

Assorted weak matrix elements involving the bottom quark*

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As part of a larger project to estimate the f_B decay constant, we are recalculating f_B^{static} using a variational smearing method in an effort to improve accuracy. Preliminary results for the static B_B parameter and HQET two point functions are also presented.

1. INTRODUCTION

The extraction of CKM matrix elements from experimental data requires the calculation of QCD matrix elements of operators involving the bottom quark [1]. For a number of years the MILC collaboration has been doing a systematic study of the f_B decay constant [2]. The difficulty of simulating the bottom quark, with its mass larger than current feasible inverse lattice spacings, was overcome by interpolating between the results of simulations using Wilson quarks with masses around the charm mass and those from static simulations (infinite mass).

Unfortunately, for some of the simulations the f_B^{static} results were not useable. These results came as a by-product of the hopping parameter expansion of the heavy quark, and for technical reasons had significant contamination by higher momentum intermediate states when the physical volume was large [2]. In addition, it is well known that the poor signal to noise ratio of the static

simulations makes it important to use an efficient smearing method. To provide f_B^{static} results on all lattices, and to reduce the errors even in the cases where the results were previously available, we have started a set of static-light simulations using a variational smearing method.

2. STATIC f_B

The computational cost of our current implementation of the FFT on parallel machines, prohibited the use of a more sophisticated smearing technique such as MOST [3], so we use the following basis of smearing functions:

$$\begin{aligned} s(\underline{r})_1 &= e^{-A*|\underline{r}|} \\ s(\underline{r})_2 &= e^{-A*|\underline{r}|} * (1 - B * |\underline{r}|) \\ s(\underline{r})_3 &= e^{-A*|\underline{r}|} * (1 - C * |\underline{r}| - D * |\underline{r}|^2) \end{aligned}$$

The parameters A , B , C and D were obtained from uncorrelated fits to the Kentucky group's measured wave functions of static-light mesons (at $\beta = 6.0$) [3]. The A , B , C and D param-

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Table 1
 Simulation parameters

β	volume	# configs	am_{sea}	# κ
5.6	$16^3 \times 32$	100	0.01	3
5.5	$24^3 \times 64$	100	0.1	3
5.445	$16^3 \times 48$	100	0.025	3

eters were scaled to the appropriate value for a given β , using the estimates of the lattice spacing. Because of uncertainties in the lattice spacing, and to have the flexibility to choose different smearing functions for each kappa value, two additional sets of the parameters were chosen. Thus a variational smearing matrix of order ten (including the local operator) is used in the simulations. The static quark is smeared *relative* to the light quark using standard FFT methods. The completed static-light production runs are shown in Table 1. The $\beta = 5.6$ configurations were generated by the HEMGCC collaboration [4]. Our analysis of the data is very preliminary—in particular we have not yet fully optimized our smearing functions. All the results presented here use a single exponential source. We will focus on comparing the new static results with the numbers from other simulations. Ultimately, all the static data will be combined with that from propagating quark simulations [2].

We do a simultaneous fit to the smeared-local and smeared-smeared correlators. To compare our raw lattice numbers with other simulations, we quote our numbers in terms of Z_L , defined by

$$C_{LS}(t) = Z_L Z_S e^{-E_{sim}t} \quad (1)$$

$$C_{SS}(t) = Z_S Z_S e^{-E_{sim}t} \quad (2)$$

Z_L is related to the decay constant, no perturbative factors are included in its definition and we assume the light quark propagator has been multiplied by 2κ . E_{sim} is the energy of the static-light meson, it is equal to the sum of the difference between the mass of the B meson and the bottom quark mass, and an unphysical $\frac{1}{a}$ renormalization factor. Correlations were included for the fits in time, but no kappa correlations were included in the chiral extrapolations.

Table 2

Z_L fit results for $\beta = 5.6$, $am_{sea} = 0.01$, with a source $\exp(-0.4 * |r|)$, and fit region 7 to 11

κ	aE_{sim}	$a^{3/2}Z_L$	χ^2/dof
0.156	0.708_{-3}^{+6}	0.312_{-4}^{+7}	4.7/7
0.158	0.684_{-4}^{+7}	0.287_{-5}^{+8}	4.9/7
0.159	0.637_{-4}^{+7}	0.274_{-5}^{+7}	5.6/7
0.16103	0.649_{-5}^{+8}	0.249_{-6}^{+7}	0.01/1

Some preliminary, static fit results for the $\beta = 5.6$ simulation are contained in Table 2. Ali Khan *et al.* [5] have also calculated Z_L on gauge configurations from the same $\beta = 5.6$ simulation, as those used here, but not on exactly the same sample of configurations. At $\kappa = 0.1585$, they quote $aE_{sim} = 0.528(5)$ and $a^{3/2}Z_L = 0.24(3)$. Although we do not have this kappa value in our simulation we can use the information obtained in the chiral fit model to estimate $a^{3/2}Z_L = 0.280_{-5}^{+7}$ and $aE_{sim} = 0.536_{-4}^{+7}$, (where we have added $\log u_0$, with $u_0 = 0.867$, to our value of aE_{sim} because Ali Khan *et al.* rescaled all their gauge fields by the tadpole improvement factor of u_0). The results from the older MILC static calculation also agree with those from the new simulations.

We have done some fits to the $\beta = 5.445$ static data. The masses obtained were consistent with the older MILC calculation. This is a large-volume case where the older method does not produce useable static Z_L factors, so no check is available there. However, the new static correlators seem reasonably consistent with the propagating quark results.

3. STATIC B_B PARAMETER

The B_B parameter is required in the extraction of the V_{td} CKM matrix element from the experimental data on $\bar{B}-B$ mixing. For the $\beta = 5.5$ and 5.6 simulations we calculated the static B_B parameter (so far we have only analyzed the $\beta = 5.6$ data). Ours is the first calculation of the static B_B parameter that includes dynamical fermions. The method used is described in references [6]

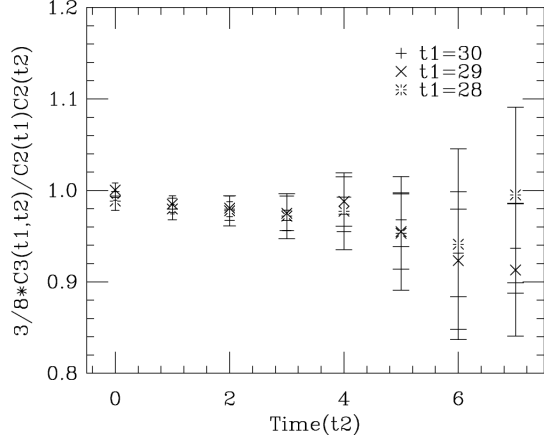


Figure 1. B_L parameter as a function of time slice, for $\kappa = 0.156$, and source $\exp(-0.4 * | \underline{r} |)$

and [7]. The same set of smearing functions used in the f_B^{static} calculations was used to smear the quarks in the external mesons. Fig. 1 shows the static B_L operator as a function of time. In Table 3 we show some preliminary results for static B_B parameter. The χ^2/dof for all the fits in Table 3 were all close to 0.5. The errors are statistical only; the systematic errors due to the choice of fit range are larger than the statistical errors. The errors should be reduced when we include the additional smearing functions in our analysis.

We used the Kentucky group's [6] organization of perturbation theory to find the required linear combination of operators to calculate $B_B(m_B)$. However we omitted some next to leading order $\log \mu/m$ terms. The missing terms have only recently been calculated [8] and are a small effect. Chirally extrapolating the $B_B(m_B)$ results to $\kappa_c = 0.16103$ and converting the results to the one loop RG invariant \hat{B}_B parameter, we obtain the preliminary value of $\hat{B}_B = 1.31_{-4}^{+3}$ (statistical error only). This result is consistent with the values obtained from quenched simulations using Wilson fermions [1] [6].

4. LATTICE HQET

To calculate the form factors of the semi-leptonic decays of the B meson, we want to follow a strategy similar to the one used in the f_B simu-

Table 3

Static B_B parameter results for $\beta = 5.6$, $am_{sea} = 0.01$, using the source $\exp(-0.4 * | \underline{r} |)$, fixing timeslice 29, and fitting times 2 to 6.

	0.156	0.158	0.159
B_L	0.98(1)	0.99(2)	1.00(2)
B_R	0.96(1)	0.94(2)	0.94(2)
B_N	1.01(2)	1.01(2)	1.01(2)
$-\frac{8}{5}B_S$	1.00(1)	1.00(1)	1.01(2)
$B_B(m_B)$	0.91(2)	0.92(2)	0.93(2)

lations, except that we will combine the results of heavy quark effective field theory (HQET) with the analogous propagating quark calculations, to allow the final results to be interpolated to the B meson mass.

As a “warm up exercise” we studied the two point function of a HQET-light meson. This exercise allows us to investigate the smearing of the quarks in the B meson and to study the nonperturbative renormalization of the velocity (a peculiarity of lattice HQET) – both of which are important prerequisites to the calculation of form factors.

We have implemented the HQET propagator equation introduced by Mandula and Ogilvie [9]. The two-point function for an HQET-light meson at finite residual momentum \underline{p} and bare velocity v is

$$C(\underline{p}, t; v) = \sum_{\underline{x}} \sum_{\underline{r}} f(\underline{r}) e^{i \underline{p} \cdot \underline{x}} \langle \bar{b}_v(\underline{r}, 0) \gamma_5 q(\underline{0}, 0) \bar{q}(\underline{x}, t) \gamma_5 b_v(\underline{x}, t) \rangle$$

When there is no excited state contamination, the correlator has the form

$$C(\underline{p}, t; v) = Z_f^v Z_L^v e^{-E(p, v^R) t} \quad (3)$$

where Z_f^v and Z_L^v are the smeared and local matrix elements. The dispersion relation for an HQET-light meson is [10]

$$E(\underline{p}, v^R) = \frac{E_{sim}}{v_0^R} + \frac{v^R \cdot \hat{\underline{p}}}{v_0^R} \quad (4)$$

where E_{sim} is equal to the energy obtained in the

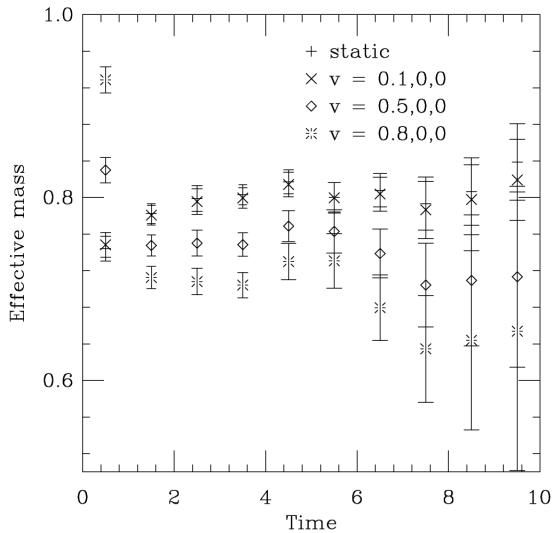


Figure 2. Effective mass plot for the HQET-light two point function, with source $\exp(-0.67 * |x|)$

static simulation. Here v^R is the renormalized velocity.

As a pilot study we generated 20 correlators at $\beta = 5.445$ with a light quark kappa value of 0.160. So far, a single exponential source, as in the static simulations, was used. Fig. 2 shows the effective mass plots for a HQET-light meson at zero residual momentum with bare velocities only in the x direction of 0.1, 0.5, and 0.8; the static effective mass plot is also included (equivalent to zero velocity). The single exponential produces almost usable plateaus for all three velocities – the plateaus should improve when we use variational smearing. The signal to noise ratio does not decrease rapidly with increasing velocity. The renormalization of the velocity is caused by the breaking of the Lorentz symmetry by the lattice. This renormalization is required in the calculation of form factors such as the Isgur-Wise function [11]. Our preferred way to extract the renormalization is to use the dispersion relation in Eq (4) with the energy taken from simulations. This approach seems to us [10] [12] to be simpler than the one used by Mandula and Ogilvie [13].

An estimate of the velocity renormalization can be obtained from the data in Fig. 2, using naive fits to the effective masses and using Eq (4) with

zero residual momentum. For the bare velocity of 0.5 (0.8) in the x direction, the renormalized velocity was approximately eighty (sixty five) percent of the bare velocity. With higher statistics and better smearing a more sophisticated analysis will be done. The perturbative calculation of Mandula and Ogilvie [13] also gave a renormalized velocity smaller than the bare velocity.

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