton. If it were not so, our pretense of nucleons should otherwise be a sham pretense.

4. Classical Quantities. It must be recognized as a philosophical principle of the highest importance that the structures present in our (mathematical) models, alone, cannot determine which elements of these models may be put to representing realities existing in the world, and which may not. Thus the structure of a Hilbert space is not, by itself, sufficient for determining which operators defined on it may be engaged for representing true-life measurable quantities and which may not. Even the most thorough inspection of the mathematical characteristics of a simple rotation operator like R (of previous acquaintance in this piece) does not reveal that measurable quantities represented by operators on the Hilbert space should not have their operations affected by those alterations (in this case rotations) brought about by R; this proposition concerns potential relations of representation between elements of the model and those portions of the world which they are put to modelling. In other words, it concerns the potential for modeling the world correctly. And thus the matter cannot be judged by inspection of the model alone; it requires inspection also of the world, either in its necessary or its contingent aspects. In the case of R, for example, the elements of the model have a particular relationship to the rotational symmetry existing in the world. And the superselection rule requiring measurable quantities to commute with R is a means of expressing the fact that R represents no procedure that could alter a system's physical condition.

The demands and realities of modelling must be substantively treated by every physical theory. And friends of the superselection treatment of measurement phenomena take modeling issues especially seriously, endeavoring, in their considerations, to accommodate the proposition that some systems exhibit so-called *classical* traits. They do not consider this a proposition belonging to logic, but one achieved through observation. And they urge that accommodation of this proposition be made in the form of superselection rules. In other words, they call on us to acknowledge the truth of the proposition that certain measurable quantities are classical, (by hand, so to speak), by introducing restrictions on those operators recognized as representing measurable quantities.[9]

If there does indeed exist a class of quantities that do not participate in quantum *entanglement* or *interference* effects,[10] the question arises whether a theory treating this class of quantities should not merely accommodate the fact, but *explain* it as well. This is something that some (for example, Bub [6] and Wan and Harrison [21]), but not all (notably, not [2]), superselectionists have aspired to do. Since, however, my objection to superselection accounts of measurement shall simple have to do with their *accommodation* of this fact, I simply pass over this question.

Superselectionists have as their primary objective to accommodate measurement episodes in a canonical quantum mechanical framework. They take this task to be comprised of two parts. In the first, a Hilbert space representations of indicator quantities, understood as *classical* quantities, are developed; this is anticipated to take the form of a stipulative and noncreative definition of the notion *classical quantity.* In the second part of the project, they propose to show that a description of the measurement process, conforming to (P1), (P2), (a) and (b) above, may be given in terms of notions defined stipulatively, and that this description shall draw on no principles external to canonical quantum theory itself—such things as, for example, projection postulates. The aim of the superselection program is to vindicate the proposition that canonical quantum theory requires neither supplementation nor modifi-

9. Superselectionists do not adhere to the proposition that *any* operator admitted by their superselection rules corresponds to a measurable quantity, since they acknowledge that there may be other considerations, entirely independent of their own, that may go against admission of certain operators into algebras of measurable quantities. In other words, nothing in the superselection treatment forbids *further* restrictions on the membership of $\mathcal{A}_S^*$. For superselectionists do not conceive of themselves as giving an account of the notion of *measurable quantity,* so much as treating one type of qualification for being a measurable quantity.

10. There are those who challenge the proposition that such a class exists: they suggest it is not impossible in principle for macroscopic quantities to enter into interference (for example, Legett [15]). SQUID experiments are thus being proposed and carried out.

cation in order to treat measurement interactions. The purpose of a superselection account of measurement is then, ultimately, to show that (P1), (P2) and (P3) are not in conflict, and to do this by demonstrating that, if we accept canonical quantum mechanics with superselection rules, the truth of (P3) shall not keep us from representing the final state of systems interacting in a measurement process via W.

There are two standard characterizations of classical quantity in general circulation. The first is formulated in terms of the eigenspaces of operators employed to represent these quantities. Suppose C is an Hilbert space operator corresponding to a classical quantity; and suppose, further, that each of the members of the set $\{\mathscr{H}_{C_i}|i = 1, 2, \ldots\}$ are eigenspaces of C. Then according to the first characterization of classical quantity, vectors drawn from distinct eigenspaces of C are nonsuperposable; in other words, the (nontrivial) superpositions of such vectors are not to be reckoned as *new* states of C. Nature, as the proposal has it, does not tolerate nontrivial superpositions of classical states as genuine states, for classical states are separated. An immediate consequence of this characterization is that classical quantities never participate in interference effects.

The second standard characterization of classical quantity is formulated in terms of relations had by them to other quantities. C, on this conception, is compatible with all other measurable quantities. This condition is, of course, the condition that C commutes with the appropriate operators. An immediate consequence of this second conception is that classical quantities are magnitude-definite in every allowed state of a system to which they belong.

I shall, without further comment, adopt the second characterization of *classical quantity:* a measurable quantity is classical, I shall say, whose operator representation commutes with all other operators representing measurable quantities. It is indeed a routine matter to show that, once superselection rules distinguishing an operator C as a superselector have been adopted, it follows that vectors drawn from distinct eigenspaces of C do not tolerate superposition; separation of the states of C is thereby ensured. Demonstration of this is just the proof that nontrivial superpositions of vectors from distinct eigenspaces of C are strictly equivalent to nontrivial mixtures of these same vectors: nontrivial superpositions of this kind are not new states of C because nontrivial superpositions assign exactly the same probability measures to prospects in the algebra of measurable quantities. But the equivalence of the two characterizations of classical quantity remains to be proven, for it has yet to be shown that when vectors drawn from distinct eigenspaces of C do not tolerate superposition, then C commutes with every member of $\mathscr{A}_S^*$. I shall take this proposition for granted, as others have done, although it is not, as far as I am aware, proven anywhere. (Nothing I assert in this essay shall rest on this presupposition, however.)

5. Measurement and Superselection Rules. The objective of superselection accounts of measurement is to give an account of the behavior, over time, of objects like detectors, through adoption of the policy that indicator states of detection devices shall be represented by eigenspaces (the supersectors) of superselectors. The first item on the superselectionist's agenda is now complete, for a general account of classical quantity has been presented. The second item on the agenda is to employ this characterization of classical quantity in mounting an account of the measurement process, again without adding to canonical quantum theory. The result shall be a theory which, while harmonizing P1, P2 and P3 (and thereby differing in empirical content from quantum theory *without* superselection rules) does not diverge from the axioms of the theory.

Let $\mathscr{H}_S$ and $\mathscr{H}_M$ be once again the Hilbert spaces of measured and measuring, respectively, with the composite system represented in the tensor product space $\mathscr{H}_{S+M} = \mathscr{H}_S \otimes \mathscr{H}_M$. We shall stipulate that if Φ_k is an indicator state of M, then the observable P_{Φ_k} projects onto a superselection subspace $\mathscr{M}_k$ of $\mathscr{H}_M$: nature, we shall say, does not tolerate superpositions of states drawn from distinct eigenspaces $\mathscr{M}_k$ of $\mathscr{H}_M$. Next, we generate the elements of the von Neumann algebra $\mathscr{A}^*_{S+M}$ of physical quantities of $S + M$. The elements of $\mathscr{A}^*_{S+M}$ will be those Hermitian operators that also commute with all superselectors defined for $\mathscr{H}_{S+M}$. Once $\mathscr{A}^*_{S+M}$ is specified, there will exist a mechanism for determining equivalence classes of state representations. Selecting among the members of each class exactly one distinguished representation that corresponds to a mixture, the superselectionist proposes to produce an argument to the effect that, under certain specified circumstances, we can expect some pure states to evolve into mixtures of the appropriate form, by evolving into states that are equivalent to these mixtures with respect to the measurable quantities.

Since $\mathscr{H}_M$ is composed of supersectors, there is some reason to suspect that any Hilbert space in which $\mathscr{H}_M$ appears as a subcomponent will also be constituted by supersectors, since it too will reflect a partly classical structure. Let $\mathscr{H}_{S+M}$, therefore, inherit the superselection subspaces of the component spaces $\mathscr{H}_S$ and $\mathscr{H}_M$ on which it is constructed, for if (for example) $\mathscr{H}_M$ comprises supersectors $\mathscr{M}_k$, then any tensor product space which features these $\mathscr{M}_k$ will also be comprised of supersectors, this time of the form: $[.] \otimes \mathscr{M}_k$.[11] For the sake of focusing

11. Superselectionists adopt this rule of inheritance with little or no careful argument,

attention on measurement phenomena, assume that $\mathscr{H}_S$ does not itself comprise supersectors. So the supersectors with which we will be concerned will be of the form $\mathscr{H}_S \otimes \mathscr{M}_k$. A measurable quantity of a system represented in such a space will be represented by operators that commute with each $I \otimes P_{\mathscr{M}_k}$. Only such operators may be admitted to the algebra $\mathscr{A}^*_{S+M}$.

Turning now to the treatment of time evolution in systems with classical quantities, consider the case when the initial state of S is an eigenstate ϕ_k of the operator representing the measured quantity. Suppose that S, under the current procedure, terminates in the state ψ_k^M, which may depend, not only on the initial state of S but also on the kind of instrument M and its tendency to disturb the states of S,[12] and that M terminates in Φ_k. The final state of the composite system will then be $\psi_k^M \otimes \Phi_k$. This final state lies in a supersector of $\mathscr{H}_{S+M}$; hence we need not replace it by any mixture.

Now consider the case in which $\psi_0 = \Sigma_i\, c_i\phi_i$. The initial state of $S + M$ will be $(\Sigma_i\, c_i\phi_i) \otimes \Phi_0$ and will evolve, by the Schrödinger equation, into the state represented by $\Lambda_f = \Sigma_i\, c_i(\psi_i^M \otimes \Phi_i)$—a vector in $\mathscr{H}_{S+M}$ with operator representation P_{Λ_f}. But now, since $\mathscr{H}_{S+M}$ comprises supersectors, we cannot be guaranteed that the final state vector corresponds to a pure state, since it may not lie in a supersector of the composite space. If the final state vector of the composite system does not belong to a supersector of the tensor product space, we shall be obliged to replace it by the canonical mixture: $W = \Sigma_i\, |c_i|^2 P_{(\psi_i^M \otimes \Phi_i)}$, since W is the only operator that commutes with every member of the algebra of measurable quantities.

Before application of superselection rules we have the following: application of the time development operator to the nontrivial superposition P_{Λ_0} results in a pure state P_{Λ_f}. But P_{Λ_f} is *not* a correct description of the final state of $S + M$; W is the correct description—according to (P2). Before adoption of the superselection policy, we have the result that P_{Λ_f} and W are equivalent, with respect only to any operators of the form $O_S \otimes I$ and $I \otimes O_M$, where O_S and O_M are operators defined on the respective component spaces alone. This is due to the fact that

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suggesting merely that since the Φ_ks of the measuring device represent classical states, then anything of the form $\alpha \otimes \Phi_k$ must also represent a classical state and therefore an eigenstate of a classical observable—a superselector. But there is a clear justification of this rule of inheritance, at least in situations in which the Φ_ks are conceived of as images, each of all the others. In that case anything of the form $\alpha \otimes \Phi_j$ will be trivially a copy of $\alpha \otimes \Phi_k$.

12. For perfectly nondisturbing measurements, replace ψ_k^M by ϕ_k.

13. These states are *bona fide* mixtures, though not mixtures susceptible of interpretation on the ignorance interpretation of mixtures. See [13, Ch. 5 and 9] for details.

P_{Λ_f} and W have the same reduced states (the same images on the component Hilbert spaces).[13] As a result P_{Λ_f} and W are equivalent with respect to the algebra generated by operators of the form $O_S \otimes I$ or $I \otimes O_M$.

There are, however, operators of the form $O_S \otimes O_M$, but not of the form $O_S \otimes I$ or $I \otimes O_M$, that represent measurable quantities. With respect to these operators, the state representations P_{Λ_f} and W are *not* equivalent, prior to adoption of the superselection policy. Now it is not at all the contention of advocates of superselection that *all* these operators are without physical significance. To the contrary: it is their contention that, with respect to all the (true) measurable quantities of the form $O_S \otimes O_M$, P_{Λ_f} and W generate identical probability assignments.[14] On the basis of equivalences between such states as P_{Λ_f} and W, the superselectionist considers the measurement problem dissolved.

6. The Contradiction. By far the most widely acknowledged infirmity of superselection accounts of measurement is the following, attributed to Hughes [13, § 9.7] and van Fraassen [20, 267]—by each to the other. Let $\mathscr{M}_k$ be a supersector of $\mathscr{H}_M$. Then $\mathscr{H}_S \otimes \mathscr{M}_k$ is a supersector of the tensor product space $\mathscr{H}_{S+M}$. Consider transformations brought about by the unitary operator $U_\tau = e^{-i\hbar H\tau}$, where H is the infinitesimal generator of this weakly continuous one-parameter unitary group of operators. Suppose H is the *Hamiltonian* of the system, and represents total energy. U_τ then represents the time evolution operator, and maps the initial state designated as Λ_0 into the one designated as Λ_f. In the cases of interest the final state Λ_f is subject to being replaced, in a superselection account, by the appropriate mixture W. Since in a large number of normal instances $\Lambda_0 \in \mathscr{H}_S \otimes \mathscr{M}_k$ but $\Lambda_f \notin \mathscr{H}_S \otimes \mathscr{M}_k$, the superselection subspaces of $\mathscr{H}_{S+M}$ do not reduce U_τ. Hence U_τ can be neither a measurable quantity of such systems, nor a function of one; thus H cannot be a measurable quantity. But according to the superselection account, all measurable quantities must be reduced by the superselection subspaces of the system. Hence superselectionists must admit either: (1) that H, the infinitesimal generator of time evolution, does *not* represent energy, or (2) that energy, represented by H, is not a measurable quantity for the systems being treated.

The Hughes-van Fraassen complaint is that the proposal to deny H its normal canonical interpretation is conspicuously undermotivated: it rejects, without good reason, the canonical practice of modeling time evolution around energy considerations. The significance of energy in quantum mechanics is due to its appearance in the rule of dynamical

14. A proof for a simple case is presented in [13, 285–287].

transformation, which is, according to Hermann Weyl, the fundamental axiom of quantum mechanics and is of universal validity [23, 80]. Thus the complaint can be put more forcefully: the superselectionist's commitment, either to denying the status of measurable quantity to the Hamiltonian or to breaking its association with total energy, necessitates a departure from canonical quantum theory. A routine application of canonical quantum theory *presumes* that a Hamiltonian exists and corresponds to energy. For the Schrödinger equation, the sole principle of time transition in a theory whose primary subject matter *is* time transformation, describes transition that conforms to a slate of desiderata, among them conditions of homogeneity of both space and time. A description satisfying these conditions exists only on the assumption that a Hamiltonian exists; and a Hamiltonian exists for a system exactly on condition that total energy has a nonchanging magnitude throughout the span of time we undertake to describe that system. If these conditions are not met, no application of the Schrödinger equation to an initial state like Λ_0 can be justified. But this application is precisely what leads to the problematic final state P_{Λ_f}—a problem solved in a superselection theory by replacement of the problematic P_{Λ_f} with the more congenial W. But if we are not to understand the Hamiltonian in the normal way, how are we to justify a routine application of the Schrödinger equation to Λ_0?

This is a grave difficulty for superselection accounts. But it is not, in principle, an insurmountable one. After all, the proposition that the Hamiltonian does not have its usual status is not so bad a thing as a contradiction. Although if one alleges that the Hamiltonian is not an observable, for example, one then owes an account of what it does represent, and why it is important to represent it, and why it plays an important role in time development. However to say that some kind of explanation is called for, is not to put forward a damning criticism. We must recognize, after all, that superselection accounts accommodating measurement interactions are not put forward as complete accounts of the measurement process, but merely as general schemata for assembling certain apparatus-oriented restrictions into the form of superselection rules. More must be said, and perhaps more physics of instrumentation be offered, before any specific applications of the theory may be approved. Each application of the superselection account must ultimately be justified on its own merits.

Nevertheless the Hughes-van Fraassen criticism does well in calling the following to attention: the simple and possibly true proposition that measuring instruments are never in nontrivial superpositions of distinct indicator states is not sufficient justification, strictly speaking, for asserting a specific superselection rule. The superselectionist must set out reasons for adopting each proposed rule. This necessity moves the superselection project from the theoretician's armchair ultimately into the laboratory. Optimism might survive: the application-specific theories, perhaps also instrument-specific, which may be entertained on behalf of application of the superselection theory in a particular case might, if superselectionists are very fortunate, *also* provide justification for either the conjecture that H does not represent the total energy for the system in the application, or the conjecture that H in that instance is not a measurable quantity despite the fact that it generates time evolution.

Wan [23, 980], acknowledging this problem, draws our attention to specialized treatments, in both classical and quantum settings (Dirac's Hamiltonian formulation of general relativity, the Gupta-Bleuler formulation of quantum electrodynamics), in which, as it is said, it is inappropriate to regard H as representing energy. This, however, does not help in contexts in which H *is* appropriately regarded as representing energy, which—if canonical quantum theory strictly applies to any system whatever—must exist. Wan's suggestion seems to be that we may acknowledge a solution to the measurement problem only in the special cases. But if this is right, then the measurement problem is not solved: it is merely confined to nonspecial cases. Another group of physicists ([4], [3], [10]) have advanced arguments, which bear a family resemblance to each other, for the claim that the Hamiltonian of a weakly-coupled many-body system, representing as it does total *microscopic* energy, is not a *genuinely* measurable quantity in certain contexts, and thus cannot be treated alongside certain other quantities—at least not in the same way. Common ground in these arguments is the assumption that the nature of a many-body system is such that to conduct a measurement of its total *microscopic* energy, in distinction from its *macroscopic* energy, is necessarily to subject the system to a process which invariably destroys interactions and correlations between components of that system. But this procedure constitutes a violence to the macroscopic system as such; the body *as macroscopic body* cannot survive such a procedure. Hence, despite the fact that microscopic energy is both defined for the system and plays a prominent and important role in moving its development forward in time, microscopic energy is not genuinely *measurable.* A superselectionist might adopt such an argument, suggesting that quantities satisfying specified con-

15. To sustain this line of argument the respondent must argue for a disanalogy between macroscopic bodies and microscopic ones. The destruction of a microscopic body in the process of a measurement of some one of its properties does not, after all, imply that the property measured by that procedure is in some way immune to measurement. The normal practices of microscopic measurement are quite to the contrary of such a conclusion. It present quite a challenge, therefore, to maintain this position.

ditions are not genuinely measurable as quantities *of the body as such.*[15] This proposal has strong Aristotelian overtones, and might be supposed to rest on the merits of the conception of 'mixt' which Pierre Duhem put forward early in this century: "Some bodies, the one different from the other, are brought into contact. Gradually they disappear, they cease to exist, and in their place, a new body is formed, distinguished by its properties from each of the elements which produced it by their disappearance. In this mixt, the elements no longer have any actual existence. They exist there only potentially, because on destruction the mixt can regenerate them."[16]

Similarly, Wan and Harrison [21] undertake to explain the nonmeasurability of the Hamiltonian as follows. They say that certain processes involve destructions of certain aspects of systems (for example, destructions of superconductivity). "Evolution from one supersector to another involves the *annihilation* of the initial system and the subsequent *creation* of a new one"—not merely ordinary changes of state (p. 6). Such a process, they say, cannot be explained by reference to internal processes, because the former are not, according to the authors, "self-generated." They continue: "We argue that the destruction of a system cannot be achieved by a self-generated process, but must be effected by external factors. So it is understandable that the generator of the evolution group cannot even represent an observable of the system involved, let alone their energy." According to Wan and Harrison, these facts should alleviate widespread unease concerning the Hamiltonian's unusual status in superselection accounts. "Despite there being no external interference during the transition between different supersectors, this 'nonequilibrium process' cannot be described solely in terms of observables of the initial and final equilibrium systems" (p. 7). This last passage, however, makes matters worse rather than better, since in it the authors are admitting that normal conditions are met for presupposing that a Hamiltonian exists and represents total energy. Why therefore are special pleadings necessary to justify the proposition that it does not count as measurable? On the other hand, the scare-quoted 'nonequilibrium process' suggests that normal dynamics are in fact *not* operative. But if they are not, how are we in good conscience to put forward the proposition that canonical quantum mechanics has been saved from the sword by the superselection account? Pitfalls spring up wherever the superselectionist treads, and apparent inconsistencies. These traps would all be explained if only we could find one clear and true contradiction in the theory. This is what I propose to reveal.

16. Duhem [11, 12], as translated and discussed by Needham [17].

I shall, first of all, explain why superselectionists sustain a need to deny observable status to the operator H (whatever, if anything, it might signify). If my explanation is correct, there is no room for extra-armchair activities in an all-things-considered superselection account of measurement. In the process we shall come to recognize that superselection, as an account of measurement, entails a contradiction. This contradiction comes about as a result of an inadmissible exploitation of excess model structure. But first: explanation of the nonobservable status of H within a superselection theory of measurement.

In order to provide for alterations over time in magnitudes of classical quantities, the superselectionist must find means of representing time transitions across supersector boundaries, for the sake of representing transitions in an indicator quantity from a ground state to another state of definite indication. How can such a transition be accommodated in a superselection account? Superselectionists propose to retain representation of time evolution by the action of a self-adjoint operator. That operator, according to the theory, will either achieve the required transition directly, or it will take advantage of the existence of superselection rules. Either way, I will argue, that operator must be such that it transgresses supersectors.

If the time evolution operator achieves representation of time transitions in indicator quantities *directly,* it must, obviously, fail to respect the invariance of supersectors, because its application to a given vector or projection operator in one supersector will (on the appropriate occasions) result in a vector or operator in a *different* supersector. But consider now the case in which the time evolution operator achieves representation of time transitions in indicator quantities by taking advantage of superselection rules—as superselectionists anticipate it will. Application of the time evolution operator results in a superposition, Λ_f or P_{Λ_f} for example, on the appropriate occasion (when Λ_0 represents the initial state). Superselection rules, then, will ensure the replacement of this superposition with a mixture, $W,$ which may be said to represent a state in which the indicator quantity takes on a different value than at the first. However since Λ_f does not lie in the same supersector as Λ_0 (because, in general, superpositions do not lie in the same subspaces as the superposed vectors), the time evolution operator must cross supersector boundaries. So, whether changes in indicator quantity are achieved directly or through superselection rules, the time evolution operator of a superselection account must violate supersector boundaries.

Thus if there are to be alterations in the magnitudes of indicator quantities in particular, and if these quantities are to be represented by superselectors, the operator representing time transitions must be allowed to transgress supersectors. And if so, it must be such as *not* to commute with all superselectors. And this fact, that the time evolution operator violates supersectors, is (to repeat for the sake of emphasis) due to precisely three conditions that a superselectionist approves:

1. Classical quantities shall be modeled by a subset of the superselectors.
2. Time evolution shall be generated by a self-adjoint operator.
3. Time evolution sometimes involves alterations in the magnitudes of classical quantities, of which indication quantities are a special species.

(1) is just the superselection policy currently on offer. (2) is that aspect of canonical quantum theory which superselectionists are determined to save. (3) is undeniable, and also part of the package (P1)–(P3) which superselectionists are determined to save; without (3) there is no (apparent) conflict among (P1)–(P3), and hence no measurement problem to be solved.

The fact that the time evolution operator $U_\tau = e^{-i\hbar H\tau}$ cannot be an observable is therefore completely explained by (1)–(3), and nothing more needs to be said. And the same exactly holds also of the nonobservable status of the Hamiltonian H. It cannot help, therefore, to look to Dirac's formulation of relativity, or the Gupta-Bleuler formulation of QED, or 'nonequilibrium processes' or anything else, for an explanation of the nonobservable status of H and U_τ. What could they add? For nothing is wanting in our explanation of the nonobservable status of H and U_τ.

Putting now to one side the meaning of H, what could U_τ, which *also* transgresses supersectors in superselection accounts of measurement, signify? Superselectionists assume it is capable of representing (and generating) the advance in time, just as it does normally. But could it? Superselection is founded on the principle that the action of an operator transgressing supersector boundaries cannot represent a change of magnitude in a measurable quantity. Justification of this principle rests on the proposition that the action of superselectors preserves the magnitudes of all measurable quantities (since it commutes with the operators representing them). The principle that a change of supersectors does not, as such, represent a change in a physical quantity, is therefore a cornerstone of *any* theory that calls on superselection rules. It can also be construed as resting on incontestable principles of modeling directed at prohibiting illegitimate use of excess model structure. This principle prohibits us representing true physical changes via an operator that does not commute with all superselectors. I shall refer to this prohibition as the *Excess Structure Principle (ESP)*. What is the status of ESP? ESP, to emphasize, is *presupposed* in each and every account which names the name of superselection rules, because it is concerned with making certain that we always treat exactly alike those state representations which agree on all probability measure assignments to the measurable quantities we wish to treat. Another way to conceive of the special status of ESP is to see it as reflecting the notion that superselection rules are a gauge of excess model structure. Upon the very clear merits of ESP we have been prepared to entertain the superselectionist's replacement proposals (swaps of problematic superpositions with unproblematic mixtures like W).

Now, in a superselection account of measurement neither H nor U_τ commutes with all superselectors, since both (as we saw) must transgress supersectors. *Hence, the superselectionist must acknowledge that the action of neither H nor U_τ may represent a change in a physical quantity.*

Could U_τ represent the advance of time, therefore, if it cannot represent an alteration in physical state? Of course it can, if advance in time does not produce a change in state. But this, exactly, is the trouble with the superselection account. For it is in especially in those instances in which U_τ transgresses supersectors that superselectionists would like to acknowledge a change of magnitude in a physical quantity—specifically, a change in the indicator quantity. Thus superselectionists aspire to violate a cornerstone of their own theory. If the cornerstone is removed, how shall the swaps, which allow superselectionists to proclaim that certain superpositions have evolved by the Schrödinger equation into mixtures, be justified?

My complaint of superselection theories is therefore this: that they are committed to both of the following two propositions: (i) that U_τ *must,* as a result of the incontestable ESP, commute with all superselectors, since it must sometimes represent transformations in physical state; and also to (ii) that U_τ *cannot* commute with all superselectors, since its action must be capable of bringing about supersector boundary transgressions (and to do so by conforming to 1–3 above). This is a contradiction.

Hence if, on the urgings of superselectionists, we are prepared to adopt ESP, the principle that the action of an operator transgressing supersectors cannot signify alteration in some physical quantity—a concession that may itself be made without peril of contradiction—we cannot also be prepared to regard an operator that transgresses supersectors as responsible for representing straightforward alterations in indicator states. We can accept a superselection account of measurement, therefore, only to the extent that we are prepared to acknowledge that, for our purposes, the initial and final states in a measurement process are one and the same physical state. But superselectionists would be the last to urge that we accept such a thing. We must therefore reject superselection *as a means of accommodating the phenomena of measurement.*

The utilization of superselection rules is legitimate only if, in tolerating the redundancies which exist when more than one element of the complicated mathematical model can represent the same system or situation, in relation to the interests identified by the theory framers, those theory framers can nonetheless forbid transitions in time out of supersectors and localize physical processes within supersectors. This prohibition is routine in treatments of protons and neutrons as "nucleons."[17] In these treatments a supersector-local description of behavior over time is prescribed and produced. Likewise we have no need for permitting a system with rotational angular momentum to evolve across supersector boundaries of the operator $e^{-i2\pi J_3}$. It should be a requirement of superselection treatments that they prohibit transformations across supersector boundaries. Therefore in a treatment of quantum measurement, in which we wish to accommodate the fact that our measuring devices make transitions to and from their various indicator states over time, it is entirely inappropriate to represent those states as eigenstates of a superselector.

The superselectionist's mistake, put in its best light, is to suppose it possible to represent familiar macroscopic quantities that enter into interactions with the nonclassical systems treated so successfully by quantum mechanics, and suffer alterations in the process, as quantities for which we shall have no need of special treatment, either because they fall conveniently under a concept we already have in ordinary quantum mechanics, or because they require no treatment at all. The superselection proposal is simply to handle such quantities simply in the *legend* of a quantum-mechanical model. But the relations between these special macroquantities on the one hand, and those microquantities treated so beautifully by quantum theory on the other, must be very complex, as well as very interesting, and do not deserve relegation to the ignominious existence of a mere inscription in the legend of a model—even if such treatment could be given without contradiction—however simple, convenient or in some other way irresistible a superselection treatment might otherwise prove to be.

7. The Special Case of Infinite Systems. It might be objected that the prohibition I am sponsoring, which disallows time transformations

17. And, as an anonymous referee pointed out, this is true also of identical particles: we cannot begin with bosons in some pure state and end with a mixture of bosons and fermions.

across supersector boundaries, is not acceptable in treatments of systems with infinite numbers of degrees of freedom, such as for example the class of systems for which Bub's solution to the measurement problem are designed. (Bub thinks of these infinite systems as an ideal or limiting case, and proposes that we think of ordinary macrosystems as approximating infinite systems.) For, unlike systems with finite numbers of degrees of freedom, which are such that all the irreducible Hilbert space representations of their observables are unitarily equivalent, systems with infinite numbers of degrees of freedom enjoy many *inequivalent* irreducible representations of their algebras of observables. And these, as the objection continues, can legitimately be utilized to represent *different* classical or macroscopic states, because there are objective grounds for discriminating among the inequivalent representations for purposes of representing different macroscopic or classical states. (Bub [9], for example, presents a treatment of spin density in an infinite 1-dimensional array of spin-$1/2$ systems which takes different values for microstates in different supersectors.) In the instance of infinite degrees of freedom there are, in other words, grounds for taking different irreducible representations to represent different states because these representations, according to the objection, are not simply copies of each other. Thus ESP does not apply. And so no contradictions like that above can be deduced.

In reply it bears pointing out, first of all, that the fact that a certain pair of representations of operator algebras are not unitarily equivalent does not, *as such,* determine that these representations are not equally suited to representing the same quantities or states in exactly the same ways. The question whether they are so suited turns on whether the differences that exist among these representations are differences which may portray true physical differences among true physical systems, and this question is beyond the scope of the present discussion. So for present purposes let us simply grant the assumption that the infinitist's operator sets have different representational powers. Let us also grant the assumption that, in an infinitist treatment of measurement phenomena, each member of the set of operators identified as classical or macroscopic can always be defined so as to commute with all operators, with the important exception of the so-called *interaction* Hamiltonian, which serves the purpose of producing transgressions of supersector boundaries in the infinitist treatment. Can we accommodate the facts of measurement in the infinitist's superselection model, without contradiction? Not if we insist on its truly being a superselection account.

The infinitist is, to be sure, rejecting the proposition that we are dealing with Hilbert spaces that have excess structure. But does this, if true, render the infinitist account immune to the prohibition I am sponsoring? Not entirely. ESP prohibits us from representing true physical changes by means of the action of an operator that transgresses supersector boundaries. Any treatment which observes the (superselectionist) policy of handling alterations in time by an operator that violates a set of supersectors, automatically violates ESP. Thus infinitists who adheres to (1)–(3), remaining loyal to the superselection approach to measurement, do *not* escape the criticism to the effect that these accounts are a breeding ground for contradictions.

Infinitists might simply reply that, since the Hilbert spaces by means of which they propose to represent classical or macroscopic states are not simply copies of each other, these spaces cannot, officially speaking, be termed 'supersectors'. (They might say that we in fact know this is the case because the operators representing classical quantities in their account do not commute with the interaction Hamiltonian, as I myself have pointed out. And so these operators must not be superselectors, officially speaking.) And since the eigenspaces of an infinitist's treatment are not, strictly speaking, supersectors, there is no violation of ESP when the interaction Hamiltonian transgresses sector boundaries. So there is no contradiction.

To be sure, infinitists who take this route will escape violating ESP. However they will at the same time cease to possess suitable qualifications as superselectionists, as they will not be availing themselves of superselection rules. (For an infinitist who takes this route out of the difficulty is claiming that the superselection portion of his or her account is disposable, and not core to the proposal on offer.) And such an infinitist will have lost an important ingredient of the original theory, namely the original grounds (spelled out above) for replacing certain superpositions with mixtures. For these replacements are predicated on the fact that there is excess structure in the models being pressed into service. The replacements depend, in other words, on the fact that certain superpositions, in the model being employed, do not represent physical states different from the states superposed (because commutation relations hold between operators representing physical magnitudes and a *superselector*), but do represent something because we make the decision to allow more than one mathematical object to represent the same physical situation. This is a heavy loss indeed, for now the infinitist faces the very large task of developing a new proposal for replacing superpositions with mixtures.

The infinitist might say that the superpositions which need replacing are meaningless—for example, a superposition of spin density states "predominantly-up" and "predominantly-down" is meaningless. And so infinitists are entitled to replace them with mixtures on the very same grounds that superselectionists are entitled to make their replacements—namely, that there is *some* excess structure. Nevertheless the infinitist may admit that there is not as *much* excess structure in the model being proposed as in the models superselection theorists typically have on their hands. This seems a promising approach. But the claim that there is *some* excess structure in the model of infinite systems, does not automatically entitle the infinitist to replace superpositions with mixtures. Entitlement to these replacements comes with being able to demonstrate that the mixture sought and the superposition it is replacing agree on probability assignments to all the prospects associated with the algebra of measurable quantities. This agreement can be demonstrated, as is done in noninfinite superselection accounts, on the assumption that all operators representing measurable quantities are reduced by certain of the model's subspaces, which fact qualifies these subspaces as supersectors. But the infinitist has just given up the right to this assumption, because (if the infinitist reply presented in the preceding paragraph is correct) there are no true supersectors in the infinitist's model. Perhaps the infinitist can make a case for replacements, but we should like to see that case made.

In conclusion, while the original, noninfinitist, superselection theories of measurements suffer from being contradictory, infinitist theories whose superselection components are either disposable or disposed of, suffer from being misnamed. What is worse, they suffer from lacking grounds for making the swaps they wish to make—swaps of superpositions for mixtures—for which a superselection proposal was presumably adopted in the first place. To be sure the infinitist has room for maneuver: he or she may offer alternative grounds for replacing superpositions with mixtures, possibly superselection-like, possibly resting on (yet undisclosed) facts about inequivalent irreducible representations. But even if the infinitist can pull this off this sizable feat, the result will not be—strictly speaking—a superselection account.

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