Electronic Raman scattering in a magnetic field

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Raman scattering in a magnetic field is proposed as a possible method to solve the problem of whether the light scattering in high-T_c superconductors comes from conduction electrons or not. The electronic Raman light scattering in a magnetic field is studied theoretically. The semiclassical approach based on the system of Boltzmann and Maxwell equations is applied. Two types of resonances in the scattering cross section are found to be from electrons moving along the semiclassical trajectories in a magnetic field. The first one, the cyclotron resonance, is associated with the time dispersion and has the symmetrical logarithmic shape. It is sharp in a clean metal. We show that in a dirty metal the relaxation continuum obtained by Zawadowski and Cardona is modified drastically into the set of smaller continua which are located at the frequencies of simple and multiple cyclotron resonances. The other resonance, associated with the space dispersion, results from the electrons moving coherently with the effective external field. It has the asymmetrical shape. The effects of skin depth and Coulomb interaction are analyzed. The strong peak associated with excitation or absorption of helicons is found in the electronic Raman spectra in a high magnetic field. This peak is considered for the case of dirty metal.

I. INTRODUCTION

The well known Raman light scattering (RS) is a successful tool for studying excitation spectra of various systems. The theory of RS from insulators is well developed. It is in good agreement with various experimental data (for the review see Ref. 1). However, the recent measurements of RS from high-T_c superconductors[2–7] are in striking disagreement with the present theory. Three main problems had arisen. First, the low-frequency transfer behavior of RS is linear and does not show any threshold at the superconducting gap. Second, the great qualitative and quantitative differences are observed in the RS spectra of light with different polarizations. Third, the high-frequency behavior for some polarizations does not demonstrate the decreasing of scattering cross section at all. It is independent of the frequency transfer right up to the large frequencies $\simeq 10^{15}\ \mathrm{s}^{-1}$.

Several attempts have been made to explain these astonishing facts. Particularly, the absence of frequency threshold is explained supposing the $d_{x^2-y^2}$ symmetry of the gap.[8] The polarization dependence was explained by the alternating layers of superconducting and normal phases.[9] At last, the high-frequency continuum was assumed to be from nesting on the Fermi surface.[10]

All of these explanations suppose RS in high-T_c superconductors as being from the conduction electrons. However, the notion exists that this is not the fact, because the high-frequency continuum is not sensitive to the carriers density.[7] That is, the problem arises to determine whether the RS is from conduction electrons or not. The theory of RS from electrons in normal metals and conventional superconductors is developed and the historical review is as follows. Since the first paper[11] considered a clean superconductor, there were papers devoted to the electron-impurity,[12,13] and electron-phonon[14,15] relaxation continua in normal metals. The dirty conventional superconductors were considered in Ref. 16. The various processes of electronic RS, with excitation or absorption of different quasiparticles, such as acoustic[13,15] and optical[17–19] phonons and plasmons,[20,21] were evaluated too.

In this paper, the possible method for determining the role of electrons in the RS is proposed. One has to investigate the RS in a magnetic field. The according theory is derived. We use the semiclassical approximation, provided that the frequency and momentum transfer are much smaller than the Fermi energy and momentum, respectively. The Boltzmann equation together with Maxwell equation are solved self-consistently in a magnetic field. The external magnetic field modifies drastically the electronic contribution to RS. Also, the low-energy excitations appear in the magnetic field — the so-called helicons. Their contribution into the scattering cross section arises from Maxwell equation, as well as the effects of Coulomb interaction of carriers. We consider the simple geometry of normal light incidence and scattering. The magnetic field is along the perpendicular to the surface. In this geometry, the surface contribution of electromagnetic excitations is absent. The surface affects the scattering only through the existence of the skin layer. The electron collisions are taken into account in the τ approximation.

The paper is organized as follows. In Sec. II, the expression for the RS cross section is derived in terms of the linear response to the effective field, using the fluctuation-dissipation theorem. The system of Boltzmann and Maxwell equations is solved with the appropriate boundary conditions for electrons. In Secs. III and IV, different terms in RS cross section are analyzed. In Sec. III, two types of resonances are found. The cyclotron resonances are located at the frequencies transfer which are multiple to the cyclotron frequency. The other, "magneto-raman" resonance (similar to the well-known magnetoacoustic[22] resonance in the sound damping) comes from the singularity in the density of states of electrons moving coherently with the effective field. In Sec. IV, the role of Coulomb interaction is analyzed. The helicon contribution is found to give a sharp peak in the strong magnetic field. The height and width of the peak are defined by the largest of two factors resulting from the skin effect and the electronic collisions.

II. THEORETICAL FRAMEWORK

One can consider the electronic RS as the scattering process in the effective external field described by the Hamiltonian,

$$H_{\text{eff}}=\frac{e^2}{mc^2}\int\frac{d^3p}{(2\pi)^3}\gamma(\mathbf{p})U(\mathbf{r},t)f_p(\mathbf{r},t), \tag{1}$$

where $f_p(\mathbf{r},t)$ is the semiclassical electronic distribution function. The effective electron-photon vertex $\gamma(\mathbf{p})$ takes into account the interband transitions and depends on light polarizations (see Ref. 23). The explicit expression for the vertex is not of interest here, so we do not derive it. The effective external field $U(\mathbf{r},t)$ is the product of vector potentials of the incident and scattered light:

$$A^{(i)}(\mathbf{r},t)A^{(s)}(\mathbf{r},t)\simeq U(\mathbf{r},t)=U(z)\exp[i(\mathbf{k}_s\mathbf{s}-\omega t)], \tag{2}$$

here, $\mathbf{s}$ is the spatial vector coordinate along the surface and z is the coordinate normal to the surface. The indices (i) and (s) denote the quantities related to the incident and scattered waves, respectively, and the frequency $\omega=\omega^{(i)}-\omega^{(s)}$ and momentum $\mathbf{k}_s=\mathbf{k}_s^{(i)}-\mathbf{k}_s^{(s)}$ transfer (along the surface) are introduced. The sample occupies the half space $z>0$. The factor, $U(z)$ describes the penetration of light into the crystal. To find this factor, one has to solve the system of Maxwell and Boltzmann equations. However, we will be interested in the case of the normal skin effect, with the usual exponential law,

$$U(z)=\exp(i\zeta z)=\exp(i\zeta_1 z-\zeta_2 z), \tag{3}$$

where ζ represents a sum of incident and scattered light wave vectors in the metal:

$$\zeta=\zeta^{(i)}+\zeta^{(s)}.$$

For example, for the isotropic case, we have

$$\zeta^{(i)2}=\epsilon(\omega^{(i)})\frac{\omega^{(i)2}}{c^2}-\mathbf{k}_s^{(i)2},$$

and the same for $\zeta^{(s)}$. The penetration is described by the quantity $\zeta_2=\operatorname{Im}\zeta$ related to the skin depth as $\delta\simeq 2\zeta_2^{-1}$.

The expression (1) is essentially semiclassical. It can be used if the frequency transfer and momentum of light are rather small: $\omega\ll\varepsilon_F;k^{(i)},k^{(s)}\ll p_F$, where ε_F,p_F are the Fermi energy and momentum, respectively. For the real experimental situation this is always the fact.

The RS cross section was obtained previously by evaluating the generalized susceptibility in the field (1), with the help of the fluctuation-dissipation theorem. It has the form[21]

$$\frac{d^2\sigma}{d\omega do}=\frac{8e^4k_z^{(s)}\omega^{(s)}}{\pi m^2c^3\omega^{(i)2}}\frac{\Sigma(\mathbf{k}_s,\omega)}{1-\exp(-\omega/T)}, \tag{4}$$

where $k_z^{(s)}$ is the z component of scattered light momentum in vacuum,

$$k_z^{(s)2}=\frac{\omega^{(s)2}}{c^2}-\mathbf{k}_s^{(s)2},$$

and $\Sigma(\mathbf{k}_s,\omega)$ is defined by

$$\Sigma(\mathbf{k}_s,\omega)=-2\operatorname{Im}\int_0^\infty dz U^*(z)\int\frac{2d^3p}{(2\pi)^3}\gamma^*(\mathbf{p})f_p(\mathbf{k}_s,z,\omega), \tag{5}$$

where $f_p(\mathbf{k}_s,z,\omega)$ is the Fourier transform of distribution function, with respect to coordinates parallel to the surface:

$$f_p(\mathbf{k}_s,z,\omega)=\int\frac{d\mathbf{s}}{(2\pi)^2}f_p(\mathbf{r},\omega)\exp(-i\mathbf{k}_s\mathbf{s}).$$

Thus, to obtain the cross section, one needs to find the electronic distribution function in the field (1). The most natural way to do this is to apply the Boltzmann equation.

Let us consider the simple geometry of normal to the surface (along the z axis) light incidence and scattering (in fact, we need $\mathbf{k}_s=0$ only). The external magnetic field $\mathbf{H}$ is assumed to be applied along the z axis. Hence, the magnetic field does not affect the electron motion along the z axis. That is why the boundary condition for electrons is not modified by the magnetic field in the chosen geometry. We take it in the form of specular reflection at the boundary $z=0$,

$$f_p(z=0,\omega)_{v_z>0}=f_p(z=0,\omega)_{v_z<0}. \tag{6}$$

It is convenient to transfer from p_x,p_y to variables ε and ϕ, which are the energy and angle, respectively:

$$\gamma(\mathbf{p})\rightarrow\gamma(p_z,\phi,\varepsilon),d^3p\rightarrow m^*(p_z)dp_zd\phi d\varepsilon,$$

with $m^*(p_z)$ being the cyclotron mass.[22]

The Boltzmann equation for electrons in the field (1) can be linearized by the substitution

$$f_p(z,\omega)=f_0[\varepsilon_0+\gamma(p_z,\phi)U(z)-\mu]+\frac{df_0}{d\varepsilon}\chi_p(z,\omega), \tag{7}$$

where the first term is the Fermi-Dirac local equilibrium distribution function. We do not write the factor e^2/mc^2 before $\gamma(p_z,\phi)$, because it is already extracted in (4). The first term cancels the collision integral. For the second term in Eq. (7), we will use the τ approximation (about the τ approximation in the RS; see Ref. 13).

To use the Fourier transform, with respect to the z coordinate in entire space, we apply the even continuation into the $z<0$ half space for $\chi_p(z,\omega),U(z),E_s(z,\omega)$ [e.g., $U(z)=U(-z)$] and the odd continuation for $E_z(z,\omega)$ [$E_z(z,\omega)=-E_z(-z,\omega)$]. Here, $\mathbf{E}$ is the electric field induced by (1)–(3). After all, the Boltzmann equation takes the form

$$-i[\omega-v_z(p_z)k+i\tau_p^{-1}]\chi_p(k,\omega)+\omega_H(p_z)\frac{\partial\chi_p(k,\omega)}{\partial\phi}$$
$$=i\omega\gamma(p_z,\phi)U(k)-e\mathbf{v}(p_z,\phi)\mathbf{E}(k,\omega), \tag{8}$$

here, $\omega_H(p_z)=eH/\mathrm{c}m^*(p_z)$ is the cyclotron frequency. The vertex $\gamma(p_z,\phi)$ and velocity $\mathbf{v}(p_z,\phi)$ are taken on the Fermi surface, since $\omega,\omega_H,T\ll\varepsilon_F$. We also suppose that v_z does not depend on ϕ for simplicity. Using the periodicity over ϕ, one can write the solution of (8) in such a way

$$\chi_p(k,\omega)=-\sum_n e^{in\phi}\frac{\omega\gamma^{(n)}(p_z)U(k)+ie\mathbf{v}^{(n)}(p_z)\mathbf{E}(k,\omega)}{\omega-n\omega_H(p_z)+v_z(p_z)k+i\tau_p^{-1}}, \tag{9}$$

where we introduce

$$\gamma^{(n)}(p_z)=\int_0^{2\pi}\frac{d\phi}{2\pi}\gamma(p_z,\phi)e^{-in\phi},\mathbf{v}^{(n)}(p_z)$$

$$=\int_0^{2\pi}\frac{d\phi}{2\pi}\mathbf{v}(p_z,\phi)e^{-in\phi}. \tag{10}$$

One can easily see that the solution (9) satisfies the boundary condition (6). The Eq. (7) should be accompanied by the Maxwell equation for the electric field:

$$\text{rotrot}\mathbf{E}(k,\omega)=\frac{4\pi i\omega}{c^2}\mathbf{j}(k,\omega), \tag{11}$$

with the electric current density being

$$\mathbf{j}(k,\omega)=-2e\int\frac{d\phi dp_z}{(2\pi)^3}m^*(p_z)\mathbf{v}\chi_p(k,\omega). \tag{12}$$

As was pointed out,[24] the contribution of filled bands $(\omega^2/c^2)\mathbf{D}$ in conductors may be omitted in the Eq. (11) .

By using Eq. (9), we obtain from Eq. (12)

$$j_i(k,\omega)=\sigma_{ik}(k,\omega)E_k(k,\omega)+e\omega F_i(k,\omega)U(k,\omega), \tag{13}$$

with

$$\sigma_{ik}(k,\omega)=ie^2\sum_n\left\langle\frac{v_i^{(n)*}(p_z)v_k^{(n)}(p_z)}{\omega-n\omega_H-v_zk+i\tau_p^{-1}}\right\rangle, \tag{14}$$

$$F_i(k,\omega)=\sum_n\left\langle\frac{v_i^{(n)*}(p_z)\gamma^{(n)}(p_z)}{\omega-n\omega_H-v_zk+i\tau_p^{-1}}\right\rangle, \tag{15}$$

where the brackets denote the integration over the Fermi surface:

$$\langle\cdots\rangle=\frac{2}{(2\pi)^2}\int dp_zm^*(p_z)(\ ).$$

Substituting Eq. (12) into the Maxwell equation (11), one receives the equation determining the electric field. Its solution has the form

$$E_i(k,\omega)=\frac{4\pi i\omega^2}{c^2}D_{ik}(k,\omega)F_k(k,\omega)U(k,\omega), \tag{16}$$

where the electromagnetic Green function $D_{ik}(k,\omega)$ is defined in the chosen geometry by the equation

$$\left(k^2(\delta_{ik}-\delta_{iz}\delta_{kz})-\frac{4\pi i\omega}{c^2}\sigma_{ik}(k,\omega)\right)D_{kl}(k,\omega)=\delta_{il}. \tag{17}$$

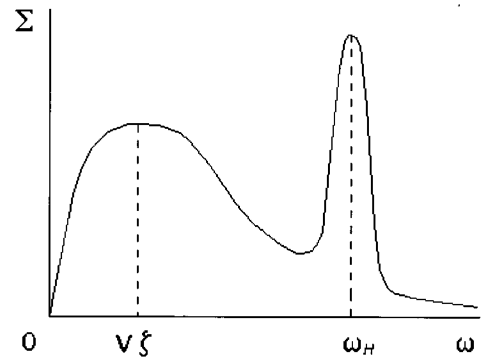

FIG. 1. The RS cross section in the clean metal $|\zeta|l>>1$[see Eqs. (21) and (22)] for the Stokes range $\omega>0$. The continuum comes from the zero-harmonic $n=0$ scattering chanel. It takes the maximum at $\omega=v|\zeta|$ and for higher frequencies decreases as $\sim\omega^{-3}$. The peak corresponds to the cyclotron resonance $(n=1)$. Its height is proportional to $\ln|\zeta|l$. The other peaks $(n>1)$ are not shown here.

Substituting the electric field (16) into the solution of Boltzmann equation (9) and then to Eqs. (7) and (5), we find the electronic RS cross section in a magnetic field

$$\Sigma(\omega)=-\ \text{Im}\int\frac{dk}{2\pi}|U(k)|^2$$
$$\times\left(\sum_n\left\langle\frac{\omega|\gamma^{(n)}(p_z)|^2}{\omega-n\omega_H(p_z)-v_z(p_z)k+i\tau_p^{-1}}\right\rangle\right.$$
$$\left.-\frac{4\pi e^2\omega^2}{c^2}F_i^*(k,\omega)D_{ik}(k,\omega)F_k(k,\omega)\right). \tag{18}$$

The rest of the paper is devoted to the discussion of different terms in the basic formula (18).

However, let us first disscuss the concrete form of effective field $U(k)$. As soon as we deal with the optical frequency range for the incident and scattered light, the normal skin effect is supposed. Then, because the light frequencies are large enough, $\omega^{(i)}\sim\omega^{(s)}\gg\omega,\omega_H,vk$, the light penetration into metal is not affected by various resonant processes, and the skin effect is static (2). The Fourier transform of $U(z)$ after the even continuation

$$U(k)=\frac{2i\zeta}{\zeta^2-k^2} \tag{19}$$

has form of the peak, located at $k=\zeta_1$ with the width ζ_2.

III. RAMAN SCATTERING FROM ELECTRON-HOLE PAIRS

The first term in the right hand side of Eq. (18) represents the contribution of usual electron-hole pairs moving along the semiclassical trajectories in the external magnetic field. We see that the zero harmonic $n=0$ is not affected by the magnetic field. It represents the well-known electron-hole continuum. In a clean metal $v\zeta\tau\gg1$,[11,21] it has the maximum at $\omega\simeq v/\delta$ and for larger frequencies transfer decreases as $\sim\omega^{-3}$ (see Fig. 1). In a dirty metal $v\zeta\tau\ll1$,[12,13] the maxi-

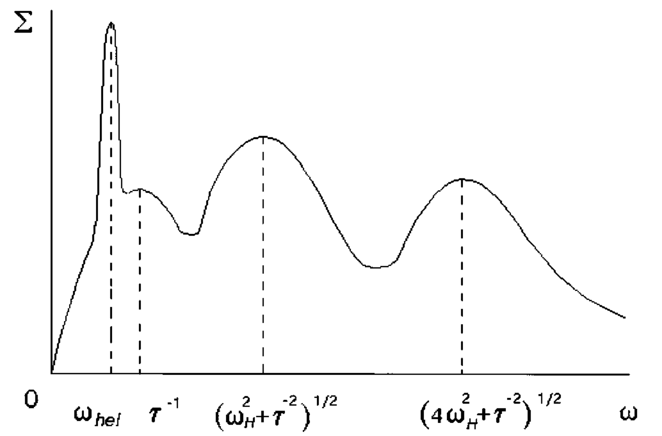

FIG. 2. The RS cross section for the dirty metal $|\zeta|l<<1$ [see Eqs. (23)] and strong magnetic field $\omega_H>\tau^{-1}$. The first continuum is the usual relaxation background $\omega\tau/(\omega^2\tau^2+1)$, first obtained by Zawadowski and Cardona (Ref. 12). The other continua have the same width τ^{-1}. Their maxima are located at $\omega=(n^2\omega_H^2+\tau^{-2})^{1/2}$. The sharp peak represents the scattering with the excitation or the absorption of helicon. It is located at $\omega=\omega_H c^2\zeta_1^2\omega_P^2$.

mum is at $\omega\simeq\tau^{-1}$ and then behaves as $\sim\omega^{-1}$ (see Fig. 2).

The nonzero harmonics $n\neq 0$ are responsible for different resonant processes, which we consider now.

A. Cyclotron resonances

The cyclotron resonances are located at $\omega=n\omega_H$, $n=\pm 1,\pm 2\ldots$. They result from the electrons moving with the angle frequency ω_H. Such the electrons can absorb and radiate electromagnetic waves with a frequency transfer multiple to ω_H. Let us consider first the case of spherical Fermi surface and suppose that $\omega_H(p_z)=$ const.

(i) In the clean metal $\tau^{-1}\rightarrow 0$, the imaginary part is due to the pass by pole,

$$\Sigma^{(n)}(\omega)=2\pi\omega\int_{k_0}^{\infty}\frac{dk}{2\pi}\frac{|U(k)|^2}{k}\times\langle|\gamma^{(n)}(p_z)|^2\delta(v_z-|\omega-n\omega_H|/k)\rangle, \quad (20)$$

where $k_0=|\omega-n\omega_H|/v$. This expression has the logarithmic singularity when k_0 approaches zero $\omega\rightarrow n\omega_H$. Evaluation of the integral (20) gives

$$\Sigma^{(n)}(\omega)\simeq\frac{\pi\omega\langle|\gamma^{(n)}(p_F)|^2\rangle}{v}\times\begin{cases}\dfrac{1}{|\zeta|^2}\ln\dfrac{v|\zeta|}{|\omega-n\omega_H|}, & v|\zeta|\gg|\omega-n\omega_H|\\ \dfrac{v^4|\zeta|^2}{|\omega-n\omega_H|^4}, & v|\zeta|\ll|\omega-n\omega_H|.\end{cases} \quad (21)$$

This expression has the form of the infinite peak with the width $v|\zeta|$. The height of the peak is determined by the finite scattering rate τ^{-1}. The formula (21) is still correct in the range, the $|\omega-n\omega_H|\gg\tau^{-1}$. In the opposite range, the $|\omega-n\omega_H|\ll\tau^{-1}$ evaluation gives the height of the nth cyclotron peak,

$$\Sigma^{(n)}\simeq\frac{\pi n\omega_H\langle|\gamma^{(n)}(p_F)|^2\rangle}{v}\frac{1}{|\zeta|^2}\ln|\zeta|l, \quad (22)$$

where $l=v\tau$. One can see the cyclotron resonances have the sharp logarithmic shape in a sufficiently clean metal $l\gg|\zeta|^{-1}$ (see Fig. 1). The relative height of the first resonance and the background is estimated as

$$\frac{\Sigma^{(1)}}{\Sigma^{(0)}}\simeq\ln|\zeta|l.$$

In the anisotropic metal, the above obtained results are still valid, except that $k_0=|\omega-n\omega_H(p_z)/v(p_z)|_{\min}$. Now the scattering cross section (20) and (21) is determined by the value of γ_n at the point p_z, where the function $|\omega-n\omega_H(p_z)/v(p_z)|$ takes the minimum.

(ii) Now let us consider the case of a dirty metal. When $\tau^{-1}\gg v|\zeta|$, we can ignore the term v_zk in the denominator (18) (the so-called zero-transfer limit),

$$\Sigma(\omega)=\zeta_2^{-1}\sum_n\langle|\gamma^{(n)}(p_z)|^2\rangle\frac{\omega\tau^{-1}}{(\omega-n\omega_H)^2+\tau^{-2}}. \quad (23)$$

The first term $(n=0)$ of this expression represents the relaxation background obtained by Zawadowsky and Cardona.[12] We see that in a magnetic field, the set of wide relaxation continua $(n\neq 0)$ appears (see Fig. 2). These continua represent the scattering from the oscillators with eigenfrequencies and damping equal to $n\omega_H$ and τ^{-1}, respectively. The maxima of the continua are determined by the according vertices $\langle|\gamma^{(n)}(p_z)|^2\rangle$ and located at $\omega=(n^2\omega^2+\tau^{-2})^{1/2}$. Their widths τ^{-1} do not depend on n at all. The maxima are resolved in a strong magnetic field $\omega_H>\tau^{-1}$. In the opposite range $\omega_H<\tau^{-1}$, they collapse together into the usual background, which is determined by the full vertex $\Sigma_n\langle|\gamma^{(n)}(p_z)|^2\rangle$.

One can propose the investigation of electron-hole contribution in a strong magnetic field (20)–(23) as a possible way to find the symmetry of electron-photon vertex function $\gamma(\mathbf{p})$. The size of nth continuum indicates the value of nth vertex harmonic.

B. Magneto-raman resonances

The cyclotron resonances are associated with the time dispersion of the electronic distribution function. They result from the small wave vectors k. We show now that there is the other type of resonances associated with the space dispersion from the finite k. These resonances have the similar origin to those in the sound damping, which are called magnetoacoustic resonances. So we call them "magneto-Raman" resonances. They can exist for the case of anisotropic Fermi surface.

There is the pole in the integrand (18) defined by the equation

$$\omega=n\omega_H(p_z^0)+v_z(p_z^0)k. \quad (24)$$

The electrons with the momentum p_z near p_z^0 (24) move coherently with the external field $U(k,\omega)$. Such electrons produce a resonance when their number is large. Thus, we are asked to seek after the singularity in the electron density of states. This happens when

$$\frac{\partial}{\partial p_z}[n\omega_H(p_z)+v_z(p_z^0)k]=0. \tag{25}$$

The strong resonance appears if ω and ω_H are chosen in the way that the condition (25) is satisfied together with (24) at the same point p_z^0. Here, we can expand the denominator (18) near p_z^0 up to the second order of $p_z-p_z^0$ and extend the integration over p_z from $-\infty$ to ∞. The smooth functions $\gamma_n(p_z)$ and $m^*(p_z)$ may be taken at p_z^0. Evaluating the integral and extracting the imaginary part, one finds the asymmetric shape of the magneto-Raman resonance,

$$\Sigma^{(n)}(\omega)\simeq\frac{\omega m^*(p_z^0)|\gamma^{(n)}(p_z^0)|^2}{2\pi\zeta_2|a_n|^{1/2}}\times\left(\frac{(\Omega_n^2+\tau^{-2})^{1/2}-\Omega_n\ \mathrm{sgn}a_n}{\Omega_n^2+\tau^{-2}}\right)^{1/2}, \tag{26}$$

where we denote

$$\Omega_n=\omega-n\omega_H(p_z^0)-v_z(p_z^0)\zeta_1,$$

$$a_n=\frac{1}{2}\frac{d^2}{dp_z^2}[v_z(p_z)\zeta_1+n\omega_H(p_z)]_{p_z=p_z^0}.$$

The expression (26) is correct, provided that $\zeta_1\gg\zeta_2$ and $v\zeta_2\ll\tau^{-1}$. Note that ζ_1 plays the role of k and has to be substituted into the conditions (24) and (25).

IV. EFFECTS OF ELECTROMAGNETIC FIELD

The second term in the right-hand side of Eq. (18) represents the effects of electromagnetic field. In the chosen model for the z component of electron velocity, we have $v_z^{(n)}(p_z)=v_z(p_z)\delta_{n0}$, while for the other components, $v_\alpha^{(0)}(p_z)=0$, $\alpha=x,y$. We see from Eqs. (15) and (14) that the Green function has the reduced form

$$D_{ik}(k,\omega)=\begin{pmatrix} D_{\alpha\beta}(k,\omega) & 0 \\ 0 & D_{zz}(k,\omega)\end{pmatrix}. \tag{27}$$

After substituting Eq. (27) into (18), the zz component represents the effects of Coulomb interaction, while the $\alpha\beta$ components result in the helicon contribution.

A. Coulomb interaction

The Coulomb effects originate from the $F^{z*}D^{zz}F^z$ term in the cross section (18),

$$\Sigma_C(\omega)=-\omega^2\frac{\left\langle\dfrac{v_z(p_z)\gamma^{(0)*}(p_z)}{\omega-v_zk+i\tau_p^{-1}}\right\rangle\left\langle\dfrac{v_z(p_z)\gamma^{(0)}(p_z)}{\omega-v_zk+i\tau_p^{-1}}\right\rangle}{\left\langle\dfrac{v_z^2(p_z)}{\omega-v_zk+i\tau_p^{-1}}\right\rangle}. \tag{28}$$

For the range $|\omega+i\tau_p^{-1}|\ll vk$, we find after some lengthy but trivial algebra that the term (28) renormalizes the first term in (18) in such a way,

$$\gamma^{(0)}(p_z)\to\gamma^{(0)}(p_z)=\gamma^{(0)}(p_z)-\langle\gamma^{(0)}(p_z)\rangle/\langle 1\rangle. \tag{29}$$

We see the Coulomb interaction screens partly the zero-harmonic ($n=0$) continuum. However, all the resonant effects described in Sec. III are not affected by the Coulomb interaction.

In the range $|\omega+i\tau_p^{-1}|\gg vk$ the expression (28) represents a small correction to the electron-hole contribution of the order of $|vk/(\omega+i\tau_p)|^2$. However, the renormalization (29) still takes place. It comes from the omitted term of collision integral in the τ approximation. The proof is straightforward and is not derived to save room. It can be made similarly to the derivation of this fact for zero magnetic field (see Ref. 13).

B. Helicon contribution

We consider the term $F_\alpha^* D_{\alpha\beta}F_\beta$ for the most interesting case of strong magnetic field and dirty metal $\omega_H>\tau^{-1}\gg\omega,v\zeta$. Under these conditions, the helicon contribution appears in the RS cross section. We suppose the spherical Fermi surface for estimates. Thus, $v_\alpha^{(n)}\neq 0$ only for $n=\pm 1$: $v_x^{(\pm 1)}=\pm iv_\perp(p_z)n/2$, $v_y^{(\pm 1)}=v_\perp(p_z)/2$. Substituting these expressions into the formula (14), we find the well-known conductivity tensor,

$$\sigma_{xx}=\sigma_{yy}=\frac{e^2N}{m^*}\frac{\tau^{-1}}{\omega_H^2+\tau^{-2}},\quad \sigma_{xy}=-\sigma_{yx}=\frac{e^2N}{m^*}\frac{\omega_H}{\omega_H^2+\tau^{-2}}, \tag{30}$$

here, the N density of electrons. With the help of Eq. (30), we obtain from Eq. (17) the Green function,

$$D_{\alpha\beta}(\omega)=\frac{1}{k^4-\omega^2\omega_P^4/\omega_H^2c^4}\begin{pmatrix} k^2 & -i\omega_P^2/\omega_Hc^2 \\ i\omega_P^2/\omega_Hc^2 & k^2\end{pmatrix}, \tag{31}$$

where we save only the leading terms in powers of τ^{-1}/ω_H and introduce $\omega_P^2=4\pi Ne^2/m^*$. The pole of the Green function defines the helicon spectrum,

$$\omega_{\mathrm{hel}}(k)=\omega_H\frac{c^2k^2}{\omega_P^2}.$$

Now let us pay attention to the oscillator strengths $F_\alpha(\omega,k)$. Only the $n=\pm 1$ harmonics contribute to $F_\alpha(\omega,k)$ for the isotropic Fermi surface. One obtains after the simple evaluation,

$$F_x=\frac{i}{\omega_H}\langle v_\perp(p_z)\mathrm{Re}\gamma^{(1)}(p_z)\rangle,$$

$$F_y=-\frac{i}{\omega_H}\langle v_\perp(p_z)\mathrm{Im}\gamma^{(1)}(p_z)\rangle, \tag{32}$$

where we have assumed that $\gamma(\mathbf{p})$ is real and $\gamma^{(-1)}(p_z)=\gamma^{(1)*}(p_z)$.

Substituting Eqs. (31) and (32) into the formula (18), we obtain the helicon contribution in RS

$$\Sigma_{\rm hel}(\omega)$$
$$=|\langle v_\perp \gamma^{(1)}\rangle|^2 \frac{\omega^2\omega_P^2}{\omega_H^2 c^2} \operatorname{Im}\int \frac{dk}{2\pi} \frac{|U(k)|^2 k^2}{k^4-(\omega+i\Gamma)^2\omega_P^4/\omega_H^2 c^4}. \tag{33}$$

The integrand (33) has a maximum at $|k|=\zeta_1$, with width ζ_2 and a maximum at $k=(\omega/\omega_H)^{1/2}\omega_P/c$. The width of the last maximum is determined by the helicon damping Γ, which results from the omitted imaginary part of conductivity tensor (30). The simple calculation gives the estimate $\Gamma/\omega_{\rm hel}(k)\simeq\tau^{-1}\omega_{\rm hel}(k)/\omega_H^2\ll 1$. Evaluation of integral (33) depends on the ratio between the widths of these maxima.

(i) If $\zeta_2 \ll \zeta_1(\omega\tau^{-1})^{1/2}/\omega_H$, we may integrate $|U(k)|^2$ only taking the remains at $k=\zeta_1$. We get

$$\Sigma_{\rm hel}(\omega)\simeq \frac{c^2\zeta_1^2|\langle v_\perp\gamma^{(1)}\rangle|^2}{2\zeta_2\omega_P^2}\frac{\omega\Gamma}{[\omega-\omega_{\rm hel}(\zeta_1)]^2+\Gamma^2}. \tag{34}$$

This expression has the form of a peak located at the helicon frequency with the width defined by the damping. The comparison of Eq. (34) with the expression for $n=0$ continuum (23) shows that the ratio $\Sigma_{\rm hel}^{\max}/\Sigma_{n=0}^{\max}\simeq c^2\zeta_1^2\omega_{\rm hel}/\Gamma\omega_P^2$. Since the damping is small this ratio demonstrates the strong peak.

(ii) If $\zeta_2 \gg \zeta_1(\omega\tau^{-1})^{1/2}/\omega_H$, we integrate the denominator taking $U(k)$ at $k=\omega_P/c(\omega/\omega_H)^{1/2}$:

$$\Sigma_{\rm hel}(\omega)\simeq \frac{c^2\zeta_1|\langle v_\perp\gamma^{(1)}\rangle|^2}{\omega_P^2}\frac{\omega\omega_{\rm hel}(\zeta_1)}{[\omega-\omega_{\rm hel}(\zeta_1)]^2+4\frac{\zeta_2^2}{\zeta_1^2}\omega_{\rm hel}^2(\zeta_1)}. \tag{35}$$

Now we find the ratio $\Sigma_{\rm hel}^{\max}/\Sigma_{n=0}^{\max}\simeq c^2\zeta_1^3/\omega_P^2\zeta_2$. This peak is sharp if $\zeta_1\gg\zeta_2$.

V. SUMMARY AND CONCLUSIONS

In the present paper, we consider the electronic Raman scattering from metals in a strong magnetic field. We show that measurements of RS spectra in the magnetic field should give many useful results. They are the origin of RS in high-T_c superconductors and the symmetry of the effective electron-photon vertex $\gamma(\mathbf{p})$. The results obtained theoretically are as follows.

(i) In the clean metal $|\zeta|l\gg 1$ (see Fig. 1), the continuum originates from usual collisionless electron-hole pairs. It is partly screened by the Coulomb interaction. The peaks are the cyclotron resonances $\omega=n\omega_H$ (21). Their heights defined by the values of $\gamma^{(n)}$ are logarithmically large (22).

(ii) In the dirty metal $|\zeta|l\ll 1$ (see Fig. 2), the known wide relaxation continuum transforms into the set of relaxation continua with the same widths. They are located at the frequencies of simple and multiple cyclotron resonances (23). The screening now comes from the electron-impurity interaction. The peak is associated with excitation and absorption of helicons (34) and (35). It is sharp in the strong magnetic field if the penetration depth of light into crystal is sufficiently large.

In real high temperature superconductors, the RS spectra are measured at the incident wavelength $\sim 5\times 10^{-5}\mathrm{cm}^{-1}$. For such frequencies, the cyclotron resonances (21) and (22) are observable if $\tau^{-1}<10^{12}\ \mathrm{s}^{-1}$. However, the various estimations[25] give for high-T_c superconductors $\tau^{-1}\sim 10^{12}-10^{13}\ \mathrm{s}^{-1}$. It shows that case (ii) takes place. The picture similar to Fig. 2 should be observed for the strong magnetic field $\omega_H\sim\tau^{-1}$. To observe the helicon contribution, one has to use the higher field $\omega_H\gg\tau^{-1}$. This requires for high T_c, too strong of a magnetic field. Another possibility is to find the compound with small electronic cyclotron mass.

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