

Scale and conformal invariance in Chern-Simons-matter field theory

Wei Chen

Department of Physics, University of Utah, Salt Lake City, Utah 84112

G. W. Semenoff

Department of Physics, University of British Columbia, Vancouver, British Columbia, Canada V6T 2A6

Yong-Shi Wu

Department of Physics, University of Utah, Salt Lake City, Utah 84112

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We study perturbative renormalization of $D=3$ Chern-Simons gauge theory coupled to scalar and fermionic matter. We show that in the non-Abelian case the coefficient of the Chern-Simons term has infinite renormalization and that both the matter and the non-Abelian gauge fields acquire nonvanishing anomalous dimensions at the two-loop level. However, the two-loop β function of the gauge coupling always vanishes, indicating that scale and conformal invariance survive quantization and infinite renormalization.

The action of a generic P - and T -violating gauge theory in $D=2+1$ dimensions contains a Chern-Simons term [1]

$$I_{CS} = -i \int d^3x \epsilon^{\mu\nu\lambda} \left[\frac{1}{2} A_\mu^a \partial_\nu A_\lambda^a + \frac{g_0}{6} f^{abc} A_\mu^a A_\nu^b A_\lambda^c \right], \quad (1)$$

where f^{abc} are structure constants of the gauge group G . This term contains no dimensionful parameters and no reference to the metric and is therefore invariant under spacetime diffeomorphism. Pure gauge theory with action (1) is known to be a finite topological field theory [2,3]. However, physical applications of Chern-Simons gauge theory almost inevitably require coupling to dynamical matter fields. (It is known that this coupling transmutes the spin and statistics of matter fields [4,5]. The resulting phenomena are conjectured to have physical realizations in the quantum Hall effect [6] and high-temperature superconductivity [7].) This coupling of ordinary matter to a topological field theory certainly breaks the diffeomorphism invariance and yields metric-dependent dynamics. A remaining issue is the fate of other symmetries such as (large) gauge invariance and, when the matter action has no dimensional parameters, scale and conformal invariance. More precisely, what is at issue is whether these symmetries survive quantum fluctuations or renormalization; for example, conjectures have been made in the literature about a possible renormalization-group flow [8] of the "statistics parameter," which clearly implies a breakdown of scale and conformal invariance by radiative corrections. To clarify this issue, unlike the familiar situations in $(3+1)$ -dimensional spacetime, one has to go to at least two loops; this is because, in $(2+1)$ -dimensional renormalizable gauge theories, superficially divergent one-loop diagrams always become cutoff independent in any gauge-invariant regularization scheme. In this Rapid Communication we report on our recent results of a systematic perturbative two-loop investigation, in particu-

lar for the non-Abelian cases. (The details need a much longer presentation and will be published elsewhere.)

A Chern-Simons gauge theory with minimal coupling to matter is power-counting renormalizable. Conventionally, the emergence of anomalous dimensions of various fields and nonvanishing β functions that lead to a running coupling constant render the naive scaling Ward identities anomalous [9]. For an *Abelian* theory coupled to a *massive* scalar and fermion, a no-renormalization theorem [10,11] assures that the Chern-Simons action is not renormalized at all beyond one loop (including the finite part). This implies that both the anomalous dimension of the gauge field and the β function for the gauge coupling must vanish. Coupling *massless* matter induces a finite correction to the Abelian Chern-Simons term at two loops [11,12]. Furthermore, as we will show below, the matter fields also acquire nontrivial anomalous dimensions at two loops.

For a *non-Abelian* theory coupled to matter, a one-loop investigation (with a bare F^2 term) has been carried out by Deser, Jackiw, and Templeton [13]. In contrast with the Abelian cases, in our recent two-loop investigation as reported below, we find nonzero anomalous dimensions for the *non-Abelian* gauge fields as well as for the scalar and fermion fields. On the other hand, the two-loop β function for the gauge coupling is shown to vanish due to delicate cancellation between renormalization constants. This indicates that the classical scale and conformal Ward identities are modified merely by anomalous dimensions. These results of ours involve only divergent renormalization parts and therefore are valid both for massless and massive matter. Other theoretical implications will be discussed later.

We first consider the Chern-Simons gauge theory coupled to a massless scalar field with the scale-invariant bare Euclidean action

$$S = \int d^3x \left[|\partial_\mu \phi_i + g_0 T_{ij}^a A_\mu^a \phi_j|^2 + \frac{1}{2\gamma} (\partial_\mu A_\mu^a)^2 + \partial_\mu \bar{c}^a (\partial_\mu c^a + g_0 f^{abc} A_\mu^b c^c) \right] + I_{CS}[A] \quad (2)$$

where $T^a = -(T^a)^\dagger$ are generators of the representation of the Lie algebra of G carried by the scalar and we have fixed a relativistic gauge $\partial_\mu A_\mu^a = 0$ with \bar{c} and c the Faddeev-Popov ghosts. This model contains no dimensional constants and is therefore strictly renormalizable and requires regularization. We introduce renormalization constants,

$$S = \int \left[Z_\phi |\partial_\mu \phi|^2 + Z'_g g [(A_\mu \phi)^* \partial_\mu \phi + (\partial_\mu \phi^*) A_\mu \phi] + (Z''_g) g^2 |A_\mu \phi|^2 - \frac{i}{2} Z_A \epsilon^{\mu\nu\lambda} A_\mu^a \partial_\nu A_\lambda^a \right. \\ \left. - \frac{ig}{6} Z_g \epsilon^{\mu\nu\lambda} f^{abc} A_\mu^a A_\nu^b A_\lambda^c + Z_{gh} \partial_\mu \bar{c}^a \partial_\mu c^a - \tilde{Z}_g g f^{abc} \partial_\mu \bar{c}^a A_\mu^b c^c - \frac{1}{2\gamma_R} (\partial_\mu A_\mu^a)^2 \right], \quad (3)$$

where all fields are now renormalized fields. The Slavnov-Taylor identities give

$$Z''_g = (Z'_g)^2 / Z_\phi, \quad (4a)$$

$$Z_A / Z_g = Z_{gh} / \tilde{Z}_g = Z_\phi / Z'_g, \quad (4b)$$

which must be obtained in any gauge-invariant regularization scheme. We will also work in the Landau gauge $\gamma \rightarrow 0$, where the gauge interactions are free of infrared divergence [1,3].

Various gauge-invariant regularization schemes can be considered. For example, one could introduce a Maxwell or Yang-Mills term $(1/4\Lambda)(F_{\mu\nu}^a)^2$, where Λ is a dimensional parameter which acts as the cutoff. This is called F^2 regularization [3,11] and renders the regularized theory super-renormalizable [13]. This regularization complicates the rather simple Feynman rules of the pure Chern-Simons term so that higher-order computations are less accessible. We will use a more elegant regularization, i.e., the regularization by dimensional reduction introduced in Ref. [3]: One performs a dimensional continuation of the integration measure in all Feynman integrals, but the vector and tensor quantities including $\epsilon^{\mu\nu\lambda}$ which appear in the numerators of Feynman integrands are always treated as if they are strictly three dimensional. Though there is no *a priori* reason to believe that this procedure is gauge invariant, in all cases we have examined it is. Previously it was shown to be gauge invariant in pure Chern-Simons gauge theory to three loops [3]. In fact, with this regularization the pure Chern-Simons theory has been proven to have no renormalization at all to three loops, and it is conjectured that it has no renormalization to all orders [3]. For the cases at hand, we have been careful to verify the Slavnov-Taylor identities whenever possible.

At one loop, we have checked explicitly that in the dimensional reduction scheme there is no renormalization of the vertices in (3), from either the matter or the gauge-field couplings. Thus all renormalization constants Z_i can be chosen as $Z_i = 1$ to one-loop order [14]. This is in agreement with Ref. [13]. In particular, with our dimensional regularization there is no shift in the Chern-Simons coefficient at all and the mass squared counterterm for the scalar field $\delta m^2 = 0$, while the former is finite and the latter is linearly divergent in, e.g., F^2 regularization.

At two loops, the Feynman graphs can be classified into three sets—those which vanish identically before performing any integrations, those which vanish or at least give finite results after performing one or more of the integrations (in all cases this happens solely because of Euclidean rotation invariance of the integral), and those which have

divergent parts. We confine ourselves to divergent graphs. The relevant Feynman diagrams are presented in Figs. 1(a)–1(d), respectively. Under the minimal subtraction they yield the divergent parts for relevant renormalization constants:

$$Z_A = 1 + \frac{g^4}{24\pi^2} C_1 C_2 \frac{1}{3-\omega}, \quad (5a)$$

$$Z_{gh} = 1 - \frac{g^4}{48\pi^2} C_1 C_2 \frac{1}{3-\omega}, \quad (5b)$$

$$Z_g = 1 + \frac{g^4}{16\pi^2} C_1 C_2 \frac{1}{3-\omega}, \quad (5c)$$

$$Z_\phi = 1 + \frac{g^4}{12\pi^2} \kappa \left(\frac{5}{2} \kappa + \frac{13}{8} C_2 + C_1 \right) \frac{1}{3-\omega}, \quad (5d)$$

$$Z'_g = 1 + \frac{g^4}{12\pi^2} \left[\kappa \left(\frac{5}{2} \kappa + \frac{13}{8} C_2 + C_1 \right) + \frac{1}{4} C_1 C_2 \right] \frac{1}{3-\omega}, \quad (5e)$$

where $C_2 \delta^{ab} = f^{acd} f^{bcd}$, $C_1 \delta^{ab} = \text{Tr}(T^a T^b)$, and $\kappa I = T^a T^a$. We normalize T^a so that $\text{Tr}(T^a T^b) = -\frac{1}{2} \delta^{ab}$ in the adjoint representation. Furthermore, we verify that \tilde{Z}_g is finite, in agreement with a general theorem [15] that the gluon-ghost-ghost vertex is finite and one can always choose $\tilde{Z}_g = 1$. These divergent parts of the renormalization constants are computed independently and respect the Slavnov-Taylor identities (4b). This nontrivial check verifies that the divergent parts of vertices which we compute by dimensional reduction are compatible with gauge invariance.

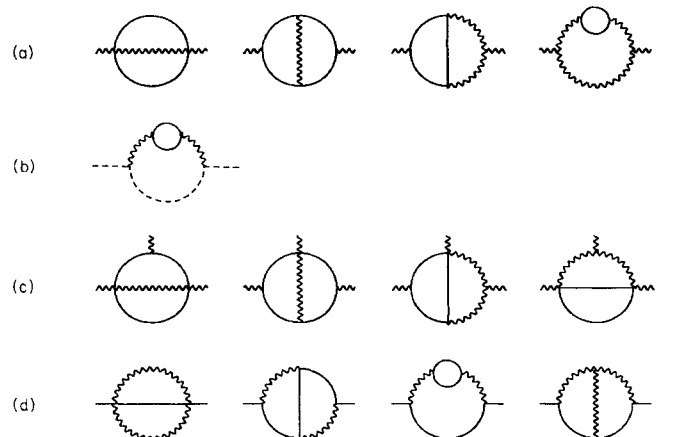


FIG. 1. Relevant divergent two-loop Feynman diagrams for (a) Z_A , (b) Z_{gh} , (c) Z_g , (d) Z_ϕ .

Thus, the bare and renormalized fields are related by logarithmically divergent renormalization. (5a) and (5d) give the corresponding anomalous dimensions

$$\begin{aligned} \gamma_A(g) &= -\frac{g^4}{12\pi^2} C_1 C_2, \\ \gamma_\phi(g) &= -\frac{g^4}{6\pi^2} \kappa \left(\frac{5}{2} \kappa + \frac{13}{8} C_2 + C_1 \right). \end{aligned} \tag{6}$$

(These quantities, though gauge dependent, will appear in the anomalous scale Ward identities and in the calculation of anomalous dimensions for gauge-invariant composite operators.) By using (5a) and (5c), we find that a cancellation leads to a vanishing β function for the renormalized coupling constant $g = g_0 Z_A^{3/2} / Z_g$:

$$\beta_g(g) = 0. \tag{7}$$

We conjecture this is true to all orders in perturbation theory. This indicates that the physical gauge coupling constant does not run at all, though the theory needs infinite renormalization. An argument similar to that in $D=4$ in Ref. [16] shows that (7) also implies the conformal invariance when charged matter is massless.

Similar results are also obtained for the Chern-Simons theory coupled to massless fermion: We find exactly the same formulas as Eqs. (5a)–(5c) for the divergent parts of renormalization constants in the fermion case, but

$$Z_\psi = 1 + \frac{g^4}{16\pi^2} \kappa \left(\kappa + \frac{5}{6} C_2 + \frac{1}{3} C_1 \right) \frac{1}{3-\omega}. \tag{8}$$

In passing we note a simple criterion for checking whether or not the β function vanishes: Since Z_i are finite in pure Chern-Simons theory, (4b) implies no infinite renormalization for g_0 if and only if the divergent contributions from matter loops satisfy

$$\delta Z_A = -2\delta Z_{gh} \text{ or } \delta Z_g = -3\delta Z_{gh}. \tag{9}$$

(At two loops only one divergent diagram contributes to δZ_{gh} .)

For an Abelian theory, we take $f^{abc} = 0$ and $T^a = -i$; then, by minimal subtraction,

$$Z_A = 1, \tag{10a}$$

$$Z_\phi = Z'_g = 1 + \frac{7g^4}{24\pi^2} \frac{1}{3-\omega}, \tag{10b}$$

$$Z_\psi = 1 + \frac{g^4}{12\pi^2} \frac{1}{3-\omega}. \tag{10c}$$

While (10a) is known before [10,11], Eqs. (10b) and (10c) are our new results.

Since ultraviolet divergences of a gauge theory are independent of whether the coupled matter is massless or massive, our results are valid also for *massive* matter, assuming there is no bare F^2 term. In fact, in a massive theory our results for Z_i are the same as if one uses Weinberg's zero-mass renormalization scheme [17]. However, note that while massless matter does not induce a finite Maxwell or Yang-Mills term, massive matter does. If an F^2 term is incorporated in the action (3), the theory becomes super-renormalizable and our results can

be used to derive the leading-logarithmic terms in the large-gauge-boson-mass limit, since the F^2 term may be viewed as a regulator for the theory without it. We also note that though in Chern-Simons theories finite radiative corrections are regularization method dependent [1,3,12,13], our result on the vanishing β function is expected to be both gauge invariant and regularization independent.

Our two-loop results have interesting and significant implications. First, Eqs. (5a) and (5c) show that the non-Abelian gauge fields acquire an infinite renormalization from coupling to either massless or massive matter. Unlike in the Abelian case, the coefficient of the Chern-Simons terms acquires infinite renormalization even from massive matter fields.

The bare non-Abelian coupling satisfies a topological quantization condition [1] $4\pi/g_0^2 = \text{integer}$, because of the invariance of (1) under large gauge transformations. It is widely believed but not *a priori* clear to us [18] that the same topological quantization condition should be respected perturbatively by the renormalized (or, more precisely, physical) coupling constant g . Though our vanishing β function is consistent with a topological quantization condition for g , the latter also requires nonrenormalization of g beyond one loop including the finite part, which is to be verified yet in Chern-Simons theory coupled to matter [19].

It is remarkable that in all cases we have examined, Abelian and non-Abelian, coupled to massless or massive matter, which can be either scalar or fermion, the β function for the Chern-Simons gauge coupling always vanishes, and the “statistics parameter” $g_0^2/4\pi$ is independent of the renormalization scale. Thus far there seems to be no unified understanding of this remarkable result: The no-renormalization theorems do not apply to the non-Abelian cases, while the topological quantization does not apply to Abelian cases. We speculate that the survival of scale and conformal Ward identities is somehow related to the fact that the kinetic Chern-Simons action (1) is a topological action. It is known that classical Abelian Chern-Simons gauge fields coupled to quantum-mechanical massive point particles are nonpropagating in the sense that, at fixed time, they can be eliminated in terms of charged currents. While the appearance of anomalous dimensions of field operators in the Abelian theory would seem to imply that at the quantum level the Chern-Simons fields cease to be entirely nondynamical when coupled to matter fields, the fact that the gauge coupling does not run indicates that the essential nondynamical role of transmuting the statistics of particles survives and is scale independent. This latter property may render these theories at least partially solvable [20].

Our results have implications for the statistical models whose critical behavior is described by the continuum matter-coupled Chern-Simons theory. Since the β function for the coupling vanishes, the critical behavior is governed by a one-parameter family of theories specified by the gauge coupling. Topological quantization implies that each integer $4\pi/g_0^2$ specifies a universality class [21]. Furthermore, the Chern-Simons interactions generate anomalous dimensions for operators and therefore modify

critical exponents of the theory. It would be interesting to compute the anomalous dimensions of composite gauge-invariant operators.

Note added. Since this paper was written there have been numerous studies of surface states and hierarchy states in fractional quantum Hall systems using non-Abelian Chern-Simons theory. These include J. Frohlich and A. Zee, Santa Barbara Report No. NSF-ITP-91-31 (unpublished), E. Fradkin and M. Lopez, University of Illinois report (unpublished), A. Balatsky and E. Fradkin, University of Illinois report (unpublished), E. Verlinde, Princeton Report No. IASSNS-HEP-90/60 (unpub-

lished), and X.-G. Wen, Princeton Report No. IASSNS-HEP-90/70 (unpublished). These models are all, in principle, coupled to matter and the renormalization flow of the couplings and induced anomalous dimensions of operators which we have computed for some examples in the present paper are important characteristics of the critical behavior of these systems.

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- [1] S. Deser, R. Jackiw, and S. Templeton, *Phys. Rev. Lett.* **48**, 975 (1982).
- [2] E. Witten, *Commun. Math. Phys.* **121**, 351 (1989).
- [3] For perturbative pure Chern-Simons theory, see W. Chen, G. W. Semenoff, and Y.-S. Wu, in *Physics, Geometry, and Topology*, Proceedings of Banff Summer School on Particles and Fields, Banff, Canada, 1989, edited by H. C. Lee (Plenum, New York, 1990), p. 553; *Mod. Phys. Lett. A* **5**, 1833 (1990); R. Pisarski and S. Rao, *Phys. Rev. D* **32**, 2081 (1985).
- [4] F. Wilczek and A. Zee, *Phys. Rev. Lett.* **51**, 2250 (1983); Y.-S. Wu and A. Zee, *Phys. Lett.* **147B**, 325 (1984); G. Semenoff, *Phys. Rev. Lett.* **61**, 517 (1988); A. Polyakov, *Mod. Phys. Lett. A* **3**, 325 (1988); J. Dunne, R. Jackiw, and C. Trugenberger, *Ann. Phys. (N.Y.)* **194**, 197 (1989).
- [5] A. P. Balachandran, M. Bordeau, and S. Jo, *Mod. Phys. Lett. A* **4**, 1923 (1989); *Int. J. Mod. Phys. A* **5**, 2423 (1990); J. Frohlich, in *Physics, Geometry, and Topology* (Ref. [3]).
- [6] S. Girvin and A. H. Macdonald, *Phys. Rev. Lett.* **58**, 1252 (1987); N. Read, *ibid.* **62**, 86 (1989); S. C. Zhang, T. H. Hansson, and S. Kivelson, *ibid.* **62**, 82 (1989).
- [7] I. Dzyaloshinsky, A. M. Polyakov, and P. Wiegman, *Phys. Lett. A* **127**, 112 (1987); Y. H. Chen, F. Wilczek, E. Witten, and B. I. Halperin, *Int. J. Mod. Phys. B* **3**, 1001 (1989); X. G. Wen and A. Zee, *Phys. Rev. B* **41**, 240 (1990).
- [8] A. M. Polyakov, in *Fields, Strings and Critical Phenomena*, Proceedings of the Les Houches Summer School, Les Houches, France, 1988, edited by E. Brezin and J. Zinn-Justin, Les Houches Summer School Proceedings Vol. 49 (North-Holland, Amsterdam, 1990); I. Dzyaloshinsky, in 1989 Summer Workshop at ICTP, Trieste (unpublished).
- [9] See, e.g., S. Coleman, *Aspects of Symmetries* (Cambridge University Press, Cambridge, England, 1985), Chap. 3.
- [10] S. Coleman and B. Hill, *Phys. Lett.* **159B**, 184 (1985).
- [11] G. W. Semenoff, P. Sodano, and Y.-S. Wu, *Phys. Rev. Lett.* **62**, 715 (1988).
- [12] W. Chen, *Phys. Lett. B* **251**, 415 (1991); W. Chen, G. W. Semenoff, and Y.-S. Wu, University of Utah report (unpublished).
- [13] S. Deser, R. Jackiw, and S. Templeton, *Ann. Phys. (N.Y.)* **140**, 372 (1982).
- [14] We therefore expect that in any gauge-invariant regularization and renormalization scheme the one-loop contribution to the constants Z_i is finite. This is known to be the case in F^2 regularization [3,10].
- [15] J. C. Taylor, *Nucl. Phys.* **B33**, 436 (1971).
- [16] B. Schroer, *Lett. Nuovo Cimento* **2**, 287 (1971).
- [17] S. Weinberg, *Phys. Rev. D* **8**, 3497 (1973).
- [18] We note that the renormalized and bare gauge transformations
- $$(A_r)^U = U(x)^{-1} A_r U(x) + \frac{1}{g} U(x)^{-1} dU(x),$$
- $$(A_0)^U = U(x)^{-1} A_0 U(x) = \frac{1}{g_0} U(x)^{-1} dU(x)$$
- are *not* equivalent to each other in a non-Abelian theory when $g \neq Z_A^{1/2} g_0$ or $Z_A \neq Z_g$. Also, Slavnov-Taylor identities do not guarantee large gauge invariance for A_r .
- [19] The null result by Y. C. Kao and M. Suzuki [*Phys. Rev. D* **31**, 2137 (1985)] is obtained with a bare F^2 but no Chern-Simons term.
- [20] E. S. Fradkin and M. Ya. Palchik, *Int. J. Mod. Phys. A* **18**, 3463 (1990).
- [21] G. Moore and N. Read, Yale University report, 1990 (unpublished).