MEAN FREQUENCY ESTIMATION OF NARROWBAND SIGNALS AND ITS APPLICATON TO DOPPLER ULTRASOUND BLOOD VELOCITY WAVEFORM ESTIMATION

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ABSTRACT

Many applications involving Doppler signals require the accurate estimation of the power-weighted mean frequency over short durations of the signal due to it's nonstationarity. This paper presents a novel algorithm for estimating mean frequencies using the eigenstructure of the covariance matrix of the Doppler data. Experimental results indicate that this method is superior to the competing non-parametric methods.

1. INTRODUCTION

Ultrasound is commonly considered to be a non-invasive, safe and cost effective practice in medical diagnosis. It is widely used in vascular Doppler and color flow imaging. In diagnostic ultrasound, parameters such as heart rate variability, peak systolic velocity, and systolic-diastolic ratio are estimated using blood velocity waveforms. The quality and usefulness of such estimates are limited by the accuracy of estimated blood velocity waveforms.

Blood velocity is directly proportional to the Doppler shift and can be estimated using the relationship

$$\nu = \frac{cf}{2f_o \cos\theta}, \tag{1}$$

where c is the velocity of ultrasound in the medium, f is the Doppler frequency, f_o is the incident ultrasound frequency and θ is the angle of incidence [1].

Doppler signals consist of a range of frequencies. In blood flow measurements, it spans from low frequencies corresponding to slow-moving blood close to the walls of the blood vessel to high frequencies corresponding to fast-moving blood at the center of the vessel. In many applications, we seek to estimate the power-weighted mean blood velocity defined as

$$\bar{\nu} = \frac{c}{2f_o \cos\theta}\bar{f}, \tag{2}$$

where

$$\bar{f} = \frac{\displaystyle\int_{f_{min}}^{f_{max}} P(f) f df}{\displaystyle\int_{f_{min}}^{f_{max}} P(f) df}, \tag{3}$$

and $P(f)$ is the power spectral density of the input signal at frequency f, and f_{min} and f_{max} are respectively the minimum and the maximum Doppler frequencies.

In current practice [2], there are three main steps in estimating the mean frequency of a Doppler signal. They are (i) estimate the power spectrum of the input signal, (ii) estimate the maximum Doppler frequency, and (iii) compute the mean frequency using (3). Generally, Doppler power spectra are estimated using fast Fourier transform (FFT) algorithms. Once the spectra is estimated, an arbitrary threshold level is chosen manually such that it is larger than the noise level. The highest frequency at which the estimated spectrum equals the threshold level is selected as the maximum frequency. Unfortunately, background noise level is not uniform throughout the waveform. Breathing movements, slight movements of arteries, etc can cause the variation in the noise level. In such situations, a single threshold in general will not produce a continuous maximum frequency waveform. Consequently, the estimated maximum frequency, and the corresponding mean frequency as well as the estimated blood velocity waveform may be erroneous.

This paper presents a new algorithm for estimating the mean frequency of the signal. This method does not require pre-estimation of either the spectrum or the maximum frequency of the input signal, and is independent of arbitrarily selected parameters. Our method is based on parametric high-resolution frequency estimation algorithms [3]

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that assumes that the input signal consists of multiple sinusoids embedded in white noise. In particular, we show that a single frequency approximation of a narrowband signal embedded in white noise is the power-weighted mean frequency. It exploits the covariance matrix models and mean frequencies are estimated using properties of the eigendecomposition of these matrices.

The rest of the paper is organized as follows. Section 2 contains the theoretical derivations for developing the mean frequency estimation algorithm. Experimental results involving both simulated and real Doppler signals, and demonstrating the capabilities of the new method are presented in Section 3. This section also compares the performance of our system with the conventional FFT-based method. Finally, concluding remarks are made in Section 4.

2. MEAN FREQUENCY ESTIMATION

Let

$$y[n] = s[n] + \eta[n], \tag{4}$$

represents the signal model where $s[n]$ is a complex valued narrowband signal and $\eta[n]$ is an additive circular white noise process with zero mean value and variance σ_η^2. The autocorrelation function $r_{ss}(k)$ of $s[n]$ is related to the power spectrum $P_{ss}(f)$ by

$$r_{ss}(k) = \int_{f_{min}}^{f_{max}} P_{ss}(f) e^{j2\pi f k} df, \tag{5}$$

where f_{min} and f_{max} are the minimum and maximum frequencies, respectively at which the signal $s[n]$ is present. Consider the 2×2-element autocovariance matrix of the input signal given by

$$\mathbf{R_{yy}} = \begin{bmatrix} r_{ss}(0) + \sigma_\eta^2 & r(1) \\ r^*(1) & r(0) + \sigma_\eta^2 \end{bmatrix}. \tag{6}$$

Direct calculations will show that the eigenvalues of $\mathbf{R}$ are

$$\begin{aligned} \lambda_1 &= r(0) + |r(1)| \\ \lambda_2 &= r(0) - |r(1)|. \end{aligned}$$

The eigenvector corresponding to the smallest eigenvalue λ_2 is

$$\mathbf{g} = m \left[-\frac{r_{ss}(1)}{|r_{ss}(1)|} \;\; 1 \right]^T, \tag{7}$$

where m is an arbitrary constant.

We now follow the methods employed by Pisarenko harmonic decomposition to estimate the frequency of a single sinusoid that best represents this covariance matrix. Let $\mathbf{w} = [1 \;\; e^{j2\pi\hat{f}}]^H$ where $\hat{f}$ represents the value of the single frequency that provides the best approximation in the problem such that $\mathbf{w}^H\mathbf{g} = 0$. It is straight forward to show that $\mathbf{w}^H\mathbf{g}$ can take the value of zero for an appropriate choice of $\hat{f}$. Substituting for $\mathbf{w}$ and $\mathbf{g}$, we get

$$[1 \;\; e^{j2\pi\hat{f}}] \left[-\frac{r_{ss}(1)}{|r_{ss}(1)|} \;\; 1 \right]^T = 0. \tag{8}$$

Replacing $r_{ss}(1)$ with (5) where $k = 1$ gives

$$-\frac{\int_{f_{min}}^{f_{max}} P_{ss}(f) e^{j2\pi f} df}{|\int_{f_{min}}^{f_{max}} P_{ss}(f) e^{j2\pi f} df|} + e^{j2\pi\hat{f}} = 0. \tag{9}$$

Dividing both sides by $e^{j2\pi\hat{f}}$

$$\frac{\int_{f_{min}}^{f_{max}} P_{ss}(f) e^{j2\pi(f-\hat{f})} df}{|\int_{f_{min}}^{f_{max}} P_{ss}(f) e^{j2\pi f} df|} = 1. \tag{10}$$

It is clear that the phase angle $\hat{\theta}$ of the right side of (10) is zero. Therefore

$$\hat{\theta} = \tan^{-1} \left[\frac{\int_{f_{min}}^{f_{max}} P_{ss}(f) \sin 2\pi(f-\hat{f}) df}{\int_{f_{min}}^{f_{max}} P_{ss}(f) \cos 2\pi(f-\hat{f}) df} \right] = 0, \tag{11}$$

which in turn implies that

$$\int_{f_{min}}^{f_{max}} P_{ss}(f) \sin 2\pi(f-\hat{f}) df = 0. \tag{12}$$

For narrowband signals $|f - \hat{f}|$ is small for $f_{min} \leq f \leq f_{max}$. Using the approximation $\sin 2\pi(f-\hat{f}) \approx 2\pi(f-\hat{f})$ in (12) yields

$$\int_{f_{min}}^{f_{max}} P_{ss}(f) 2\pi(f-\hat{f}) df \approx 0. \tag{13}$$

Solving (13) for $\hat{f}$ results in

$$\hat{f} = \frac{\int_{f_{min}}^{f_{max}} P_{ss}(f) f df}{\int_{f_{min}}^{f_{max}} P_{ss}(f) df}. \tag{14}$$

This results shows that the single frequency estimation for a narrow band signal using the second order covariance matrix is equal to its power weighted mean frequency. Application of the above results for the estimation of mean blood velocities of simulated and real Doppler signals is described in the next section.

3. EXPERIMENTAL RESULTS

Doppler signals are generated using the algorithm described by Sirmans et al in [4]. For the experiment, the signal spectrum was

$$P(f) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_f^2}} e^{-\frac{(f_n - \bar{f})^2}{2\sigma_f^2}} & ; f_{min} \leq f \leq f_{max} \\ 0 & ;\text{otherwise.} \end{cases} \tag{15}$$

where, $\bar{f}$ and σ_f were constant parameters of the model. The signal was then corrupted by additive white, gaussian noise with zero mean value and variance σ_η^2. In our experiment the sampling frequency was 12000 Hz, $\sigma_f = 300$ Hz, $\bar{f} = 1500$ Hz, $f_{min} = \bar{f} - 1.0\sigma_f$ and $f_{max} = \bar{f} + 2.5\sigma_f$. For this choice, the power-weighted mean frequency was $f_m = 1580.5$ Hz. The bandwidth of the signal is 1050 Hz. Figure 1 depicts the estimated spectrum of a sample data set when the SNR is 13 dB.

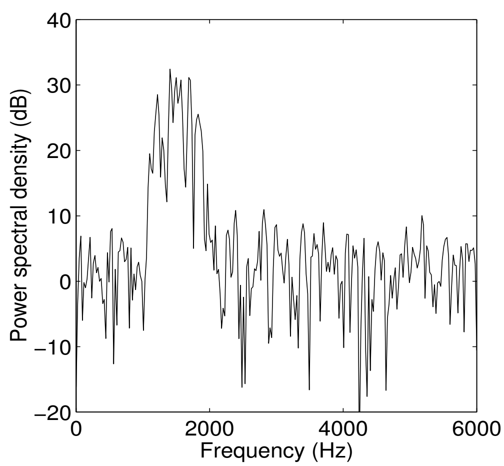

Fig. 1. Estimated spectrum of the simulated Doppler signal for SNR=13dB

The mean frequency was estimated from 512 samples of the data and 1000 independent experiments were performed at each SNR. Figure 2 displays the comparison of the normalized root-mean-square (rms) error for the method described in the paper and the conventional FFT-based algorithm [2]. The conventional method employs 512-point FFT, Hann windowing and manually selected thresholds to estimate the mean frequencies. The normalized rms error σ_e is estimated as estimated as

$$\sigma_e = \frac{\sqrt{\sum_{i=1}^{1000} |\hat{f}_i - f_m|^2}}{f_m}, \tag{16}$$

where $\hat{f}_i$ is the frequency estimate for the i^{th} run.

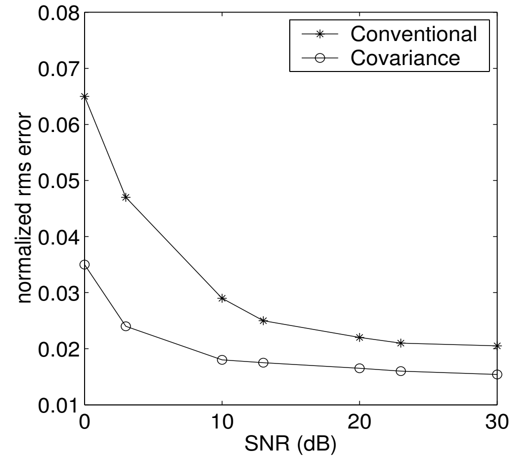

Fig. 2. Root mean-square error for the covariance method and the conventional method

We can see that the performance of the covariance-based mean frequency estimation method is superior to that of the conventional method.

3.1. Mean Frequency Estimation of Real Valued Signals

Experimentally, we have observed that similar results hold for real-valued signals. In such situations, we use a 3×3-element autocovariance matrix and the frequencies are estimated as the solution to the relationship $\mathbf{w}^H \mathbf{g} = 0$, where $\mathbf{g}$ is the eigenvector corresponding to the smallest eigenvalue of this matrix and $\mathbf{w} = [1 \; e^{j2\pi\hat{f}} \; e^{j4\pi\hat{f}}]^H$. We validate the above statement using the following experiment.

The Doppler signals were generated using the same algorithm, but only the real part of the signal was considered. Mean frequency values were estimated using 3×3-element autocovariance matrix as explained above. The conventional method employed the same parameters. Figure 3 shows the variation of the normalized rms error for different SNR values for these two algorithms.

From this results we can see that the estimation of mean frequencies of real-valued Doppler signals using the covariance based mean frequency estimation method outperforms the conventional method.

Finally, we present the result for reconstructing Doppler ultrasound blood flow measurements performed at the Department of Obstetric and Gynecology, Academic Hospital, The Netherlands. The experimental set-up was described in detail in [2]. This study was approved by the Hospital Ethics Committee at the university. All Doppler studies were performed for 14-50 seconds with women in a semi-recumbent position and during fetal apnea. Angle of incidence was less

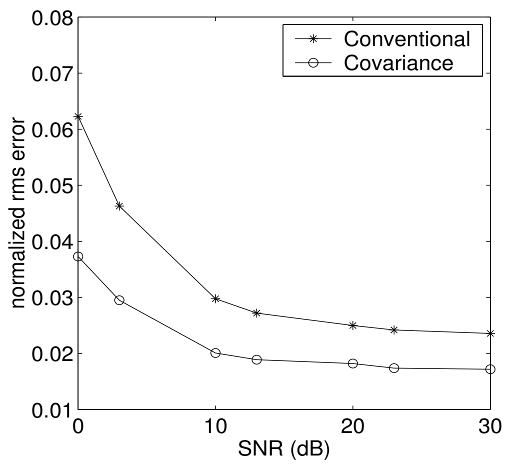

Fig. 3. Comparison of the normalized rms error with that of the conventional method for real-valued Doppler signals

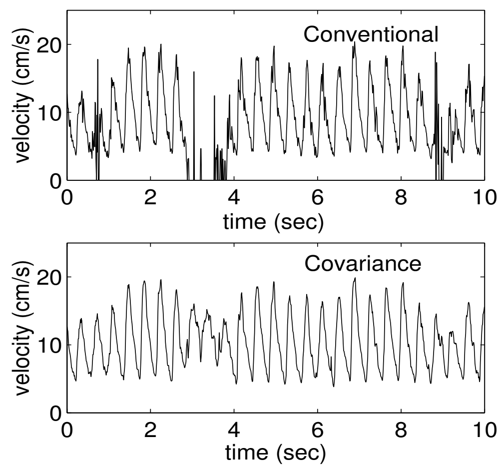

Fig. 4. Comparison of mean velocity estimation with that of the conventional method for real Doppler recordings

than 20^o and flow velocity waveforms were obtained from the free floating loop of the umbilical artery.

Figure 4 shows a portion of the mean blood velocity waveform estimated for the above recording using the two methods explained in this paper. Once the mean frequencies are estimated, blood velocity values were computed using (2). In the reconstruction process, the velocity of ultrasound in a soft tissue was considered as $c = 1540$ m/s and the frequency of the incident ultrasound beam was $f_o = 3.5$ MHz. The angle of incidence, θ was assumed to be negligibly small.

Variations of the SNR are inevitable in all measurements. In practice, threshold level cannot be varied with the variations of SNR for reconstructing blood velocity waveforms. At low SNR values, the conventional method is unable to estimate blood velocity values. From the figure, it can be seen that the reconstructed mean blood velocity waveform using the covariance-based mean frequency estimation algorithm is more robust and is less affected from the variations in the environmental noise.

4. CONCLUDING REMARKS

This paper showed that the single frequency approximation of a narrowband signal with closely spaced sinusoids using covariance matrix methods is the power-weighted mean frequency. This result was employed to develop a method for estimating the mean blood velocity waveforms from Doppler measurements. Experimental results for the reconstruction of mean velocity waveforms from synthetic and real ultrasound signals indicate that this method is superior to the current techniques for Doppler ultrasound signal reconstruction. This method has the potential to provide more robust and accurate medical diagnostic information than presently used algorithms.

5. REFERENCES

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