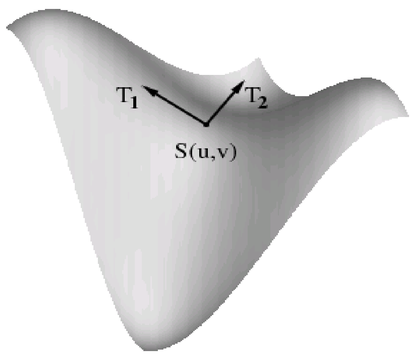
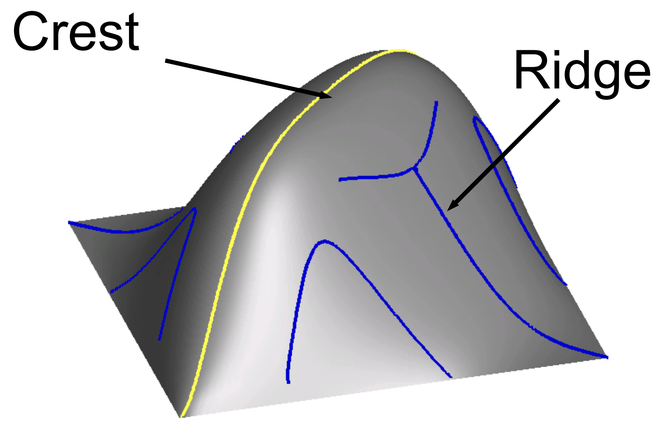
Tracing Ridges on B-Spline Surfaces*

Suraj Musuvathy[1], Elaine Cohen[1], Joon-Kyung Seong[2], James Damon[3]
[1]School of Computing, University of Utah
[2]Korea Advanced Institute of Science and Technology
[3]Department of Mathematics, University of North Carolina, Chapel Hill



Definition of Ridge:
Locus of extremum points of principal curvatures along principal directions

κ_1-ridge	$\phi_1(u,v) = < \nabla\kappa_1, t_1 > = 0$
κ_2-ridge	$\phi_2(u,v) = < \nabla\kappa_2, t_2 > = 0$
κ_1-crest	$t_1^T H_{\kappa_1} t_1 < 0 \quad , \quad \lvert\kappa_1\rvert > \lvert\kappa_2\rvert$
κ_2-crest $\rightarrow$ ravine or valley	$t_2^T H_{\kappa_2} t_2 > 0 \quad , \quad \lvert\kappa_1\rvert < \lvert\kappa_2\rvert$
	Hessian, $H_{\kappa_i} = \begin{bmatrix} \kappa_{iuu} & \kappa_{iuv} \\ \kappa_{iuv} & \kappa_{ivv} \end{bmatrix}$



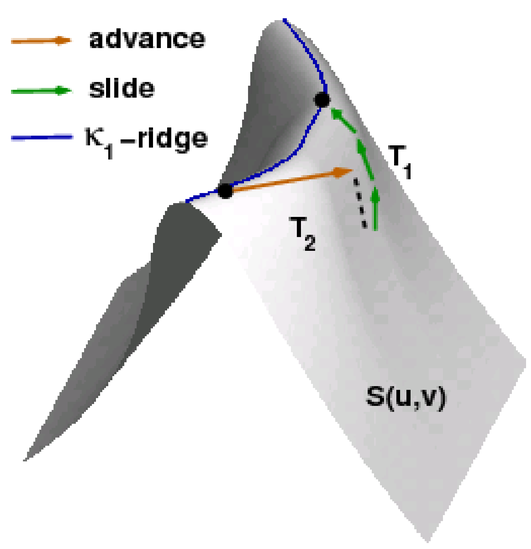
$\kappa_1 \geq \kappa_2$: principal curvatures
t1, t2 : principal directions
(tangent space coefficients)

Goals

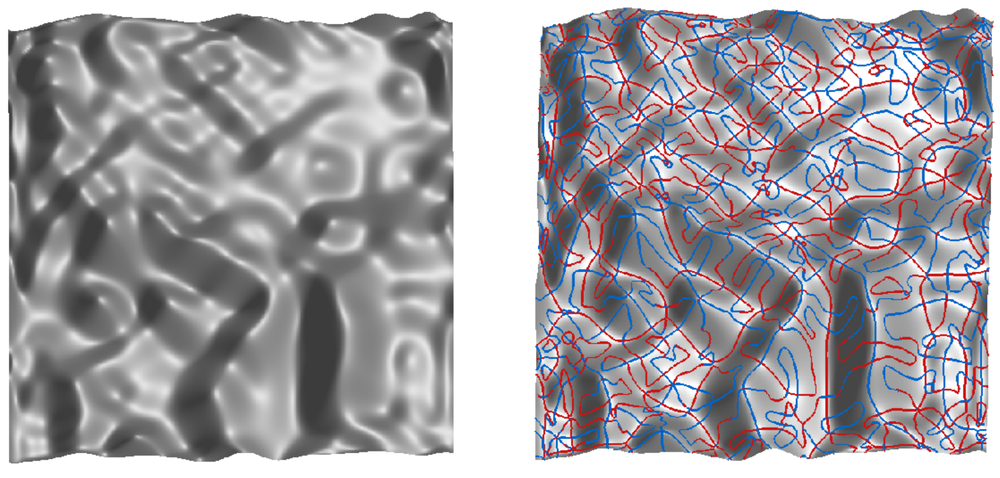
- Compute directly on B-Spline surfaces
- Robust algorithm
- Accurate
- Extract connected curves
- Address umbilics, turning points
- Computationally suitable

Approach

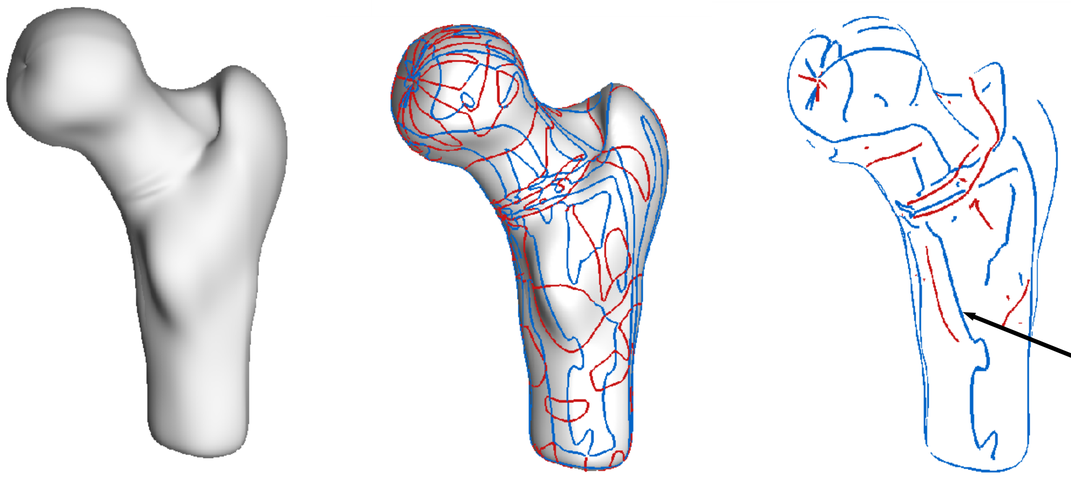
- Seed points (curvature extrema, umbilics)
- Advance + slide
- Robust step sizes
- Umbilics, turning points



Results (blue → κ_1 ridges, red → κ_2 ridges)



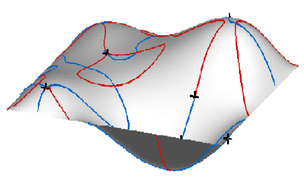
Utah terrain
20 x 20
Biquartic
B-Spline



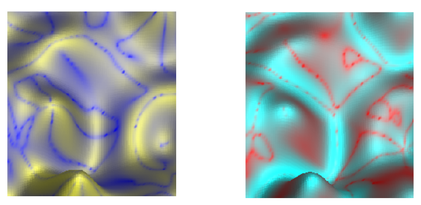
Femur
22 x 21
Biquartic
B-Spline

Crests
only

Previous Work



<u>Lattice method</u>
Low degree Bezier
Computationally expensive



<u>Sampling methods</u>
Discrete ridge points
Inaccurate
Incorrect topology

Applications

- Surface registration
- Quality control
- Visualization
- Terrain analysis

** Won Best Paper Award at SIAM/ACM Joint Conference on Geometric and Physical Modeling, San Francisco, 2009*