

# Three-dimensional analysis of particulates in mineral processing systems by cone beam X-ray microtomography

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## Abstract

*In general, X-ray-computed tomographic (CT) techniques are capable of providing three-dimensional images of the internal structure of opaque materials in a nondestructive manner. The unique cone beam geometry allows acquisition of all two-dimensional projections with only one rotation of the sample, thus providing for fast data acquisition and better X-ray utilization, as a complete two-dimensional detector array receives the cone-shaped flux of rays. Thus, an isotropic three-dimensional volume can be reconstructed without the mechanical translation and the stacking of sequential slices, as is the case for more conventional CT scanners. In this regard, a state-of-the-art, custom designed X-ray microtomography facility to provide very detailed three-dimensional spatial analysis of packed beds of multiphase particles was installed and is in operation at the University of Utah. The reconstructed three-dimensional tomographic volume allows for spatial resolution as small as 5  $\mu\text{m}$  for sample dimensions of up to 40 mm in diameter. Utilization of this custom-designed cone-beam X-ray microtomography facility for the analysis of particulate systems in three dimensions is discussed. Applications described include coal washability analysis for the design and operation of coal-preparation plants, liberation analysis for the evaluation of grinding practice and separation efficiency, mineral exposure analysis for the prediction of the ultimate recovery from heap-leaching operations, three-dimensional particle shape analysis to classify particle populations and, finally, analysis of the pore-structure network of packed particle beds for simulation of flow through such porous structures as encountered in filtration and heap leaching. The initial results from these studies demonstrate the potential utility of detailed three-dimensional microCT information for improved design and operation of mineral processing methods.*

**Key words:** Cone beam CT, X-ray microtomography, Liberation analysis, Particle shape, Heap leaching, Exposure analysis, Coal washability

## Introduction

The analysis of multiphase particles and particle beds is of considerable importance for mineral processing/extractive metallurgy applications. Applications include coal washability analysis, mineral liberation analysis, mineral exposure analysis, particle shape analysis and analysis of the pore-structure network of packed particle beds. In each case, it is desired to obtain detailed three-dimensional information regarding size, shape, composition, texture, etc.

For liberation analysis, generally, measurements are made on polished sections of carefully mounted particle samples. From the polished section, a portion of the internal structure of the particles is exposed for textural characterization and the determination of mineral liberation. The spatial interpretation of this one- and two-dimensional information extracted from such cross sections can be accomplished by means of a variety of stereological procedures developed in recent years (Miller and Lin, 1988; Barbery, 1991; Schneider et al., 1991; King and Schneider, 1993; King, 1994). The polished-section analy-

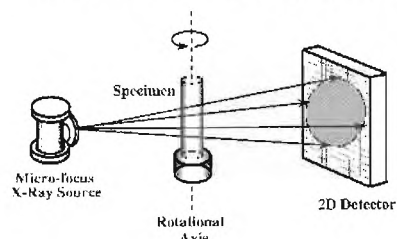
sis with stereological correction is a time-consuming process. Furthermore, to provide the stereological correction, assumptions must be made based on either textural information or on geometrical probability. Finally, extension of the stereological correction for more than two phases is limited.

The question of ultimate recovery in heap-leaching operations is always of particular concern with respect to economic considerations. Of course, the particle size distribution is a critical factor that determines ultimate recovery and that must be established based on a balance between the extent of mineral exposure and transport phenomena. In the copper heap-leaching process, inclusions of the desirable valuable minerals (copper-bearing minerals) are to be extracted from ore particles. The copper-bearing minerals have some unknown grain-size distribution, texture/exposure and spatial distribution in the ore particles. The procedure is to crush the ore so that the valuable mineral grains are exposed and can be extracted during the heap-leaching process. If the relationship between mineral exposure and particle size can be established

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**X-Ray Microtomography System**



**Figure 1** — Photograph of the University of Utah cone-beam X-ray microCT system and drawing illustrating the principle of cone beam X-ray microCT.

for different ore types, then the ultimate recovery in the heap-leaching process can be predicted for a specific particle size distribution. It is, therefore, extremely important to determine the percentage of exposed valuable mineral grains in the ore as a function of particle size. However, the percentage of exposed valuable grains in the ore cannot be determined using conventional polished section analysis such as typically practiced in the mining industry.

Filtration of fine particles involves filter-cake formation and the removal of surface moisture by drawing air through the porous particle-bed structure. Accurate assessment of the transport properties of porous media (filter cake in this case) is of major importance in the development of improved filtration processes. Implications from these studies are important in the design and operation of filtration equipment to enhance the efficiency of this important solid-liquid separation process. The microstructure and connectivity of pore space are important features necessary to describe detailed fluid flow phenomena in filter cake during fine particle filtration. In the same way, flow phenomena in heap leaching systems can be described. It is evident that three-dimensional characterization of pore structure is most useful to describe the three-dimensional multiphase flow through packed particle beds.

The ability to obtain accurate three-dimensional imaging and geometrical and textural information for a bed of multiphase, irregularly shaped particles is an important tool that can provide information to describe the performance of the various processes mentioned above in the mineral processing and extractive metallurgy industries. At the University of Utah, a new technique called cone beam X-ray microtomography has been used to nondestructively produce accurate three-dimensional images of packed-particle beds. X-ray microtomography is not subject to the same limitations as the polished section technique. In this paper, the authors present information regarding the use of this new facility and review potential applications for this advanced analytical system.

## Methods

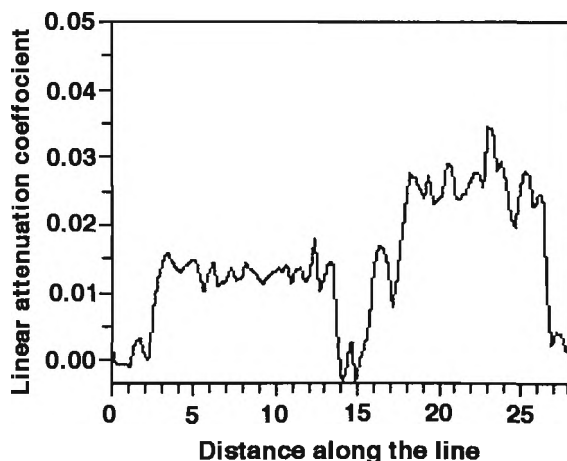
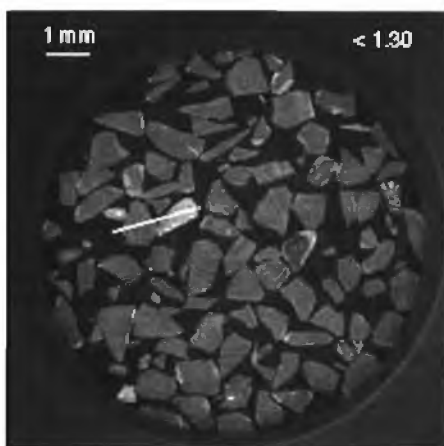
**High-resolution cone-beam X-ray microtomography system.** In general, high-resolution X-ray microtomographic systems use penetrating X-ray radiation to image an opaque

object and determine its internal features. The custom-designed micro-CT system recently installed at the University of Utah (Lin and Miller, 2001) uses cone beam geometry to obtain data over an entire object volume (Fig. 1). This enormously speeds up the process of imaging complete objects and provides data with the same spatial resolution in all directions. The system was designed based on the following considerations:

- a system geometry optimized to obtain high-resolution images of small samples, but with the flexibility to examine larger objects as required;
- a detector with the resolution, efficiency and dynamic range required to obtain high-quality data from a broad spectrum of samples;
- a reliable X-ray source that allows for focus on research rather than maintenance; and
- a high-accuracy positioning system required to maintain the spatial resolution provided by the X-ray imaging component.

The essential feature of X-ray tomographic imaging is the determination of material density (more accurately attenuation coefficient) of a small region of three-dimensional space called a voxel. Tomography can determine the density of all voxels in the three-dimensional region of the scan. Of course the position of each voxel is known precisely. It is desired to determine the geometric characteristics of any region of space that is subject to variations in density. In particular it is possible to determine precisely the shape, and therefore the volume, as well as the mass of each individual phase within the target volume.

The University of Utah micro-CT system can achieve 2,048 x 2,048 pixel reconstructions over a 10-mm-diameter object, while also allowing for the imaging of objects 40-mm in size. When operated at highest resolution, the smallest voxel is 5  $\mu\text{m}$ , which corresponds to quantitative spatial resolution of about 12  $\mu\text{m}$ . The dimensional accuracy achieved with this system is about 5  $\mu\text{m}$  over 10 mm. This system is capable of imaging through high-density minerals, even minerals having a density as high as 8 g/cc. The resulting images are digitized with 16 bit-depth and this provides a large



**Figure 2** — Cross-sectional image (left) from three-dimensional X-ray microtomography reconstruction of coal particle bed (0.850 x 0.589 mm, density < 1.3 g/cm<sup>3</sup>) and (right) X-ray attenuation coefficient of individual voxel along the line shown in the reconstructed microtomography image.

dynamic range that is required to image microsystems that are composed of a variety of minerals, such as coal macerals and pyrite. A photograph of the University of Utah cone beam X-ray microtomography system is shown in Fig. 1.

**Sample preparation.** The effectiveness of the three-dimensional X-ray cone-beam micro-CT for characterization and measurement of mineral liberation, mineral exposure and the pore network microstructure of a packed particle beds has been demonstrated for various samples. Typically, the particle sample is simply packed in a cylindrical plastic container up to 40 mm in diameter and mounted on the micro-CT stage. Special sample-preparation procedures are not necessary. A complete scan at 20- $\mu$ m resolution can be achieved within 40 minutes.

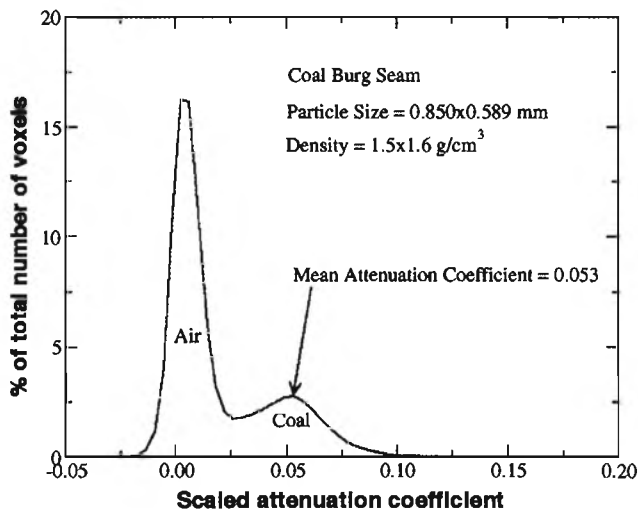
### Coal characterization and washability analysis

Application for the characterization and washability analysis of fine coal are reviewed below. The reconstructed cone-beam CT image consists of a three-dimensional array of X-ray attenuation coefficients, each associated with a finite-volume cube (voxel) of the sample. The attenuation coefficients are a function of the average density and composition of the material in any given voxel (Lin and Miller, 2001). Consider a selected two-dimensional X-ray tomographic slice from the reconstructed three-dimensional digital mapped X-ray attenuation coefficients of a packed bed of coal particles (particle size = 0.850 x 0.589 mm, density < 1.3 g/cm<sup>3</sup>), as shown in Fig. 2. X-ray attenuation coefficients of individual voxel along the line are shown on the left-hand side of Fig. 2. The coal constituents clearly exhibit a different absorption and scattering behavior for the X-rays. A plot of the attenuation coefficient histogram is a measure of the density variation throughout the sample. Usually, the attenuation coefficient histogram obtained from known densities of a coal particle bed (such as the each density fraction obtained from sink-float tests) can provide the underlying component densities (Lin et al., 2001). By way of example, Fig. 3 shows the attenuation coefficient histogram corresponding to the reconstructed three-dimensional microtomography image of the packed bed of coal particles. The attenuation coefficient histogram consists of an overlapping bivariate distribution as

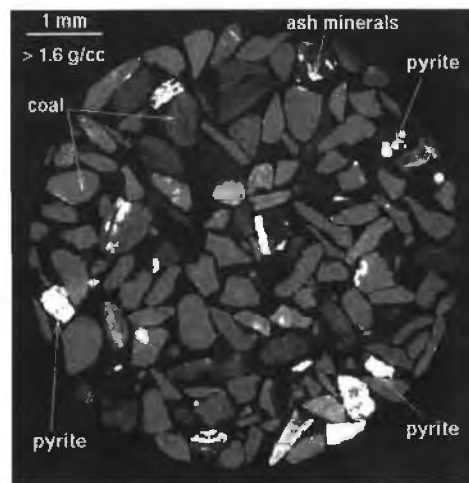
shown in Fig. 3. The peak with the higher attenuation coefficient value (~ 0.053) is associated with the coal phase having a density between 1.50 and 1.60 g/cm<sup>3</sup>. The peak corresponding to the lower attenuation coefficient value (~0) is associated with the external airspace surrounding the sample.

Cone-beam X-ray microtomography can be used to measure and quantitatively characterize fine coal particles directly. High spatial resolution and the direct processing of raw volumetric data are the two important benefits offered by this new method. To illustrate the ability of the high-resolution X-ray microtomography technique for quantitative characterization and analysis of fine coal particles, coal samples from Pittsburgh No. 8 seam were selected. Details of the liberation characteristics of pyrite and other ash-forming minerals from these samples have been investigated using an SEM system equipped with automatic image analysis (King, 1999).

The high-resolution X-ray microtomography technique used for this study is capable of imaging through high-density material, and the resulting three-dimensional digital images are quite suitable for the characterization of fine coal particles, which have a wide density range of components ranging from organic macerals to mineral matter, including pyrite. Figure 4 illustrates the ability of the high resolution X-ray microtomography technique for quantitative analysis to determine the three-dimensional spatial distribution of coal particles. Consider a selected two-dimensional X-ray tomographic slice from the reconstructed three-dimensional digital-mapped X-ray attenuation coefficient of the packed bed of Pittsburgh No. 8 coal particles (0.500 x 0.355 mm, specific gravity > 1.6), as shown in Fig. 4. In this image, the pyrite phase has the highest X-ray attenuation coefficient and is shown as white. Other ash-forming minerals (most of them are silicate-based minerals) are light gray, and coal constituents are dark gray. The background (air) is black. Here the grayscale levels of the images are based on the relative X-ray attenuation coefficient and are indicative of different mineral phases present in the sample. For quantitative analysis, the attenuation coefficient of each individual voxel of the three-dimensional digital map can be used to process and to classify each particle and the different mineral grains inside each particle. This section is taken from the three-dimensional image; the image elements in the reconstruction are cubic (resolution 10  $\mu$ m).



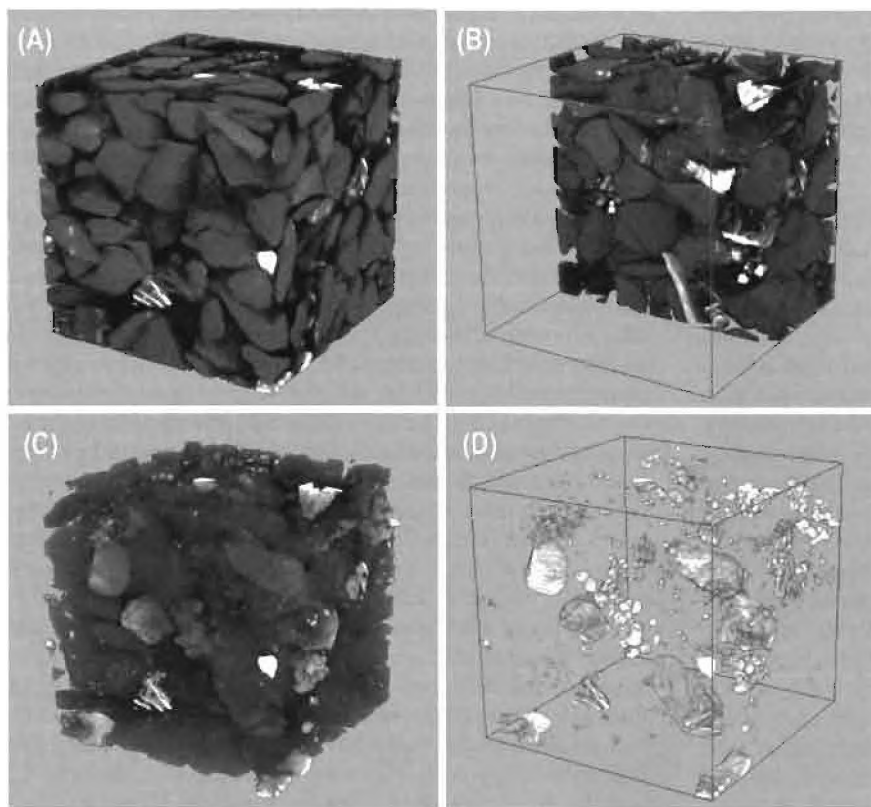
**Figure 3**—The X-ray attenuation coefficient histogram for the three-dimensional X-ray microtomography analysis of sample from the Coal Burg seam.



**Figure 4**—Coal constituents, pyrite and other ash-forming mineral phases identified from selected cross-sectional image of the three-dimensional reconstruction of multiphase coal particles (Pittsburgh No. 8, 0.500 x 0.355 mm, specific gravity >1.6).

Pyrite has different crystalline forms (vein, nodule and cluster), as shown in Fig. 4. In fact, the three-dimensional morphology and grain-size distribution of pyrite provides valuable information for processing considerations. A volume-rendered image from a subset of Pittsburgh No. 8 sample (256 x 256 x 256) is shown in Fig. 5 (A). The width

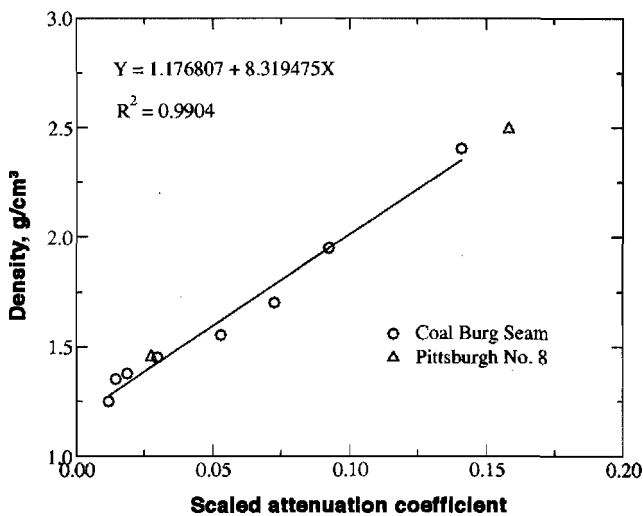
of the cube is 2.56 mm. Figure 5 (B) shows sections of the particle bed with one half of the volume removed. The corresponding three-dimensional view of the subset is shown in Fig. 5 (C) and reveals details of the distribution of ash-forming minerals (in gray) in a packed bed of coal particles (host coal constituents is set as transparent green). With the use of three-dimensional image-processing techniques, one can remove all the host coal phase and reveal only the ash-forming mineral grains, as shown in Fig. 5 (D). Distinct three-dimensional morphology of different crystalline forms of pyrites is clearly distinguished in Fig. 5 (D). It is evident that the grain size distribution of the ash-forming minerals can be determined. In summary, it is concluded that the utilization of X-ray microtomography not only allows for quantitative analysis of multiphase systems but also allows for textural characterization and the determination of phase continuity.



**Figure 5**—(A) Volume rendering image from a subset (256 x 256 x 256) of the packed bed of coal particle (Pittsburgh No. 8, 0.500 x 0.355 mm, specific gravity >1.6). (B) Section views of (A) by removing half of the volume. (C) Semitransparent volume rendering of (A). (D) Ash-forming mineral grains through the removal of coal phase.

X-ray CT scanning of single-size/single-density coal particle beds is a suitable method to obtain a statistical picture of the overall behavior of the attenuation coefficient with respect to density and size. Correlation between the density and X-ray attenuation coefficient can be obtained through the analysis of the peaks in the coal spectrum (Lin et al., 2001). Figure 6 presents a plot of attenuation coefficient vs. density. It is noted that the density calibration curves fit very well for both coal samples (Coal Burg and Pittsburgh No. 8 seams).

A fine-coal sample (0.850 x 0.589 mm, multiple density fractions, Coal Burg seam) was prepared for washability analysis using three-dimensional X-ray



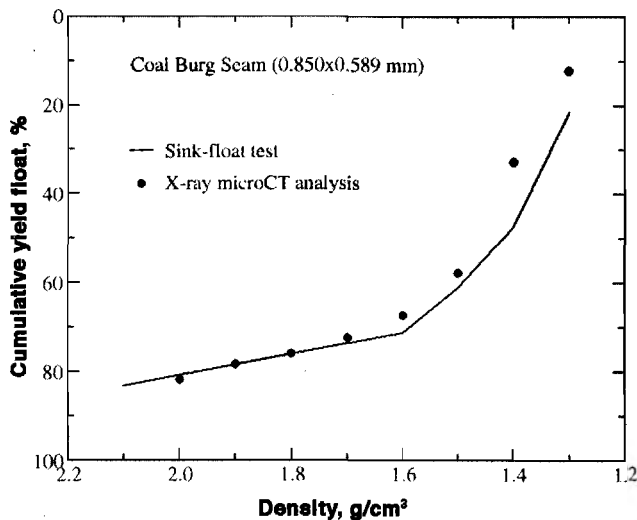
**Figure 6** — Three-dimensional X-ray microtomography density calibration for the single-size/single-density coal particles (packed bed).

microtomographic technique. Under carefully controlled laboratory conditions, coal particles from each density fraction (out of eight density classes) were collected and mixed. Using a plastic container (9-mm inside diameter, 20-mm long), this mixed coal sample was scanned by three-dimensional X-ray microtomography. The three-dimensional reconstructed image set contains 512 x 512 x 912 voxels (voxel resolution = 20  $\mu$ m). A volume-rendered image from a subset of this sample (512 x 512 x 180) is shown in Fig. 7 (A). Figure 7 (B) shows sections of the particle bed with half of the volume removed. Coal particles of different density are clearly distinguished from these images.

The washability curve (yield/density) for this carefully prepared sample is shown in Fig. 8 based on sink-float data. Previous study with coarse coal samples (Lin et al., 2001) indicated that such washability curves can be constructed based on X-ray CT analysis using a successive subtraction process. Based on this procedure, the washability curve can be established with the use of scaled attenuation coefficient histogram and density calibration curve (Fig. 6). Figure 8 illustrates the constructed yield/density curves based on the sink-float data and on the three-dimensional X-ray CT data.

### Mineral liberation analysis

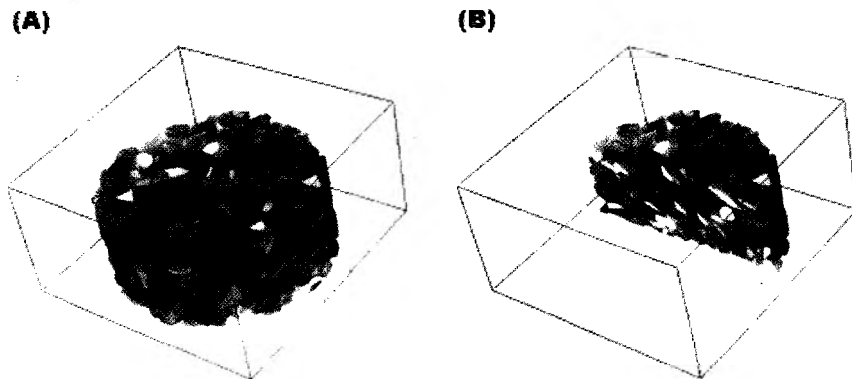
For detailed liberation analysis, the volumetric grade distribution of multiphase mineral particles can be measured directly by cone-beam X-ray microtomography, as described in a preliminary study (Lin and Miller, 1996). High spatial resolution and the direct processing of raw volumetric data are the two important benefits offered by this new method. Figure 9 illustrates the ability of the high-resolution X-ray microtomography system for quantitative analysis to determine the three dimensional spatial distribution of mineral phases in multiphase particles. Four cross sections (from a total of 300



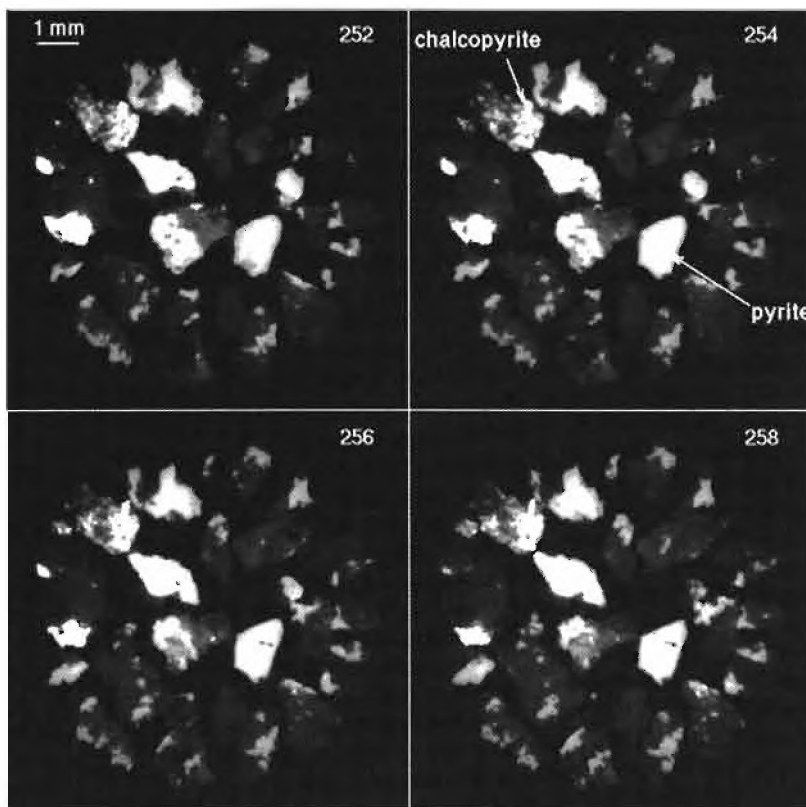
**Figure 8** — The yield/density curve for mixed coal particles from eight different density fractions (Coal Burg Seam, 0.850 x 0.589 mm) as determined by sink-float data compared with the results from three-dimensional X-ray microtomography analysis.

sections) along the Z-direction are shown as established from the three-dimensional reconstruction of a copper ore sample (2.00 x 1.18 mm, specific gravity 3.5 x 3.1). It should be noted that these sections are taken from the three-dimensional image. The image elements in the reconstruction are cubic, so the spacing between the planes equals the resolution, which in this case corresponds to 20  $\mu$ m. Here, the grayscale levels of the images indicate the relative attenuation coefficient present in the bulk of the sample. Based on X-ray attenuation coefficient, differentiation of mineral phases within the sample is possible as indicated in Fig. 9 for pyrite and chalcopyrite.

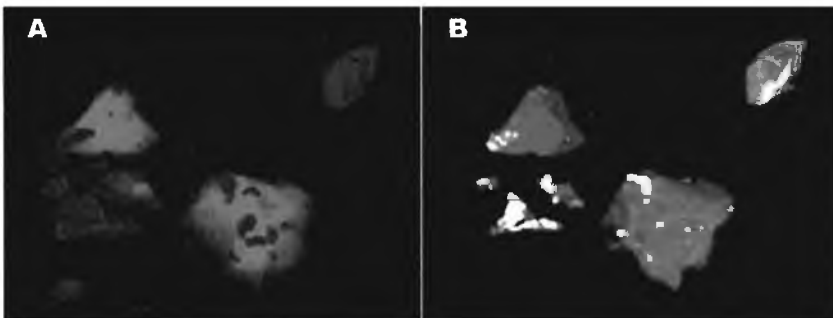
For quantitative determination of the volumetric grade distribution, suitable volumetric image analysis algorithms, such as three-dimensional segmentation and classification, will be necessary to treat the three-dimensional array. In this regard, a volume-rendered image from a sphalerite/dolomite sample is shown in Fig. 10 (A). Figure 10 (B) shows sections of the particles with a portion of the volume removed. The sphalerite (in white) and dolomite (gray) phases are clearly distinguished from this image. Table 1 shows the results for particles with a larger volume.



**Figure 7** — (A) Volume rendering image from a subset (512 x 512 x 180) of a packed bed of coal particles (Coal Burg Seam, 0.850 x 0.589 mm, eight density fractions). (B) Sectional view of (A) by removing half of the volume.



**Figure 9** — Cross-sectional images from the three-dimensional X-ray microtomography reconstruction of multiphase copper ore particles (2.00 x 1.18 mm, specific gravity 3.5 x 3.1). The dark colored grains distinguish the pyrite, chalcopyrite mineral phases in a silicate matrix.



**Figure 10** — (A) Volume rendering image from sphalerite/dolomite particles, sphalerite in red and dolomite in green. (B) Sectional view of (A) by removing portion of the volume.

**Table 1** — Size and grade by volume for selected locked dolomite/sphalerite particles (Fig. 10) as measured by three-dimensional X-ray microtomography.

| Particle number | Volume, mm <sup>3</sup> x 10 <sup>4</sup> | Equivalent size, μm | Volume grade, % | Number of grains |
|-----------------|---|---------------------|-----------------|------------------|
| 1               | 5,645.3                                   | 826.5               | 4.60            | 31               |
| 2               | 3,562.6                                   | 708.9               | 4.79            | 15               |
| 3               | 1,252.4                                   | 500.3               | 49.89           | 1                |
| 4               | 723.3                                     | 416.6               | 66.41           | 6                |
| 5               | 558.6                                     | 382.3               | 18.20           | 18               |
| 6               | 166.1                                     | 255.2               | 25.68           | 1                |
| ...             | ...                                       | ...                 | ...             | ...              |
| 26              | 1.76                                      | 56.0                | 1.92            | 1                |

Three-dimensional liberation analysis by microtomography provides an excellent opportunity to overcome many of the limitations of currently used polished section techniques. With the X-ray microtomography system, complete accounting of the three-dimensional spatial distribution of mineral phases in each particle is possible, including grain size distribution, interfacial area, shape features and textural information.

### Mineral exposure analysis for evaluation of heap-leaching operations

Figure 11 further illustrates the ability of the high-resolution XMT system for the quantitative mineral exposure/liberation analysis. Four cross sections (from a total of 512 sections) along the Z-direction are shown as established from the three-dimensional reconstruction of the packed bed of copper ore particles. Exposed grains can be identified only through the analysis of the three-dimensional data set for the packed bed of particles. Using a three-dimensional image-analysis algorithm, the overall copper-bearing grains and internal/exposed grains can be identified, as shown in Fig. 12. It must be emphasized that mineral exposure analysis can only be determined from the complete three-dimensional data set.

The ultimate recovery in the heap-leaching process can be predicted for a specific particle size distribution if one can determine the relationship between the percentage of the exposed valuable mineral with respect to particle size for different ore types (Miller et al., 2003). It is, therefore, extremely important to characterize the percentage of the exposed valuable mineral grains in the ore as a function of particle size. In this regard, two types of copper ore from different parts of the deposit were collected, sampled and sized to ten size intervals.

Representative samples of particles from different size intervals were taken and put into a cylindrical container for XMT analysis. Scanning time was varied depending on the resolution, number of views. Figure 13 shows the relationship between the percent of copper exposed and particle size for both Composites 4 and 6. As expected, the exposure decreases with an increase in particle size. The slope of the curve is much more pronounced below 10 mm, indicating that the exposure can be rapidly raised just by increasing the amount of material in the middle size classes. Composite 4 shows a better exposure than Composite 6 for coarse sizes; for fine sizes, the exposure is very similar. A better recovery should be expected for Composite 4 at equivalent PSD and appropriate chemical conditions.

Many large, almost completely liberated copper mineral grains (clusters of grains) were found in particle size classes of 3.18 x 1.7 mm (1/8-in. x 10 mesh) and 1.7 x 0.425 mm (10

x 40 mesh). Figure 14 shows cross-sectional CT images for different particle size classes.

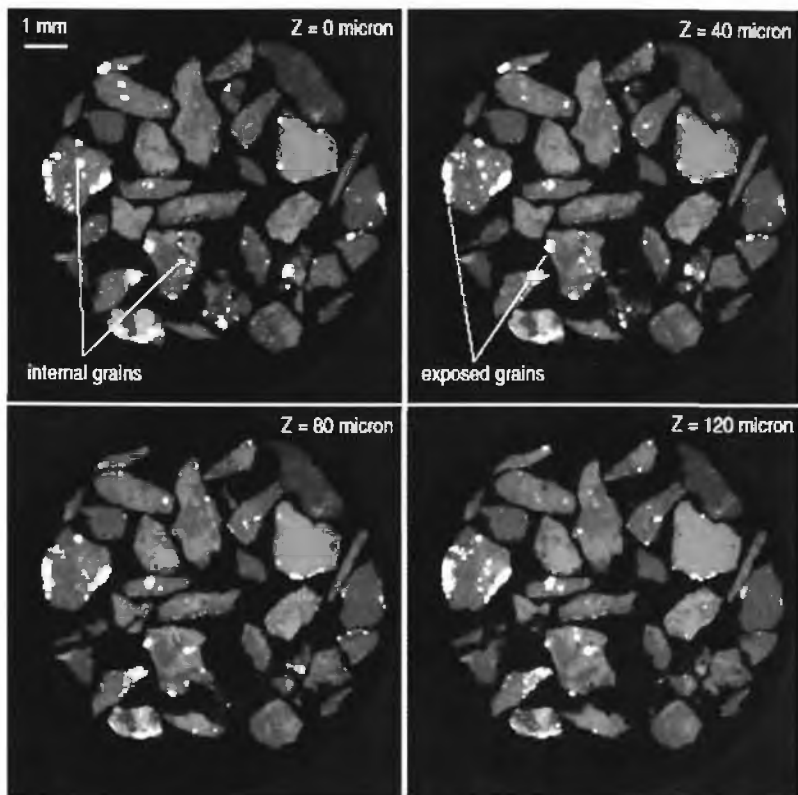
The relationship between the percent of copper exposed and particle size provides the basis for the prediction of copper recovery with known particle size distribution. In this regard, three particle size distributions were selected and 1.5-m column tests were run for about 70 days. Figure 15 shows the predicted copper recovery based on the exposure analysis and actual copper recoveries from these column tests with different particle size distributions for Composite 4. As expected, good agreement between the predicted and actual recovery was obtained.

As expected, the smaller the particle size the higher the percentage of copper mineral exposed. However, in addition to exposure analysis, the fluid-flow phenomena inside the packed bed of particles is an important issue that needs to be investigated. It is known that the smaller the particle-size the lower the permeability of the packed particle bed. Once these issues (how particle size distribution influences both mineral exposure/liberation and flow behavior) are resolved, optimal chemical conditions can be established and the chemistry designed for the optimum utilization of the various reactants. In any event, it is evident that ultimate recovery for any specific particle size distribution can be estimated from mineral exposure analysis by XMT.

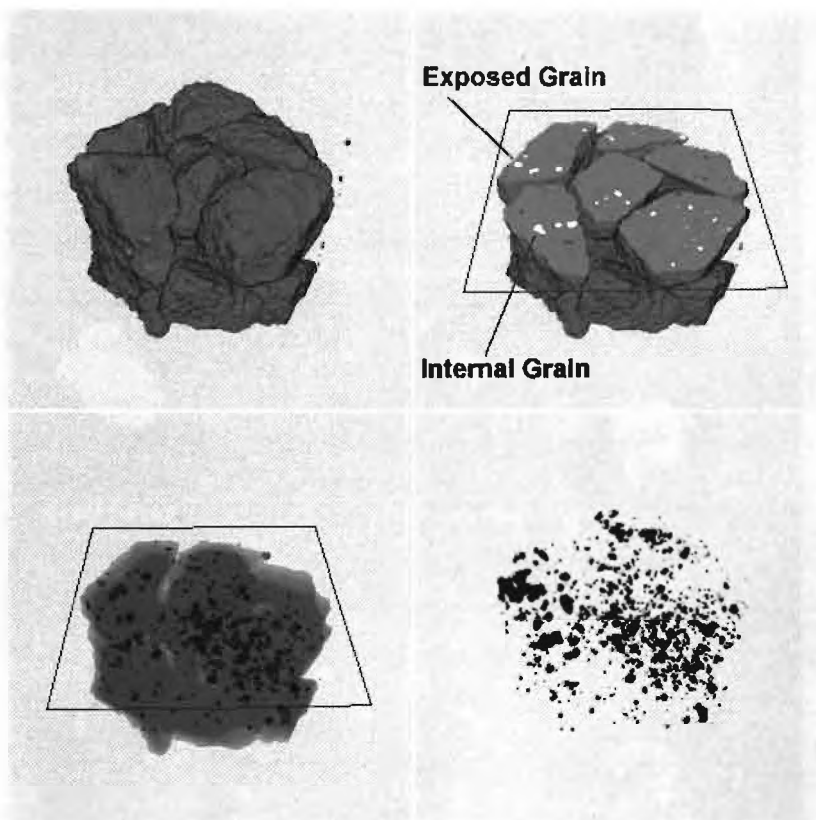
### Particle shape analysis

The analysis of mechanical and transport problems involving particulate materials will generally use a multitude of correlations to predict the behavior of particles with regular shape, particularly in the case of spheres. In practice, the particulate process industries are concerned with particles that are far from being of a regular shape and generally cover a wide range of particle size.

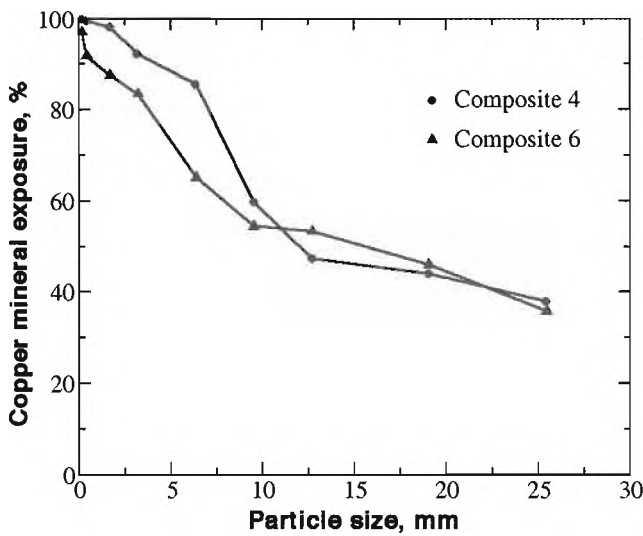
Based on regularly shaped particles, the correlations developed to predict the behavior of particulate material are very approximate. Remarkably few techniques exist to predict the mechanical and hydrodynamic behavior of single irregular particles or assemblages of irregular particles. The lack of models and correlations can be attributed to the wide variety and complexity of particle shapes, the difficulty of defining shape descriptors suitable for modeling, the limitations of measuring shape and the lack of classifying techniques to characterize particle shape. It is difficult to measure and to



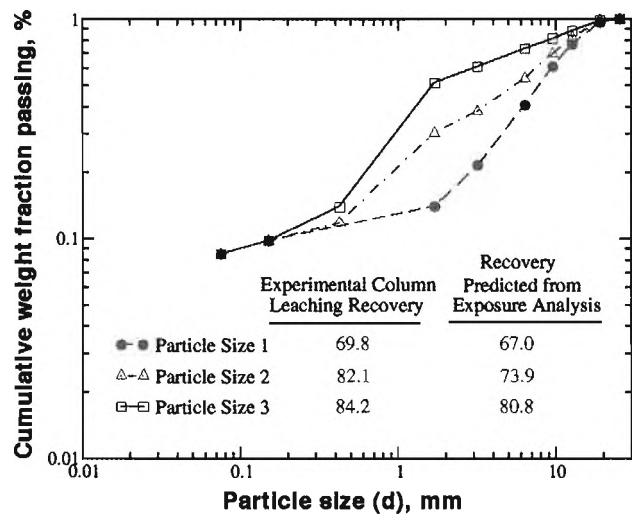
**Figure 11** — Cross-sectional images from the three-dimensional XMT reconstruction of a packed bed of multiphase particles. The white regions represent cross sections of copper mineral grains and the extent of their exposure at the particles' surfaces.



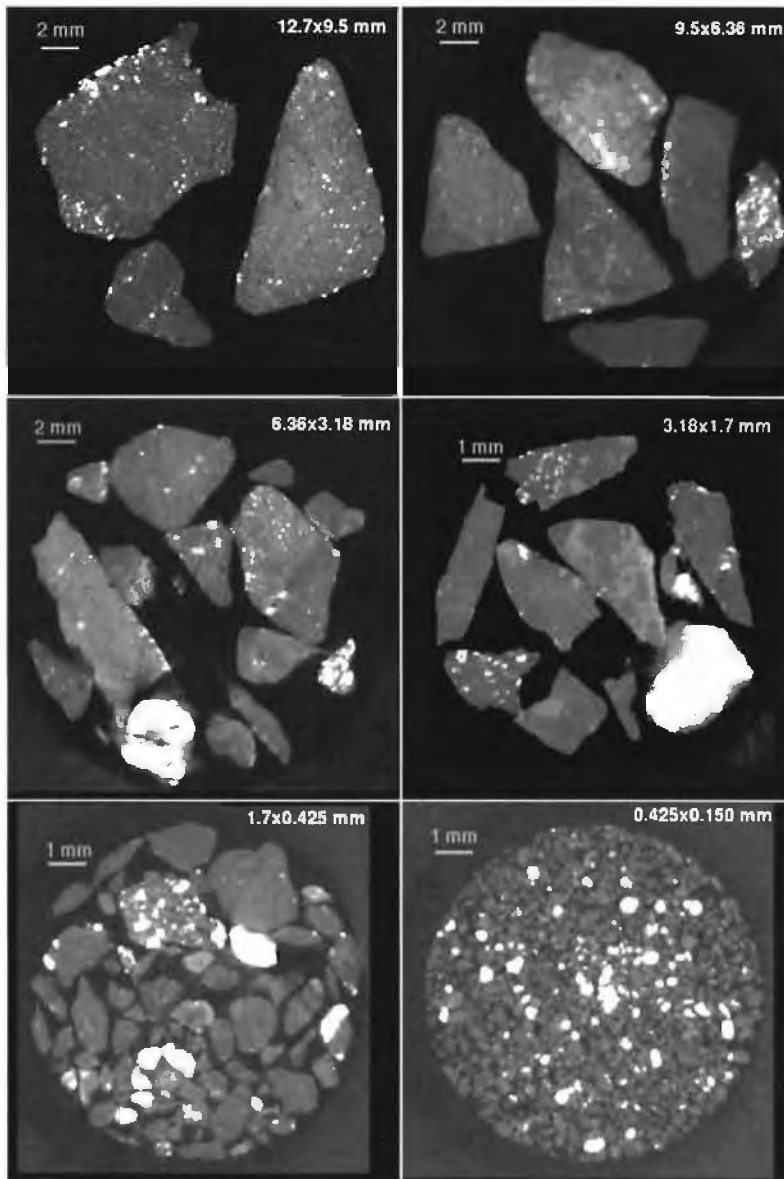
**Figure 12** — Three-dimensional mineral exposure analysis by X-ray microtomography (XMT). The internal (blue)/exposed (red) copper bearing grains obtained from three-dimensional image analysis and distinguished with different colors.



**Figure 13** — Relationship between the exposed valuable minerals with respected to particle size for Composites 4 and 6.



**Figure 15** — Comparison of experimental column recovery with that predicted from mineral exposure analysis by XMT for three different particle size distributions for Composite 4.

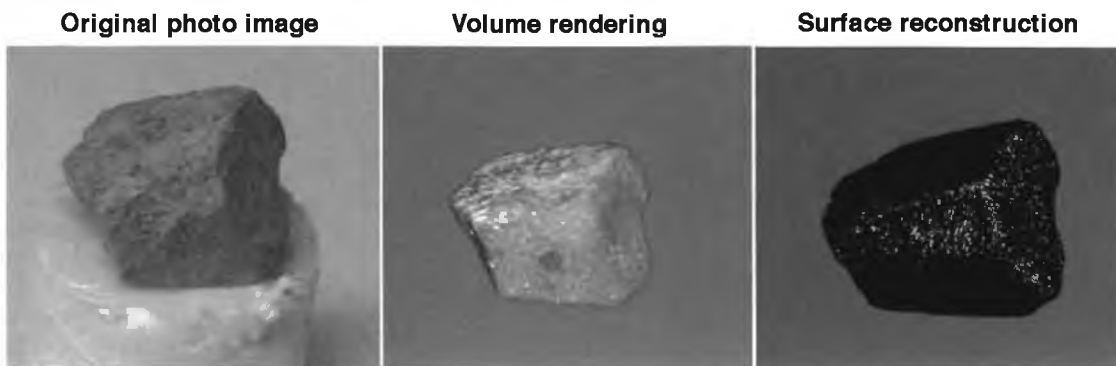


**Figure 14** — Selected cross-sectional CT images for different particle size classes of Composite 4.

classify three-dimensional particle shape, and only recently has it been possible to obtain the spatial surface data for a large numbers of particles in a relatively short time.

In the processing industries, it is known that particle shape has a close relationship to the behavior function of bulk material. A list of some technological parameters and powder properties that may be affected by particle shape is given in Table 2 (Hawkins, 1993). For example, recent studies (Ting et al., 1995) indicate that increasing particle roundness leads to a substantial decrease in observed shear resistance. In addition, increasing particle roundness results in a decreased resistance to liquefaction, decreased range of void ratios between the densest and loosest conditions, low strain at peak strength, lower average void ratios and less variation in void ratio throughout the sample. Preparing particles of the desired shape will enhance the function of the particulate material.

Particle shape is also of importance in the particulate separation processes encountered in the pulp/paper industry, the mineral industry and other industries. For example, in deinking flotation for cellulose recovery, toner particle shape is a critical issue. Similar particle shape dependence is evident in mineral systems. Already, the utility of X-ray microtomography has been demonstrated to be an effective diagnostic tool to describe the spatial distribution of mineral phases (Miller et al., 2003), and it is expected that such great utility will be possible in the analysis of particle shape. In this regard, it seems that three-dimensional quantitative particle shape analy-



**Figure 16** — Comparison of surfaces of an irregularly shaped particle using data obtained from X-ray microtomographic measurement for volumetric rendering and for reconstruction from surface data points.

sis will be quite useful for an improved understanding of particle separation processes.

With the advance of computer technology, digitization and image-processing technologies, more sophisticated descriptors for particle characterization have been used (Mandelbrot, 1977; Beddow, 1981; Clark et al., 1989; Leurken, 1991). These shape descriptors include Fourier coefficients, fractal dimensions and persistence of polygonal harmonics. It is noted that most of these shape descriptors are two dimensional in nature, and no advantages are gained in this methodology for the three-dimensional study of particle motion in a fluid or in the study of fluid flow in packed-particle beds.

Although mathematical theory is available for the three-dimensional description of particle shape, current measuring techniques for three-dimensional analysis of particle shape are few. The three-dimensional laser range scanner (Hoppe et al., 1992; Levoy et al., 2000) and X-ray microtomography (Lin and Miller, 2001) are two instruments that can be used to acquire surface information that can be used for three-dimensional particle shape analysis. In addition, a hand-held digitizer has been used to capture surface data points for relatively large objects. Due to the nature of the operation, the three-dimensional laser range scanner can be used to acquire the surface data points one particle at a time. However, X-ray microtomography is suitable for surface analysis and the acquisition of three-dimensional-shape information from a particle assemblage.

Surface reconstruction is an important step in geometric modeling (three dimensional-shape analysis) for generating surfaces from data points captured from real irregular particles. Preliminary three-dimensional particle-shape analysis results show that a directed three-dimensional digital map of the density spectrum of a particle or particle population can be obtained using X-ray microtomography. Surfaces of an irregular particle directly from volume rendering with tomographic data and from reconstruction with surface data points are shown in Fig. 16.

In the last decade, there has been considerable research concerning surface reconstruction (Hoppe et al., 1992; Edelsbrunner and Mucke, 1994; Garland and Heckbert, 1997; Levoy et al., 2000; Amenda et al., 2001; Dey and Giesen, 2001), which still remains a very active research topic. Robust and simplified surface-reconstruction processes provide one with a valuable tool for measuring the three-dimensional individual particle shapes of particle populations. In this regard, simplified surface-reconstruction processes, based on

the Garland and Heckbert (1997) method, were used to evaluate the effect of number on mesh surface and three-dimensional particle shape. Preliminary results indicate that the number of mesh surfaces from surface reconstruction can be significantly reduced from 140,668 to 200 and still keep the three-dimensional particle shape, as shown in Fig. 17.

The significance of three-dimensional particle-shape research is realized from the above discussions. However, for the study of mechanical and flow behavior of particulate material, three-dimensional particle-shape descriptors, which can be used to analyze and to classify actual irregularly shaped particles need to be established to simulate and to reconstruct these particles. In this regard, study of three-dimensional particle shape should focus on the development of analytical techniques that can be used to measure, describe, classify and reconstruct irregularly shaped particles.

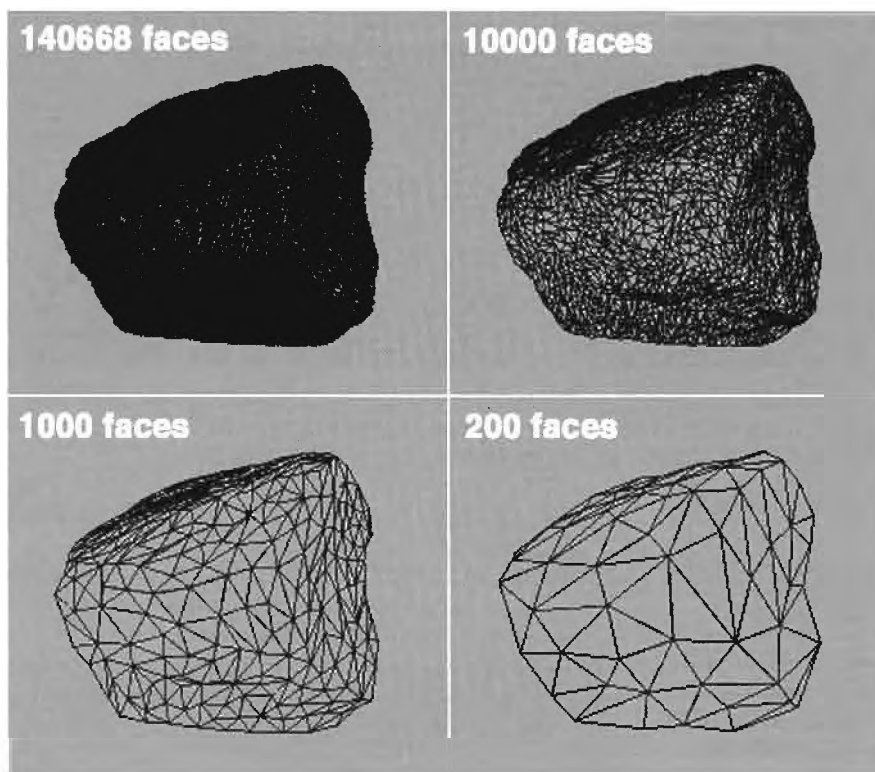
For the three-dimensional particle-shape study, geometric parameters of interest need to be established. In this regard, both simulations to generate random irregularly shaped particles and classification procedures will be needed. In general, a simplified representation of three-dimensional particle shape should include every aspect of external morphology, i.e., the form, the sharpness of any edge or corners (or roundness) and the texture of the surface as shown in Fig. 18 (Barrett, 1980). The independence of the three aspects is clear. In this regard, research is currently focused on finding the characteristic particle shape representation based on three-dimensional analysis using X-ray microtomography.

### Flow through packed particle beds

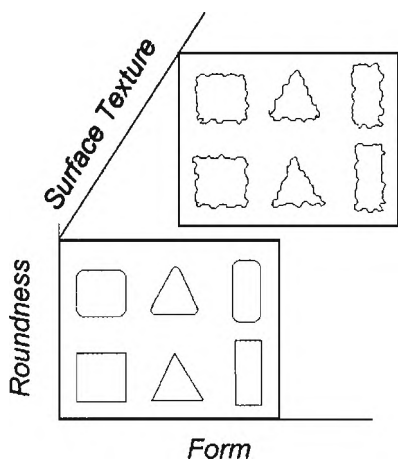
Continuous filtration of fine particles involves filter cake formation and removal of surface moisture by drawing air

**Table 2** — Physico-mechanical properties of powders that depend in some degree on particle shape. (Hawkins, 1993).

|                 |                      |
|-----------------|----------------------|
| Agglomeration   | Packing              |
| Aggregation     | Permeability         |
| Bulk density    | Porosity             |
| Cohesivity      | Reactivity           |
| Combustion      | Segregation          |
| Compressibility | Separation           |
| Explosion       | Shear behavior       |
| Flowability     | Suspension stability |
| Gasification    |                      |



**Figure 17** — Comparison of three-dimensional particle shape using different simplified surfaces from surface reconstruction method by Garland and Heckbert (1997).



**Figure 18** — A simplified particle-shape representation of form, roundness and surface texture to illustrate their independence (Barrett, 1980).

through the porous structure. Accurate assessment of the transport properties of porous media (in this case filter cake) is of major importance in the development of improved filtration processes. Implications from these studies are important in the design and operation of filtration equipment to enhance the efficiency of this important solid-liquid separation process. The microstructure and the connectivity of the pore space are important to describe fluid flow in filter cake

during fine-particle filtration. In this regard, characterization of pore structure based on parameters permitting inferences on the fluid balance is of particular interest. The pore structure has to be described by parameters that are of special relevance for the interpretation of fluid-transport phenomena. These parameters should be based on directly measured variables of the pore system and not on indirect variables (such as those determined empirically from transport processes) valid only for a particular pore structure. In this way, fundamental relationships between pore structure and fluid transport at the microstructure level can be described. Thus, it is desired to be able to directly measure the three-dimensional interconnected pore structure of filter cake.

Most present methods used to characterize the pore microstructure and its completed interconnected network rely on the microscopic observation of a series of thin or polished sections of the porous media. These data sets are then used to reconstruct and to display the three-dimensional image of the porous system with the help of advanced computer graphic techniques. Complete analysis of the three-dimensional porous system from serial sections is a tedious and time-consuming process. In addition, for a completely interconnected porous system, pore size distribution is not a well-defined parameter (Lin and Miller, 2000).

A packed bed of iron ore particles ( $180 \times 106 \mu\text{m}$ ) was prepared for the study of complex filter cake pore structure using high-resolution three-dimensional X-ray microtomography. Figure 19 (A) shows one slice from the volume data set for the packed bed of iron ore particles. A volume-rendering image from a subset of this sample ( $256 \times 256 \times 128$ ) is shown in Fig. 19 (B). The voxel size for this sample is  $10 \mu\text{m}$ . Connectivity is an important concept when flow problems are

considered. Fluid flow can occur between two points only when the pore space is connected. In this regard, overall pore structure is extracted from Fig. 19 (A), and the surface-rendering image is shown in the Fig. 20 (A). Results from a three-dimensional connection analysis indicates that the overall pore structure is composed with several independent major networks (Fig. 20 (B)) and some isolated pores.

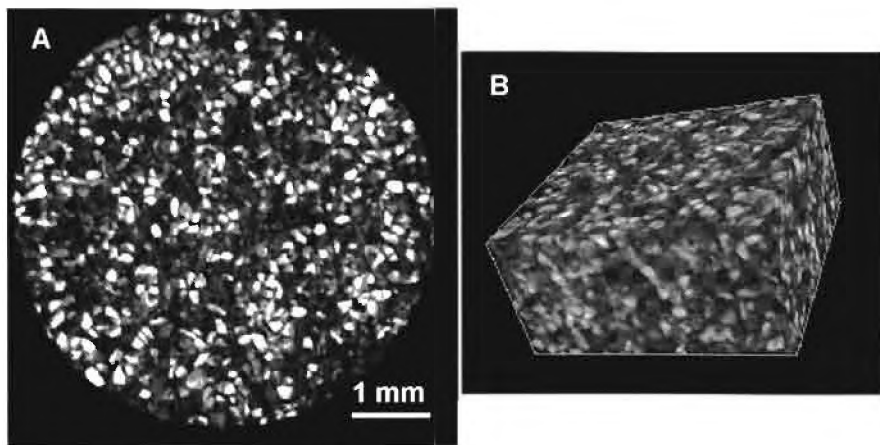
The fundamental relationship between pore microstructure and effective transport coefficients can be established based on flow simulation through the porous media and has been discussed in a separate paper (Lin and Miller, 2002). Figure 21 illustrates the feasibility of the Lattice Boltzmann (LB) simulation for fluid flow through the three-dimensional reconstructed pore space of packed particle beds (with particle size of  $210 \times 150 \mu\text{m}$ ). Once one removes the solid particle phase, Fig. 21 (B) shows the spatial variations in velocity and the nature of the flow channels. The main thing to notice is that most of the flow occurs in a small fraction of the available pore space. The estimated permeability for this sample is  $4.02 \times 10^{-7} \text{ cm}^2$ .

### Summary and conclusions

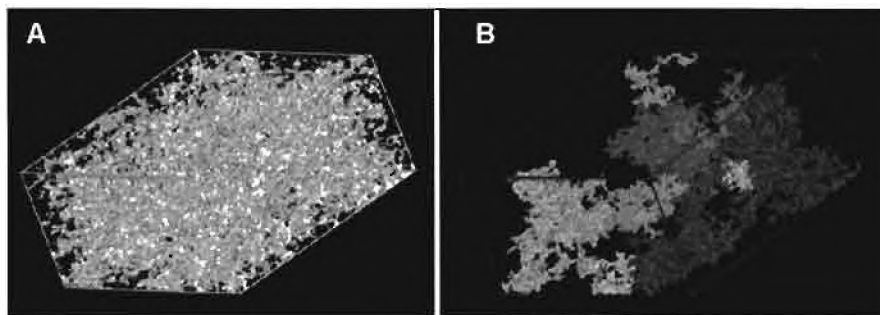
The ability to obtain accurate three-dimensional imaging, geometrical characteristics and textural information for a packed particle bed of multiphase, irregularly shaped particles is an important tool that can provide the basis to evaluate the performance of the various mineral processing/extractive metallurgical processes mentioned above. In this regard, the utilization of X-ray microtomography not only will allow for quantitative analysis of multiphase systems, but it will also allow for textural characterization and the determination of phase continuity. Potential applications for the advanced analytical system were reviewed and include particle composition distribution (coal washability/three-dimensional liberation analysis), mineral exposure analysis for heap-leaching operations, three-dimensional particle-shape analysis, and pore structure analysis for flow simulation through packed particle beds such as encountered in filtration and heap-leaching operations.

### Acknowledgments

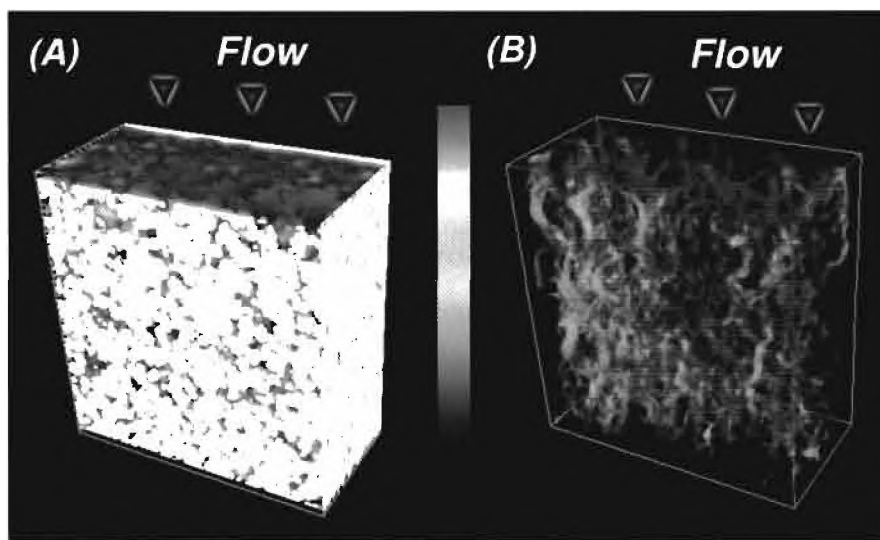
This work was supported by NSF Grant No. CTS-9724315, DOE Grant No. DE-FG22-96PC961215 and Compañía Minera Zaldivar/Placer Dome.



**Figure 19** — (A) Selected cross-sectional image from a packed bed of iron ore particles ( $180 \times 106 \mu\text{m}$ ). (B) Volume rendering image from a subset ( $256 \times 256 \times 128$ ) of the packed bed of iron ore particles.



**Figure 20** — (A) Surface-rendered image of the pore structure from a subset ( $256 \times 256 \times 128$ ) of the three-dimensional Micro-CT data set (Fig. 19 (B)). (B) Overall pore structure is composed with several independent major networks.



**Figure 21** — Three-dimensional views of LB simulated flow through a packed particle bed (with particle size of  $210 \times 150 \mu\text{m}$ ): (A) with solid phase as white and (B) with transparent solid phase.

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