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Excited States in Staggered Meson Propagators

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We report on preliminary results from multi-particle fits to meson propagators with three flavors of light dynamical quarks. We are able to measure excited states in propagators with pion quantum numbers, which we interpret as the pion $2S$ state, and argue is evidence of locality of the action. In the a_0 (0^{++}) propagators we find evidence for excited states which are probably the expected decay channels, $\pi + \eta$ and $K + \overline{K}$.

1. INTRODUCTION

Extracting ground state hadron masses from lattice operator correlation functions has become a standard exercise, and results generally agree with experimental measurements. (See, for example [1,2,3,4,5,6].) However, determining excited state masses on the lattice (e.g. [7,8,9,10,11]) is more difficult. Such determinations can provide tests of the particular lattice approach, can be used to study decays of unstable hadrons and, in principle, could help clarify quark model interpretations of higher mass hadrons.

Table 1

$\beta = 10/g^2$	am_q, am_s	$L^3 \times T$	$n_{\rm config}$
6.96	0.100, 0.100	$20^3 \times 64$	339
6.85	0.050, 0.050	$20^3 \times 64$	414
6.83	0.040, 0.050	$20^3 \times 64$	275
6.81	0.030, 0.050	$20^3 \times 64$	564
6.76	0.010, 0.050	$20^3 \times 64$	658
6.76	0.007, 0.050	$20^3 \times 64$	443
6.76	0.005, 0.050	$20^3 \times 64$	159
7.18	0.0310, 0.031	$28^3 \times 96$	496
7.11	0.0124, 0.031	$28^3 \times 96$	527
7.09	0.0062, 0.031	$28^3 \times 96$	592

2. SIMULATION

We measured meson propagators on lattices generated using the "Asqtad" action[12] with 2+1 flavors of dynamical quarks, that is, one flavor with mass approximately equal to the physical strange quark mass and two lighter flavors with degenerate masses. (See Table 1.) We describe the $20^3 \times 64$ lattices as "coarse" lattices with lattice spacing of $a \approx 0.12$ fm, while the $28^3 \times 96$ "fine" lattices have a spacing of $a \approx 0.09$ fm. More detailed descriptions of these simulations and analysis procedures can be found in Refs. [1,2]. In this report we discuss excited states in the pseudoscalar (0^{-+}) and scalar (0^{++}) isovector meson propagators.

3. RESULTS AND CONCLUSIONS

3.1. 0^{-+} states

In the staggered fermion formulation propagators generally appear with an oscillating component that represents a parity partner state. There is no partner to the taste-pseudoscalar pion and $s\bar{s}$ states, hence these propagators contain contributions only from $J^{PC} = 0^{-+}$ states. Moreover, the signal to noise ratio is large at all time separations, so it is relatively easy to measure the

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[*] Presented by E.B. Gregory

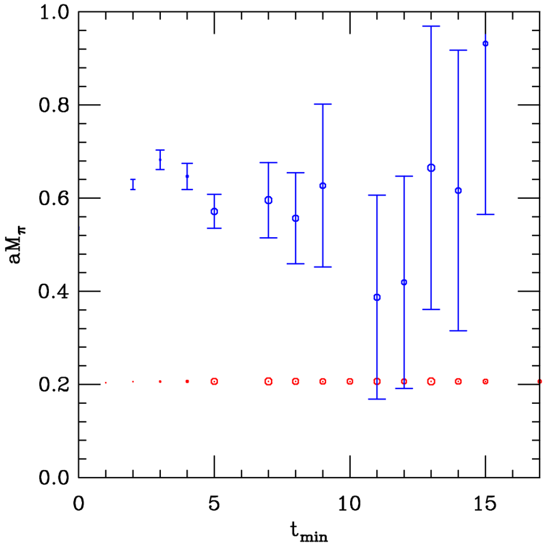

Figure 1. Pion fits for $10/g^2 = 7.11$, $t_{\max} = 28$. $t_{\min} = 5$ was selected. Symbol size is proportionate to confidence level.

contribution of excited states.

We fit 0^{-+} propagators to the form:

$$P(t) = A_0(e^{-m_0 t} + e^{-m_0(T-t)}) + A_1(e^{-m_1 t} + e^{-m_1(T-t)}), \quad (1)$$

where T is the length of the time dimension of the lattice, t is the time separation of the source and sink operators and m_0 and m_1 are the masses of the ground state and lowest excited states. We look for high-confidence, low-error fits that fall in plateaus of stability of the fit parameters with respect to the limits $t_{\max}$ and $t_{\min}$ of the propagator included in the fit. Figure 1 shows a set of pion fits with varying $t_{\min}$, and $t_{\max}$ fixed at 28.

Figure 2 shows the results of this fitting procedure for all of the data sets. The darker error bars are statistical, and lighter error bars represent systematic error due to fit choice. We plot the product of ground state and first excited state masses with r_1, a length scale derived from the static quark potential, against the ratio $(m_\pi/m_\rho)^2$, a measure of the quark mass. At the physical value of $(m_\pi/m_\rho)^2 = 0.03$ we plot the experimental mass of the states $\pi^0(135)$ and the $\pi(1300)$ scaled with $r_1 = 0.317(7)$ fm[13]. Also, an excited 0^{-+} $s\bar{s}$ state, an average of measurements on all of the fine lattices, is compared to the experimental mass of the $\eta(1440)$, with which agreement is very marginal.

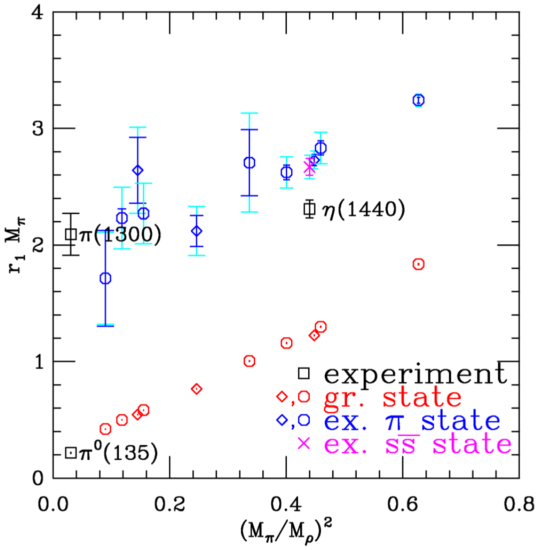

Figure 2. Ground state and excited state pion masses. Diamonds are fine lattice data; octagons are coarse lattice data. $s\bar{s}$ states are shown at $(M_{s\bar{s}}/M_\phi)^2 = (680/1020)^2 = 0.44$, where $M_{s\bar{s}} = \sqrt{2M_K^2 - M_\pi^2} = 680$MeV.

We feel that the consistency of the simulation results with the $\pi(1300)$ is an important indication that the fourth root of the determinant in the action does not introduce a non-locality with scale larger than the $\pi(1300)$ Compton wavelength: 0.15 fm.

3.2. 0^{++} states

The a_0 meson ($J^{PC} = 0^{++}$) propagator occurs as the oscillating parity partner in the taste-scalar pion propagator. With states of both parities contributing to the propagator, it is more difficult to resolve excited states. We fit these $a_0 - \pi$ propagators to the form:

$$\begin{aligned} P(t) &= A_0(e^{-m_0 t} + e^{-m_0(T-t)}) \\ &+ A_1(e^{-m_1 t} + e^{-m_1(T-t)}) \\ &+ A_2(-1)^t(e^{-m_2 t} + e^{-m_2(T-t)}) \\ &+ A_3(-1)^t(e^{-m_3 t} + e^{-m_3(T-t)}). \end{aligned} \quad (2)$$

Here m_2 and m_3 are the 0^{++} masses. In one case, $\beta = 7.09$, we were able to unambiguously discern an excited 0^{++} state. See Figure 3.

The 0^{++} states are interesting in that several of the lowest lying states are likely not pure $q\bar{q}$ states because the two meson states to which the a_0 decays are accessible in our simulations. In Figure 4, the straight line is an extrapolation of the heavier-m_q quenched a_0 masses, and is an expectation of the mass of a $q\bar{q}$ state with 0^{++}. The curved line, which crosses below the "a_0-line" at low m_q traces the sum of the π and η ground state masses[1]. The fitted full-QCD 0^{++} ground state masses roughly trace the $\pi + \eta$ line, possibly displaying the lower half of an avoided level crossing, an indication of the decay of the pure $q\bar{q}$ a_0. The measured 0^{++} excited state (burst) is not consistent with the straight line either, but relatively close to the experimental value of the mass of a $K\overline{K}$ molecule. Future efforts will try to fill out Figure 4 with more 0^{++} excited states.

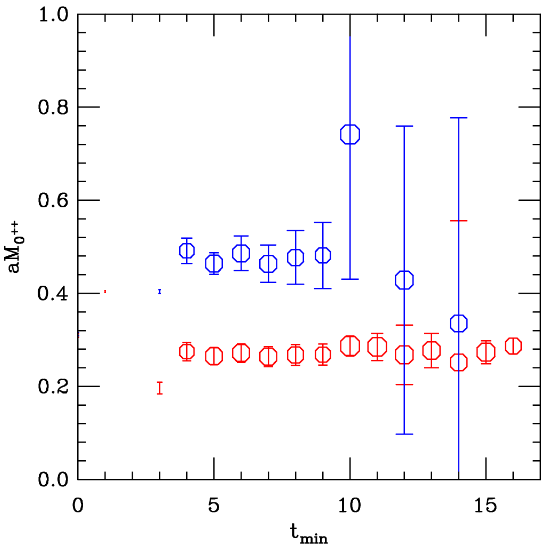

Figure 3. Fits of 0^{++} states for $10/g^2 = 7.09$ $t_{\max} = 35$. Symbol size is proportional to confidence level. $t_{\min} = 5$ was selected.

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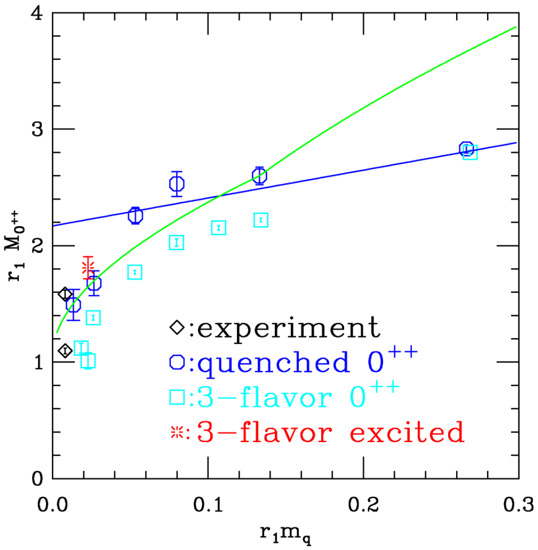

Figure 4. Mass of 0^{++} states as a function of $r_1 m_q$, with $r_1 = 0.317(7)$ fm. Straight line is a fit to mass a_0 with heavier quenched quarks. Curved line is the mass of $\pi+\eta$. Physical states are $\pi+\eta$ (682MeV) and $a_0/K\overline{K}$ (980MeV).

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