

Heavy-light decay constants—MILC results with the Wilson action*

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We present the current status of our ongoing calculations of pseudoscalar meson decay constants for mesons that contain one light and one heavy quark ($f_B, f_{B_s}, f_D, f_{D_s}$). We are currently generating new gauge configurations that include dynamical quarks and calculating the decay constants. In addition, we have several new results for the static approximation. Those results, as well as several refinements to the analysis, are new since Lattice '96. Our current (still preliminary) value for f_B is $156 \pm 11 \pm 30 \pm 14$ MeV, where the first error is from statistical and fitting errors, the second error is an estimate of other systematic errors within the quenched approximation and the third error is an estimate of the quenching error. For the ratio f_{B_s}/f_B , we get $1.11 \pm 0.02 \pm 0.03 \pm 0.07$.

1. INTRODUCTION

Heavy-light meson decay constants such as f_B and f_{B_s} are of great interest because they are necessary to interpret current and future measurements of B - \bar{B} and B_s - \bar{B}_s mixing. Only with knowledge of the decay constants and corresponding B -parameters can we extract the Cabibbo-Kobayashi-Maskawa matrix elements and begin to decipher the fundamental information about the Standard Model (or physics beyond it) that is encoded in the CKM matrix. Recent reviews of this active field can be found in Ref. [1]. Here we will concentrate only on the recent work of the

MILC Collaboration.

We have been involved in the calculation of pseudoscalar meson decay constants for some time now [2]. Our initial goal was to calculate f_B in the quenched approximation. This calculation was done using gauge couplings between 5.7 and 6.52. Our current goal is to estimate the error that comes from the quenched approximation; however, we have also been refining our analysis.

To estimate the quenching error, we have calculated decay constants on lattices generated with two flavors of dynamical Kogut-Susskind quarks. The gauge coupling ranges from 5.445 to 5.7 and a variety of quark masses have been explored. (Table 1 summarizes the lattices that have been used in our calculations.) To go from an estimate of the quenching error to an actual calculation

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Table 1

Lattices used in our calculation. (Lattice Q is used only to study finite volume effects and is not included in the final results.)

Quenched			
Name	$6/g^2$	size	#
A [†]	5.7	$8^3 \times 48$	200
B	5.7	$16^3 \times 48$	100
E [†]	5.85	$12^3 \times 48$	100
C [†]	6.0	$16^3 \times 48$	100
Q	6.0	$12^3 \times 48$	235
D [†]	6.3	$24^3 \times 80$	100
H [†]	6.52	$32^3 \times 100$	60
Dynamical, $N_f = 2$			
Name	$6/g^2, am$	size	#
F**	5.7, 0.01	$16^3 \times 32$	49
G ^{†*}	5.6, 0.01	$16^3 \times 32$	200
L	5.445, 0.025	$16^3 \times 48$	100
N	5.5, 0.1	$24^3 \times 64$	100
O	5.5, 0.05	$24^3 \times 64$	100
M	5.5, 0.025	$20^3 \times 64$	100
P	5.5, 0.0125	$20^3 \times 64$	100

[†] Approximately same physical volume

** from Columbia group

* from HEMCGC

with dynamical quarks will require much work. It will be necessary to extrapolate in the dynamical quark mass to the u/d mass. It will also be necessary to extrapolate results to zero lattice spacing. The dynamical strange quark, which does not appear in our calculations, will have to be included. Also, it would be nice to use the same lattice formulation of the dynamical and valence quarks. At the coupling $6/g^2 = 5.5$, we have results for four quark masses. We plan to have the same number at 5.6, so we will be able to attempt extrapolation in the light quark mass for at least two couplings. We have no current plans to include the effects of a dynamical strange quark.

There have been five significant changes between the results we presented at Lattice 96 [3,4] and those presented here. We now extrapolate

to a physical κ that gives the correct pion mass rather than to κ_c . We now evaluate the perturbative renormalization factors in a more consistent way. We have explored more fit ranges and this has helped us to refine our estimate of the statistical/systematic errors of the fits. We have new results for the decay constants in the static approximation for several additional configuration sets. We are now able therefore to determine physical quantities at the b -quark mass, on essentially all data sets, by interpolation between the static limit and propagating quarks that are lighter than b . (On two data sets, M and P, we still are forced to extrapolate at the present time; new static results on these sets will be completed shortly.) We have also refined our analysis of the π mass and f_π so that our chiral fits are better understood and controlled. Each of these issues is discussed in a subsequent subsection. We then present a summary of our results and, finally, our conclusions.

2. CHANGES TO THE ANALYSIS

2.1. Chiral extrapolations

For the results presented at Lattice 96, all our chiral extrapolations were done to κ_c . We now determine the light quark mass from

$$m_\pi(\kappa_l) = m_{\pi\text{phys}},$$

where $m_{\pi\text{phys}}$ is the physical pion mass, and f_π is used to set the scale. This has resulted in a small reduction of f_B of 2–6 MeV, for various cases. Physically, this change comes about because af_π^{latt} is now bigger, which results in a larger value for a , and hence a smaller value for f_B since af_B changes less than af_π .

2.2. Perturbative corrections

There are several improvements to the way we calculate the perturbative corrections to the renormalization constants of the axial-vector current. Tadpole improvement is applied in a consistent manner[6], and the remaining perturbative corrections are calculated using a boosted coupling constant $g_V^2(q^*)$. The scale q^* has been calculated for Z_A^{stat} by Hernandez and Hill to be $2.18/a$ [5]. For propagating quarks, the value $1/a$

has been suggested by Lepage and Mackenzie [6] and this value is used in the analysis presented here.

Table 2
 Differences (new – old) in values of f_B from current tadpole improvement

Quenched	
$6/g^2$	difference (MeV)
5.7	+5–8
5.85	+3
6.0	+2
6.3	± 1
6.52	-0.2
Dynamical, $N_f = 2$	
$6/g^2$, mass	difference (MeV)
5.445, 0.025	+3
5.5, various	4–7
5.6, 0.01	+2
5.7, 0.01	-0.1

Results presented at Lattice 96 followed an approximate method of Bernard, Labrenz and Soni [8] that was designed to reproduce the results of tadpole improvement at $6/g^2 = 6.3$. However, that method differs from the more consistent current method, and the differences increase with the lattice spacing. The differences are shown in Table 2. We note however that a very recent lattice perturbation theory calculation by Bernard, Golterman and McNeile gives $q^* \sim 2.3/a$ [7] for the Z_A for Wilson quarks. When this value is put into the analysis, the differences listed in Table 2 will decrease, making the analysis closer to that of Lattice 96.

2.3. Alternative fit ranges

To determine the pseudoscalar mass and decay constant, it is necessary to fit hadron propagators. Of course, it is not known *a priori* what range of distances to use in the fit. It is appropriate to consider the variation resulting from picking different ranges as part of the statistical error. At Lattice 96 we had a limited choice of alternative

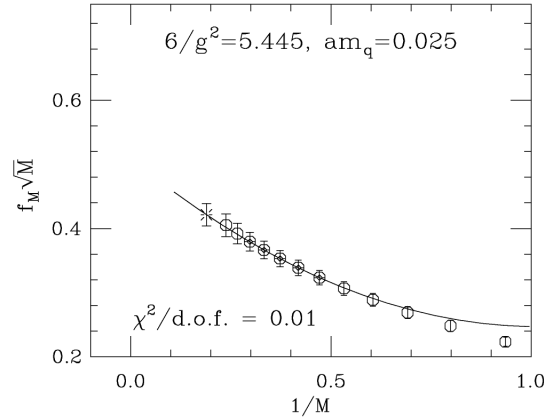


Figure 1. $f_M \times M^{1/2}$ vs. $1/M$ for $6/g^2 = 5.445$ and $am_q = 0.025$ (data set L) with no static result.

ranges used to estimate this error. We have had the opportunity to explore additional ranges, and this has changed some of our statistical errors. For example, for $6/g^2 = 6.0$ we had thought that the variation from changing the range was ± 15 MeV. We currently estimate it to be ± 8 MeV.

2.4. New static f_B calculations

All the improvements described above can be implemented after the bulk of the computation is completed. However, the calculation of improved values for f_B in the static approximation involved a reanalysis of many of our lattices. In our original calculations, the f_B^{static} results were a byproduct of the propagating quark calculation. On large volumes, however, they were subject to contamination from states with non-zero momentum, and therefore were left out of the analysis. On small volumes the energy gap between the ground state and states with momentum $2\pi/L$ was sufficient to suppress the non-zero momentum states.

Our recent calculations with dynamical quarks have been on larger volumes than the early ones, and we have done new calculations to determine the static decay constants. In some cases, we also

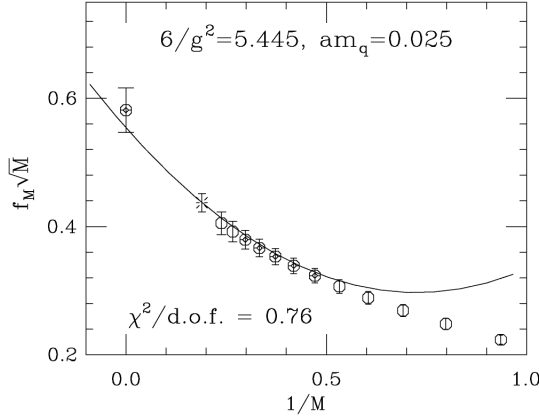


Figure 2. $f_M \times M^{1/2}$ vs. $1/M$ for $6/g^2 = 5.445$ and $am_q = 0.025$ (data set L) including the static result.

improved the accuracy of the quenched results. At Lattice 96, Craig McNeile [4] described our method and some of the initial results. The key point is that the static quark is smeared relative to the propagating quark and the basis set includes about 10 different smearing functions. The analysis of the static f_B data is still being refined.

With more accurate results in the static case, we are able to improve some of our fits for the decay constant vs. $1/M$, where M is the heavy meson mass. Let's consider some examples.

Data set L with $6/g^2 = 5.445$ and $am_q = 0.025$ is a case where we had no previous static result. In Fig. 1, we see $f\sqrt{M}$ plotted vs. $1/M$. The curve is fit to the five decorated octagons. The burst is the *extrapolated* value of the decay constant and corresponds to 203(8) MeV. An alternative fit to the points with $1/M > 0.5$ gives 200(7) MeV. In Fig. 2, we see that including the new static value results in an *interpolated* value of 210(7) for f_B . Using the points to the right, we find $f_B = 209(7)$ MeV. The results for the two choices of fitting range are more consistent and the error is slightly reduced.

As another example, for case G with $6/g^2 = 5.6$ and $am_q = 0.01$ our old estimate of f_B^{static} was 265(12)(10), where the first error is statistical for

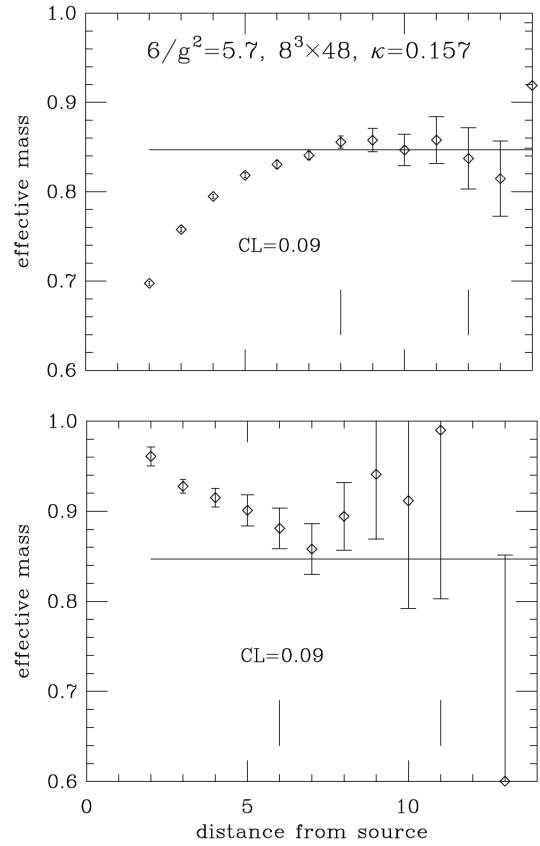


Figure 3. Old static results on data set A for effective masses and fitted masses. (a) smeared-local propagator, (b) smeared-smeared propagator.

a fixed fit range and the second is the variation from the fit range. Our new result is 285(4)(1). The errors are considerably reduced and the result is somewhat higher.

Finally, we consider a quenched example, case A. In this particular case, although the volume is small enough to avoid contributions from higher-momentum states, we had poor plateaus in our effective mass plots. Figures 3 and 4 show effective mass plots for the old calculation and the new static calculations. In each case we show two effective mass plots, one for the smeared source and local sink and one for smeared source and sink. The vertical bars near the bottom of the graph

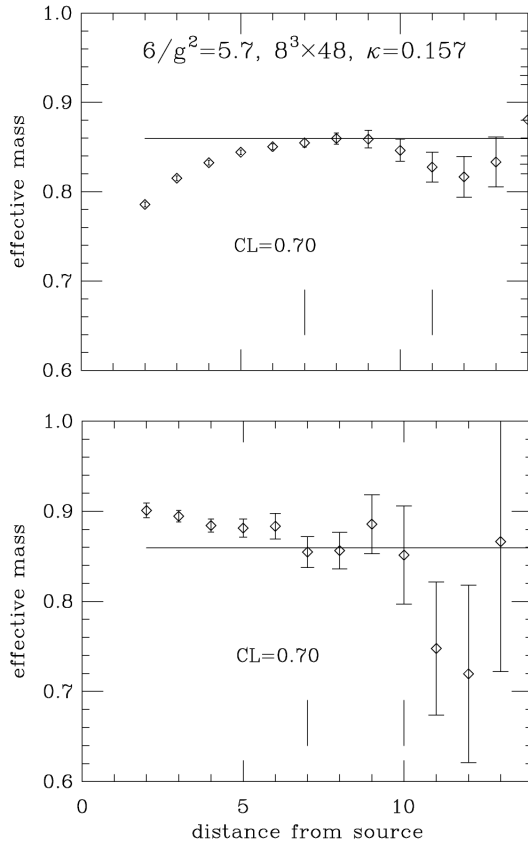


Figure 4. New static results on data set A for effective masses and fitted masses. (a) smeared-local propagator, (b) smeared-smeared propagator.

show the fitting range. The horizontal lines give the fitted mass. The confidence level of a fit including the old static result was just 0.09. With the new results it is 0.7. The new results approach the asymptotic value more quickly and the errors in the effective masses are smaller. The old value for f_B^{static} was 216(27) MeV; the new value, 283(18). Using the old static value to determine f_B , we would have gotten 201(13) MeV. Without the static value included in the fit, we obtained 227(18) MeV. (This was the fit chosen at the time of Lattice 96.) Including our new static value gives 229(12) MeV, in agreement with the

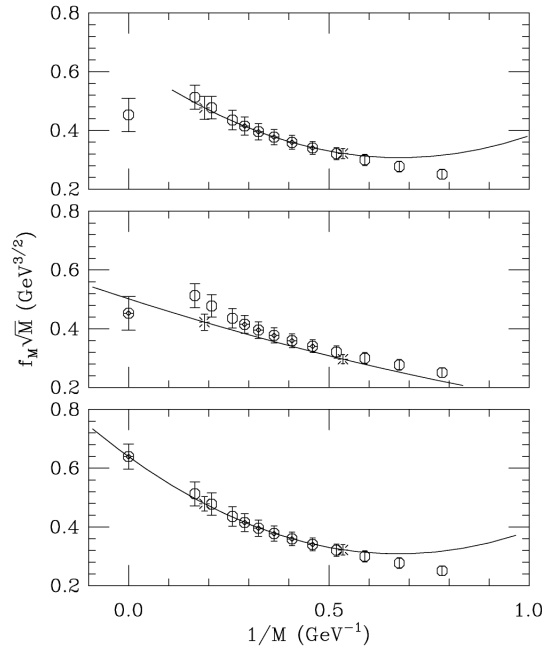


Figure 5. Decay constant *vs.* the inverse meson mass on data set A. In the top graph (from data analysis presented at Lattice 96) the static result is ignored. In the middle graph the old static result is included in the fit. In the bottom graph our new static result is included.

Lattice 96 result, but with a smaller error. In Fig. 5, we see all three fits. The points determining the curve are the decorated octagons. The bursts show both f_B and f_D . [Note: we quote statistical errors only for f_B here. Choices of different fitting ranges result in additional errors of order 15 MeV. However, the relations among the various treatments of the static point remain essentially unchanged.]

2.5. Scale determination

The final improvement we have made to our analysis involves the determination of the lattice spacing from f_π . As mentioned at Lattice 96, we often had a difficult time getting good linear fits for m_π^2 or f_π *vs.* $1/\kappa$. We used quadratic fits to estimate the systematic error in the scale, resulting in variations from 8 to 25%. However, our earlier fits for m_π^2 and f_π were based upon a fixed fitting range for all three light κ values. We have

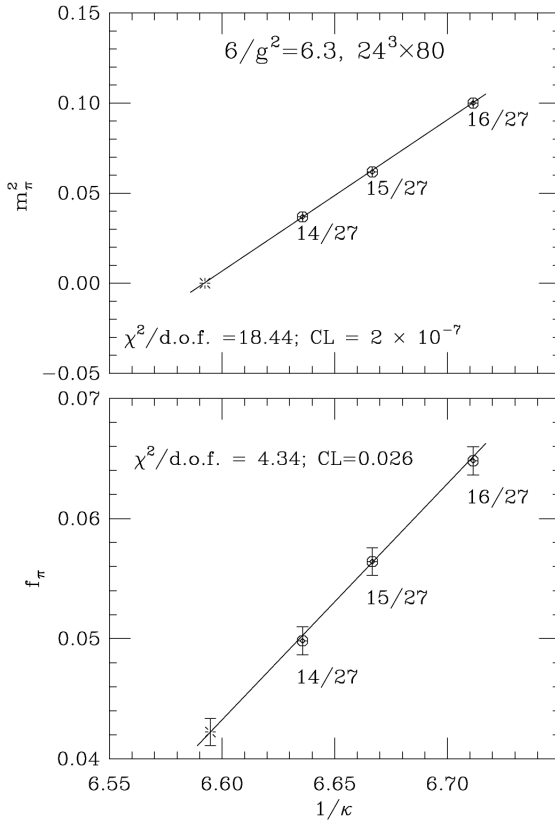


Figure 6. Chiral extrapolation for m_π^2 and f_π on data set D with a fixed fitting range

relaxed this constraint and let the fitting range vary. This has certainly helped to improve the confidence level of the chiral extrapolation. As an example, for case D, we show both m_π^2 and f_π in Figs. 6 and 7. The individual points in the plot are labeled with $\chi^2/(\text{degrees of freedom})$ for the fits to the pion propagators as a function of distance from source. We see, particularly at the lightest κ , that we are able to improve the fit by adjusting the range. The confidence level of the linear chiral fits is determined by the χ^2 of the full covariance matrix. For the fixed fit ranges, we have confidence levels of 2×10^{-7} and 0.026 for linear fits to m_π^2 and f_π , respectively. When we allow the fitting range to vary, the confidence levels improve to 0.20 and 0.56, for the

corresponding cases. We have applied the same procedure to case O, and the linear fits in $1/\kappa$ are improved, but not yet good. We have not yet implemented this for every case.

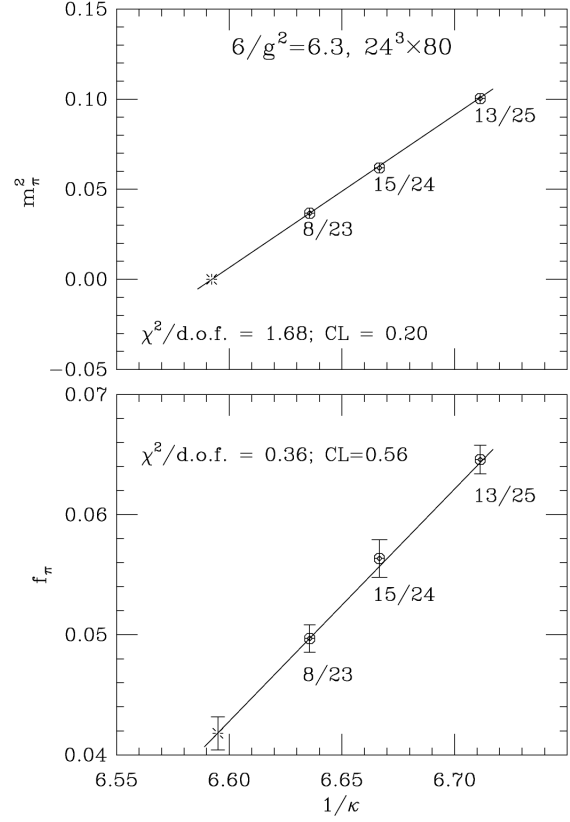


Figure 7. Chiral extrapolation for m_π^2 and f_π on data set D with different fitting ranges

We have both case A and B in order to study finite volume effects on f_B ; however, f_π also varies. We have recently completed production running on a large scale finite volume study of the light quark spectrum at the same gauge coupling [9]. We studied six quark masses on four lattice sizes from $N_s = 12$ to 24. We should be able to study both the linearity of the chiral extrapolation and finite volume effects to higher accuracy.

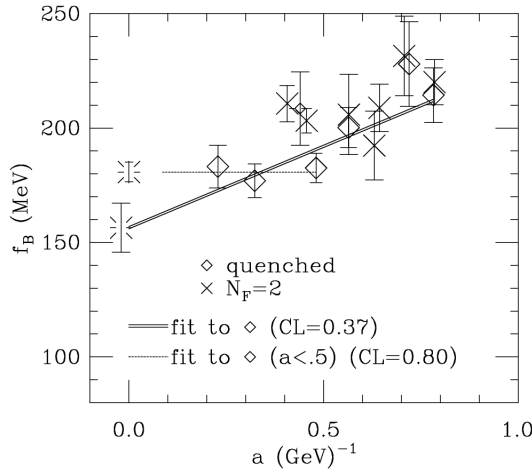


Figure 8. f_B vs. lattice spacing.

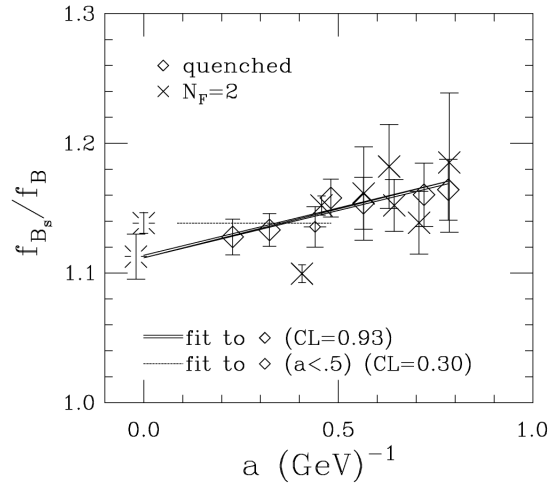


Figure 9. f_{B_s}/f_B vs. lattice spacing.

3. RESULTS

Having carefully fit the hadron propagators and done the mass extrapolations, we are now ready to address the lattice spacing dependence of our results. We have results for f_B (f_D) and f_{B_s} (f_{D_s}) and their ratios. Figure 8 shows our latest results for f_B . The diamonds are the quenched results and two fits are shown for their a dependence. The horizontal line is a constant fit to the three smallest lattice spacings. There is also a linear fit to all of the diamonds. Both fits have quite reasonable confidence levels. The bursts plotted near $a = 0$ are the extrapolated values. In Table 3, we list central values coming from the linear fit. The errors in the graphs are based upon statistical and fitting errors, and are the first errors listed in the table. The second error is our estimate of the systematic errors within the quenched approximation. The chief error here is that from the ambiguity in how to extrapolate in a . Other sources of error include the chiral extrapolation, finite volume effects, weak coupling perturbation theory, the extrapolation in $1/M$ and large ma effects. How we deal with each of these effects has been discussed in Ref. [2]. We must admit that we are wavering between taking our central value from the constant and the linear fit to the lattice spacing dependence. Since we are currently us-

ing the value from the linear fit, the second error is more likely to be positive than negative. The error from using the quenched approximation we consider to be rough at present. We now have results for seven sets of dynamical quark parameters. They are denoted by crosses in the figures. When we have finished our calculations with coupling 5.6, we will attempt an extrapolation in the dynamical quark mass.

In Fig. 9, we show the ratio f_{B_s}/f_B as a function of lattice spacing. One expects that in a ratio many of the systematic errors will be common to the two quantities and that the ratio can be calculated more accurately than either quantity. Here there is better agreement between the constant fit to the three weakest couplings and the linear fit to all the quenched couplings. There is one point that seems to stand out from the others. This is case G, an analysis of $16^3 \times 32$, $6/g^2 = 5.7$ dynamical quark lattices that were provided to us by the Columbia group [10]. These lattices are physically the smallest lattices we have studied. We tried to test whether we might be seeing a finite size effect by doing a new quenched calculation on a small volume. Our most comparable quenched coupling is 6.0 where we studied $16^3 \times 48$ lattices, so we reduced the spatial size to 12 for this calculation (lattice Q). This smaller volume cal-

Table 3
 Preliminary results for meson decay constants

f_B	$= 156 \pm 11 \pm 30 \pm 14$	MeV
f_{B_s}	$= 173 \pm 9 \pm 40 \pm 18$	MeV
f_{B_s}/f_B	$= 1.11 \pm 0.02 \pm 0.03 \pm 0.07$	
f_D	$= 192 \pm 9 \pm 18 \pm 10$	MeV
f_{D_s}	$= 209 \pm 7 \pm 27 \pm 12$	MeV
f_{D_s}/f_D	$= 1.09 \pm 0.02 \pm 0.05 \pm 0.05$	

ulation is plotted in Figs. 8 and 9 with the fancy diamond. Although it appears a little lower than the larger quenched volume, the effect is not as pronounced as for the Columbia lattices. Thus, we do not have a credible explanation of this as a systematic effect.

Of particular note among our results is the value of f_{D_s} . The world average experimental result has recently been reported in a review talk by J. Richman [11]. The result, $241 \pm 21 \pm 30$ MeV compares quite reasonably with ours.

4. CONCLUSIONS

We feel that our calculations of the meson decay constants have progressed to the stage that we can control the sources of systematic error within the quenched approximation. At this point, the major source of uncertainty within that approximation is whether to use a linear extrapolation for all of our couplings or a constant fit to the weakest ones, to extract the continuum results.

Our calculations with dynamical quarks are now giving us a rough bound on the error from quenching, but we are not yet in a position to extrapolate in dynamical quark mass or lattice spacing. We are still running on lattices with $6/g^2 = 5.6$. Beyond these calculations, one can imagine putting in a dynamical strange quark, or repeating this calculation with dynamical Wilson quarks. Such ambitious plans are not on our agenda at the present time.

The apparent agreement between lattice calculations of f_{D_s} and the recent experimental results

is encouraging, but the errors on both are still rather large. With new experimental facilities in Japan and the United States coming on-line in the near future, and with continuing improvement in the lattice results, we expect increasing interplay between experimentalists and the lattice community.

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