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Improving flavor symmetry in the Kogut-Susskind hadron spectrum

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We study the effect of modifying the coupling of Kogut-Susskind quarks to the gauge field by replacing the link matrix in the quark action by a ''fat link,'' or sum of link plus three-link paths. Flavor symmetry breaking, determined by the mass difference between the Goldstone and non-Goldstone local pions, is reduced by approximately a factor of 2 by this modification. [S0556-2821(97)50103-2]

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I. INTRODUCTION

One of the major technical challenges facing lattice QCD is the extraction of continuum physics from lattices with few enough lattice sites that computations are feasible. Recently there has been progress in developing ''improved actions,'' which give better approximations to continuum physics with large lattice spacings [1]. Starting points for improving the gauge field action include cancellation of lattice artifacts in an expansion in powers of a (Symanzik actions) [2,3] or renormalization group ideas (perfect actions) [4]. Improved fermion actions which improve the quark dispersion relation have been introduced for Wilson quarks (''clover'' action) [5] and for Kogut-Susskind quarks [6]. In particular, the Naik action for Kogut-Susskind quarks adds a third-nearest-neighbor term to remove an unwanted term of order a^2, relative to the desired term, from the dispersion relation. This produces improvements such as energy and baryon number susceptibilities at high temperature that quickly approach the free field (Stefan-Boltzmann) limits, and has been applied to QCD thermodynamics [7]. Another of the major problems with Kogut-Susskind quarks is the breaking of flavor symmetry. In particular, only one of the pions is a true Goldstone boson at nonzero lattice spacing, and at the lattice spacings where simulations are done the differences in the pion masses are large. Preliminary results suggest that improving the gauge action helps this problem, but the Naik improvement of the fermion action does not [8]. This is not surprising, since the motivation for the Naik action is the improvement of the free quark dispersion relation rather than the flavor symmetry.

Here we explore a simple modification of the Kogut-Susskind fermion action. In particular, we replace the link matrices in the fermion matrix by weighted sums of the simple link plus three-link paths, or ''staples,'' connecting the points. This is a simple modification that is consistent with gauge invariance and the hypercube symmetries of the lattice. Modifications such as this are likely to arise in any renormalization group motivated improved fermion action —this is just the simplest possible addition. In simulations with dynamical fermions, simplicity will be important since the computation of the fermion force in the molecular dynamics integration could become impossibly complicated. Of course, this modification of the quarks' parallel transport can, and probably should, be used in concert with improvements in the gauge field action and the Naik (third-nearest-neighbor) improvement to the quark dispersion relation.

In this paper we report on a study of the quenched meson spectrum using the ''fat link'' quark action. Quenched spectrum calculations are easy to do, especially when stored configurations are available, and there are many results in the literature using the conventional fermion action for comparison. We measure masses of the ''local'' mesons, which include the Goldstone pion and one non-Goldstone pion, the ''SC'' pion, or ''π_2'' [9]. The splitting between these two pions is a good indicator of flavor symmetry breaking, since the calculations that have measured other pion masses find that all of the non-Goldstone pions are nearly degenerate [10,11].

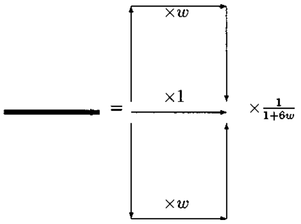
II. FORMALISM

We modify the standard Kogut-Susskind Dirac operator by replacing each link matrix $U_\mu(x)$ by a ''fat link''

$$U_{\mathrm{fat},\mu}(x)=\frac{U_\mu(x)+w\Sigma_{\nu\neq\mu}[U_\nu(x)U_\mu(x+\hat{\nu})U_\nu^\dagger(x+\hat{\mu})+U_\nu^\dagger(x-\hat{\nu})U_\mu(x-\hat{\nu})U_\nu(x-\hat{\nu}+\hat{\mu})]}{1+6w} \tag{1}$$

where w is an adjustable weight for the staples.

Symbolically, this is just



Smeared gauge links made from combinations of different paths on the lattice have been used in constructing source operators for both glueballs and ordinary hadrons [12]. In these studies only the operators were changed; the propagator computation, and hence the masses of the hadrons, were left unchanged. In contrast, changing the masses of the hadrons is exactly what we want to do. Also we should note that we do not project the fat link back onto the gauge group.

Replacement of a link by a weighted average of links displaced in directions perpendicular to the direction of the link amounts to including second derivatives, $\partial^2 A_\mu/\partial_\nu^2$, with $\nu\neq\mu$. Expressing this in a gauge and Lorentz covariant form, to lowest order in a this modifies the action by including a term (expressed in four-component notation, before the Kogut-Susskind spin diagonalization)

$$\bar{\psi}\left(\gamma_\mu\left[D_\mu+\frac{a^2}{6}D_\mu^3+a^2\frac{w}{1+6w}(D_\nu F_{\nu\mu})\right]+m\right)\psi, \tag{2}$$

where w is the staple weight. The factor of a^2, required by the dimensions of second derivative, is the expected power of the lattice spacing for effects of lattice artifacts with staggered fermions. (In contrast, the clover term for Wilson quarks is an order a modification.) The D_μ^3 term, which violates rotational invariance, is unchanged by the replacement of the ordinary link with a fat link. It is this term that is canceled in the ''Naik'' (third-nearest-neighbor) derivative.

Another way to think about the use of fat links in a spectrum computation is to consider it as the result of a modified action for the gauge fields. That is, we can ask what gauge action would produce links with the probability distribution of the fat links. In general, this gauge action will be nonlocal, involving loops of all sizes. Also, the fat links are not unitary matrices, so this will be an unconventional gauge action. For small w we could construct this gauge action as a power series in w. To first order in w the effect is the same as using a gauge action consisting of the sum of the plaquettes minus $2w$ times the 2×1 planar loops and the 2×1 bent loops. (The 2 is a combinatorial factor arising because each two-plaquette loop can be generated in two ways, by adding a staple to either of the two plaquettes in the loop.) These considerations indicate that the ''fat link'' modification of the quark action will interact with improvements in the gauge action, and it would be dangerous to assume that the same fattening parameter that is optimal for the Wilson gauge action will be optimal for an improved action.

III. RESULTS

As a dimensionless measure of flavor symmetry breaking we use the quantity $\Delta_\pi=(m_{\pi2}-m_\pi)/m_\rho$. We will also use the quantity am_ρ to define the lattice spacing, and the dimensionless quantity m_π/m_ρ as a measure of the quark mass. Ideally, to compare with the conventional quark action we would compare simulations at the same quark mass and lattice spacing. This requires tuning of parameters or interpolation among various data points.

We began with a series of tests using a set of quenched lattices with the standard Wilson gauge action at $6/g^2=5.85$ [13]. The lattice size is $20^3\times48$. Local meson propagators were calculated from wall sources, using four sources in each lattice. Because we are interested in surveying various masses and smearing weights, only a small fraction (30 lattices) of our stored lattices were used. The resulting masses and mass ratios are tabulated in Table I. To better expose the effect of fattening the link, we have generally chosen the same fitting range for all the values of w at a given quark mass. (There are some exceptions where the fitting program did not converge for the desired fit range.) The final column of this table is the number of conjugate gradient iterations required to converge the quark propagator calculation to a residual of 0.00005. Table I also contains a selection of masses with the conventional fermion action for comparison. We also include a result at $6/g^2=6.5$ to point out that at this lattice spacing the flavor symmetry breaking has become very small [15].

It is clear from this table that the use of fat links reduces the flavor symmetry breaking, mostly by making the π_2 lighter. As an added benefit, the smoother fat links require fewer iterations of the conjugate gradient algorithm. There are no obvious effects on the nucleon to ρ mass ratio. More accurately, any change in the nucleon to ρ mass ratio is of

TABLE I. Masses and mass ratios with fat link fermion action, and comparable spectrum results with the conventional fermion action. The simple plaquette gauge action was used. All of the fat link masses were run on the same set of configurations, so all of the masses are strongly correlated. The $6/g^2=6.0$ masses are from Refs. [11,14], and the 6.5 masses from Ref. [15]. A ''NA'' indicates that the mass is not published.

Fat link masses

w	m_π	$m_{\pi 2}$	m_{VT}	m_{PV}	m_N	m_π/m_ρ	m_N/m_ρ	Δ_π	CG
				$6/g^2=5.85$, $am_q=0.01$					
0.00	0.273(1)	0.435(15)	0.60(2)	0.61(3)	0.88(2)	0.448(22)	1.44(8)	0.266(28)	1413
0.10	0.246(1)	0.316(10)	0.57(2)	0.56(1)	0.80(2)	0.439(14)	1.43(4)	0.125(18)	1008
0.20	0.239(1)	0.292(7)	0.57(2)	0.56(1)	0.80(2)	0.427(8)	1.43(4)	0.095(13)	892
0.30	0.236(1)	0.290(5)	0.55(1)	0.56(2)	0.79(2)	0.421(15)	1.41(6)	0.096(10)	871
0.40	0.237(1)	0.288(4)	0.56(2)	0.56(2)	0.80(2)	0.423(15)	1.43(6)	0.091(8)	867
				$6/g^2=5.85$, $am_q=0.02$					
0.00	0.379(1)	0.494(8)	0.675(8)	0.676(14)	0.998(17)	0.561(7)	1.48(3)	0.170(12)	617
0.10	0.342(1)	0.399(3)	0.617(12)	0.618(12)	0.903(13)	0.553(11)	1.46(4)	0.093(5)	463
0.20	0.332(1)	0.380(3)	0.607(12)	0.616(14)	0.885(13)	0.547(11)	1.46(4)	0.079(5)	427
0.30	0.330(1)	0.374(3)	0.604(11)	0.609(13)	0.880(13)	0.546(10)	1.46(3)	0.074(5)	417
0.40	0.329(1)	0.371(3)	0.603(11)	0.605(13)	0.878(13)	0.57(2)	1.48(5)	0.069(6)	413
				$6/g^2=5.85$, $am_q=0.033$					
0.00	0.480(1)	0.599(9)	0.768(20)	0.781(26)	1.11(2)	0.625(16)	1.45(5)	0.155(12)	371
0.10	0.437(1)	0.487(2)	0.699(11)	0.689(15)	1.00(2)	0.625(10)	1.43(4)	0.072(3)	271
0.20	0.425(1)	0.466(2)	0.684(11)	0.684(15)	0.97(2)	0.621(10)	1.42(4)	0.060(3)	250
0.30	0.422(1)	0.459(2)	0.679(11)	0.682(15)	0.96(2)	0.622(10)	1.41(4)	0.055(3)	245
0.40	0.421(1)	0.456(2)	0.678(11)	0.682(14)	0.96(2)	0.621(10)	1.42(4)	0.052(3)	243
0.50	0.421(1)	0.455(2)	0.677(10)	0.683(14)	0.96(2)	0.622(9)	1.42(4)	0.050(3)	242

Conventional masses

$6/g^2$,am_q	m_π	$m_{\pi 2}$	m_{VT}	m_{PV}	m_N	m_π/m_ρ	m_N/m_ρ	Δ_π
5.85,0.02	0.3802(1)	0.5075(12)	0.6936(19)	0.682(7)	1.003(5)	0.549(1)	1.45(1)	0.184(2)
5.85,0.01	0.2736(1)	0.4144(17)	0.6306(42)	0.624(7)	0.888(8)	0.434(3)	1.41(2)	0.223(3)
5.95,0.025	0.3875(7)	0.4512(20)	0.5954(28)	NA	0.893(10)	0.651(3)	1.50(2)	0.107(4)
5.95,0.01	0.2501(9)	0.3215(44)	0.5159(40)	NA	0.725(29)	0.485(4)	1.41(6)	0.138(9)
6.00,0.02	0.335(1)	0.383(2)	0.520(3)	0.522(3)	0.762(5)	0.643(4)	1.46(1)	0.092(4)
6.00,0.01	0.241(3)	0.291(7)	0.471(6)	0.475(3)	0.673(14)	0.511(7)	1.43(4)	0.107(2)
6.15,0.02	0.2967(3)	0.3171(9)	0.426(1)	0.427(1)	0.632(2)	0.696(2)	1.48(3)	0.048(2)
6.15,0.01	0.2098(4)	0.2331(8)	0.372(1)	0.373(2)	0.532(2)	0.564(2)	1.43(7)	0.063(2)
6.5,0.01	0.1575(3)	0.1604(6)	0.239(1)	0.239(1)	0.354(3)	0.659(3)	1.48(1)	0.012(3)

the same order as the flavor symmetry breaking in the ρ masses, and therefore cannot be disentangled from changes in the flavor symmetry breaking for the ρ's. It can also be seen that the improvement is quite insensitive to the exact value of w. Studies of the dependence of the flavor symmetry breaking of the local pions on the lattice spacing with the conventional action can be found in Refs. [16,17]. In Fig. 1 we plot the squared masses of these two pions as a function of quark mass at this fixed lattice spacing, and show linear extrapolations of the π_2 mass to zero quark mass. Note that the intercept as well as the slope of the π_2 squared masses is lower with the fat links, showing that improvement in flavor symmetry persists in the chiral limit $m_q \rightarrow 0$.

Because the ρ and nucleon masses are reduced when w is turned on, part of the improvement in flavor symmetry breaking can be attributed to a smaller effective lattice spacing. To separate this effect from a ''real'' improvement, we want to compare calculations with the same lattice spacing, which we define by the ρ mass, and the same physical quark mass, defined by m_π/m_ρ. In looking through the available quenched spectrum calculations with the conventional action, we find simulations at $6/g^2=5.95$ and 6.0 with $m_\pi/m_\rho \approx 0.65$. We therefore chose a quark mass, 0.033, which gives a similar ratio using fat links. We can then compare the fat link spectrum with $m=0.033$ and $w=0.4$ to these two conventional calculations in Table II. In this table we see that even though m_π/m_ρ is slightly smaller for the fat link calculation (Δ_π increases as $m_q \rightarrow 0$) and the lattice spacing (from am_ρ) is larger, Δ_π with the fat link action is about half that for the conventional action.

Motivated by the interaction of fat links and improvement of the gauge action discussed above, as well as the fact that our studies of quenched Kogut-Susskind spectroscopy indicate that improvement of the gauge action results in some reduction of flavor symmetry breaking, we calculated the fat

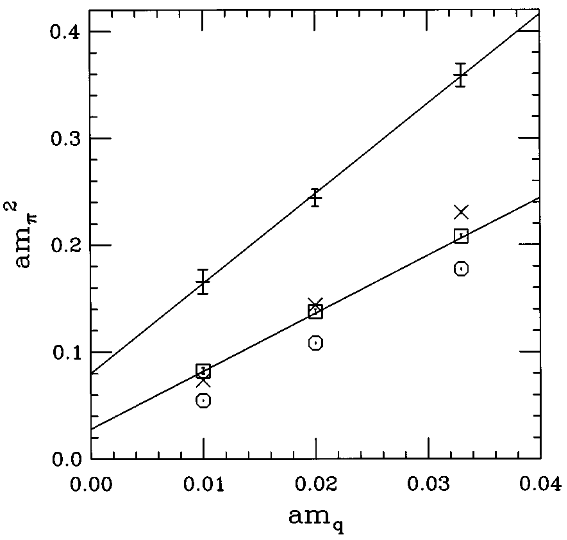


FIG. 1. The squared pion masses versus quark mass for the Wilson gauge action at $6/g^2=5.85$. The plusses and crosses are the π_2 and Goldstone π masses with the conventional fermion action, and the squares and octagons are the π_2 and Goldstone π masses with a staple weight $w=0.4$. The lines are linear fits to the squared non-Goldstone pion masses.

TABLE II. Mass ratios for the fat link action at $6/g^2=5.85$ and approximately matched conventional calculations. The conventional calculations are at similar m_π/m_ρ, but at smaller lattice spacing as defined by the ρ mass. However, the dimensionless flavor symmetry-breaking parameter is considerably smaller with the fat link action.

$6/g^2$	am_q	w	am_ρ	m_π/m_ρ	Δ_π
5.85	0.033	0.40	0.678(11)	0.621(10)	0.052(3)
5.95	0.025	0	0.5954(28)	0.651(3)	0.107(4)
6.00	0.02	0	0.520(3)	0.648(5)	0.090(7)

link spectrum on a set of stored configurations with an improved gauge action [3]:

$$S_g=\frac{\beta}{3}\left\{\sum(\text{plaquettes}) - \frac{1}{20u_0^2}(1+0.4805\alpha_s)\sum(2\times1\text{ loops}) - \frac{1}{u_0^2}0.03325\alpha_s\sum(1\times1\times1\text{ loops})\right\}. \tag{3}$$

Results are in Table III. Again we see a dramatic improvement in the flavor symmetry breaking. Curiously, in this case the Goldstone pion mass increases with fattening, while it decreased in the $6/g^2=5.85$ calculation.

It is also interesting to combine the fattened links with Naik's improvement to the fermion action. The Naik improvement removes the order a^2 (relative to the leading term) error in the quark dispersion relation. It simply consists of replacing the nearest neighbor term in D_μ, $U_\mu(x)\psi(x+\hat{\mu})$ with a combination of nearest and third-nearest-neighbor terms:

$$\tfrac{9}{8}U_\mu(x)\psi(x+\hat{\mu})-\tfrac{1}{24}U_\mu(x)U_\mu(x+\hat{\mu})\times U_\mu(x+2\hat{\mu})\psi(x+3\hat{\mu}). \tag{4}$$

This produces a rapid convergence of quantities like the free field energy and baryon number susceptibility as a function of the number of time slices, and has been used in a high temperature QCD simulation by the Bielefeld group [7]. However, in our quenched spectrum calculations [8], addition of the third neighbor term had little effect on the flavor symmetry breaking. Since high temperature calculations are typically done with larger lattice spacings than zero temperature calculations, it is especially important to improve the actions here. We may hope that a combination of the Naik improvement with fat links or similar improvement in the coupling of the quarks to the gauge fields could produce a simulation with an accurate continuum free field behavior at high temperature and a gas of the right number of light pions at low temperature. As a first step, we calculated the spectrum at $am_q=0.02$ using a third nearest neighbor term. There are several decisions to be made here. How should tadpole improvement be applied to the third neighbor term when fat links are used? Should the fat link in the third neighbor term be the product of three of the single fat links, or some other weighted combination of paths? We began with a spectrum calculation using just the product of three fattened links, with the unimproved coefficient $-1/24$ for the third nearest neighbor, using the same $6/g^2=5.85$ lattices. Results are in Table IV.

IV. EXTENSIONS AND CONCLUSIONS

We find that the replacement of the simple gauge link by a fattened link in the fermion action reduces the flavor symmetry breaking of the local pions by roughly a factor of 2 for the parameters used here. Since flavor symmetry breaking is

TABLE III. Masses and mass ratios for fat link quarks with an improved gauge action.

$6/g^2=7.40$, $am_q=0.04$, improved gauge action									
w	m_π	$m_{\pi2}$	m_{VT}	m_{PV}	m_N	m_π/m_ρ	m_N/m_ρ	Δ_π	CG
0.00	0.5347(3)	1.06(3)	1.23(1)	1.37(2)	1.81(4)	0.435(4)	1.47(3)	0.427(25)	298
0.10	0.5627(3)	0.894(13)	1.17(1)	1.22(1)	1.73(2)	0.481(4)	1.48(2)	0.283(11)	225
0.20	0.5719(3)	0.842(7)	1.16(1)	1.19(1)	1.70(1)	0.493(4)	1.47(2)	0.233(6)	206
0.30	0.5776(4)	0.824(6)	1.15(1)	1.18(1)	1.68(1)	0.502(4)	1.46(2)	0.214(6)	201
0.40	0.5818(4)	0.817(5)	1.15(1)	1.18(1)	1.68(1)	0.506(4)	1.46(2)	0.205(5)	198

TABLE IV. Masses and mass ratios for a fat link action including the Naik third-nearest-neighbor term. The gauge configurations are the same as in Table I.

				$6/g^2=5.85$, $am_q=0.020$, with Naik derivative					
w	m_π	$m_{\pi 2}$	m_{VT}	m_{PV}	m_N	m_π/m_ρ	m_N/m_ρ	Δ_π	CG
0.00	0.3652(7)	0.487(8)	0.683(20)	0.70(4)	0.983(17)	0.535(15)	1.44(5)	0.178(13)	733
0.10	0.328(1)	0.387(3)	0.611(13)	0.65(5)	0.890(14)	0.537(12)	1.46(4)	0.097(6)	529
0.20	0.319(1)	0.368(3)	0.600(13)	0.66(5)	0.872(14)	0.532(12)	1.45(4)	0.082(6)	488
0.30	0.316(1)	0.362(3)	0.597(13)	0.66(4)	0.867(14)	0.529(12)	1.45(4)	0.077(6)	476

expected to be proportional to a^2, this corresponds to a modest increase in the lattice spacing at which simulations of a prescribed quality can be carried out. However, the computer time required is proportional to a large power of the lattice spacing, so a small increase in lattice spacing can translate into a large gain in computer time.

The action considered here was primarily motivated by its simplicity and consistency with the lattice symmetries. It is plausible that this works because using the fat links in the fermion action smooths out the effects on the quarks of ultraviolet fluctuations in the gluon field, and the flavor symmetry breaking is less severe on the smoother configurations. (For comments on the effects of smoothing the gluon field seen by the quarks on the tadpole contributions, see Sec. 3.2 in Ref. [18].) Clearly a better theoretical understanding is wanted. In particular, a computation of the optimum staple weight is needed. However, it is likely that for the relatively large lattice spacings at which one would like to perform simulations with improved actions a nonperturbative (i.e., empirical) determination of the coefficients will be necessary. It would also be very interesting to see how this action affects rotational invariance.

We expect that an improvement of flavor symmetry in quenched spectrum calculations is a strong indication that dynamical simulations with this action will better reproduce the physics of a pion cloud. This needs to be tested.

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